

Little's law basics

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Different times used in Little's law

- Lead time (LT)
- Flow time (FT)
- Cycle time (CT)

It is **essential** to define precisely above mentioned times in order to better understand principles of Little's law

$$WIP = TH \times CT$$

Other two variables of Little's law

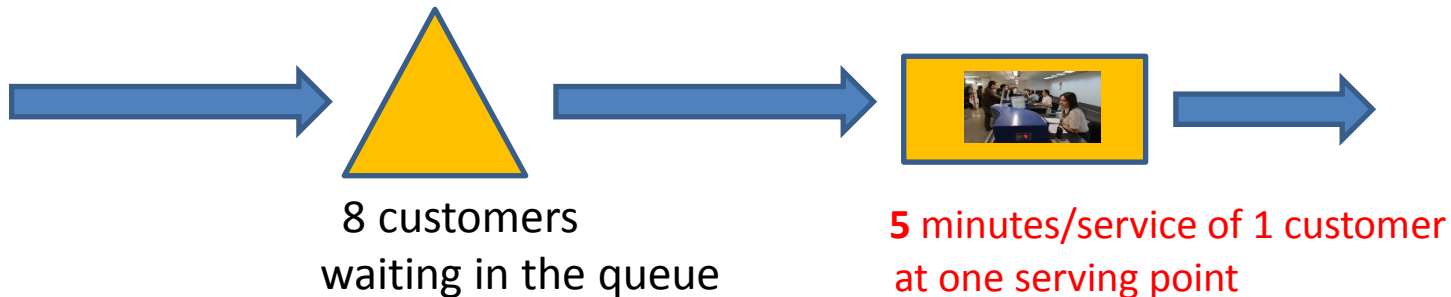
- **WIP**= Work in Process (Work in Progress)
- **TH**=Throughput=Throughput Rate = average output of production process (machine, workstations) per unit time

Definitions

- **CT**=average time from when the job is released into the station (machine or line) to when it exits
- **LT**=management constant indicating the time allotted for production of a part on a given routing
- **CT =FT** (in different publications they use FT instead of CT), where FT stands for=Flow Time
- **CT**=Throughput Time (in different publications they use Throughput Time instead of CT)

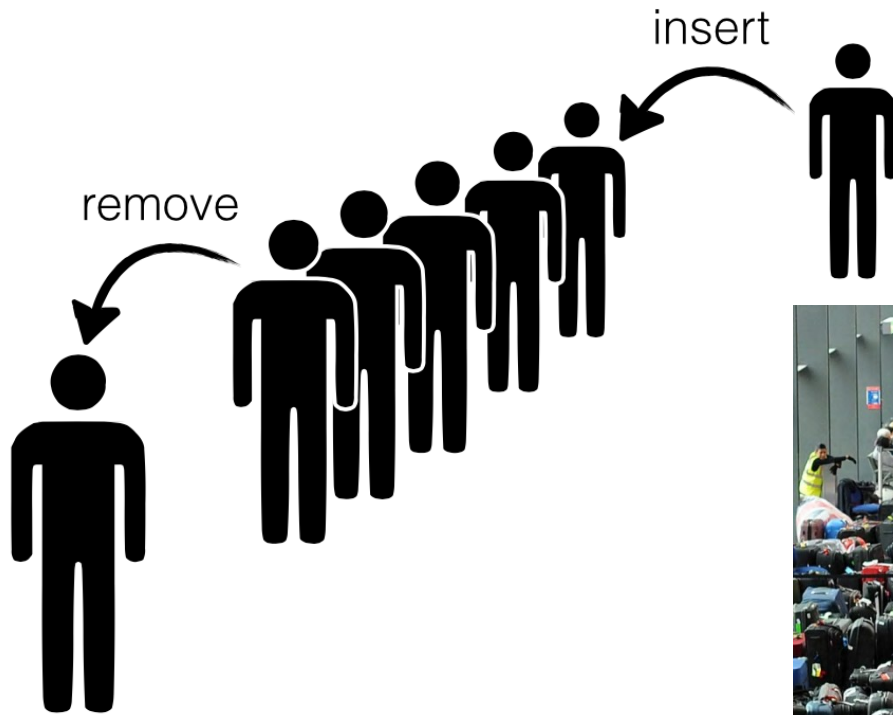
Example to be solved

- 30 customers/hour – (max capacity of the facility). The Facility may be a hairdresser, fast food, bank counter, airport checking...
- 8 customers waiting in the queue (buffer) – **see next slide**
- **5 minutes take service of one customer**



- You need to remove all times that do not add value to the process
- 1 serving point = 12 customers per hour -> (60 minutes/**5** = 12), so for 30 customers/hour the needed $2.5 = 30/12$ of serving points





Buffering

A [buffer](#) is used to temporarily keep data while it is being moved from one place to another. A buffer often adjusts timing by implementing a queue or **FIFO** algorithm in memory, simultaneously writing data into the queue at one rate and reading it at another rate.

Questions

- How long does the customer wait in the queue?
- How many people on an average can be served at once?
- How many customers are in the facility just in time (both pending and those served by facility staff)?
- What is the average time of the "flow" of the customer by the facility (wait and service)

Simplifying conditions :

"Input flow" (average) = "Output flow" (average)- **steady flow** (see video at the end of this PWP)

We do not consider fluctuation due to averaging (see flip a coin situation)

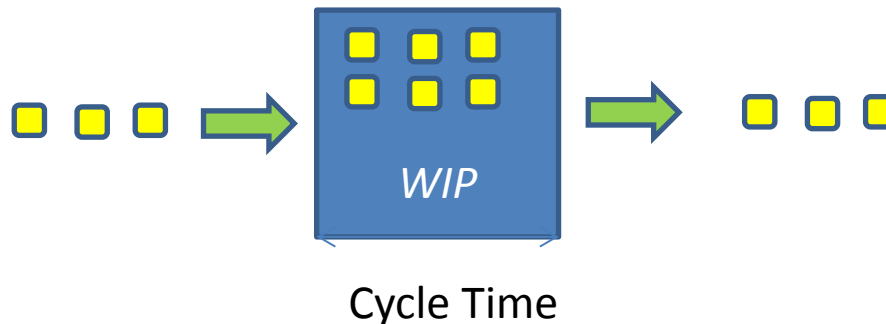
Key metrics and variables (completion of definitions)

- **CT = Cycle Time** (how long takes the whole process) = **5 minutes** **take the service of on customer**
- **Work In Process = WIP** (how many units are in process at any moment = **Work In Process**)
- **Throughput = TH** (how many customers/time unit) – e.g.. *30/hour in our case = 30/60*

• These metrics are tied together by Little's law **WIP = TH x CT**

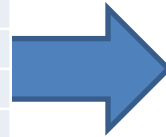
- Our example : **TH=30 customers/hour** , service of one takes **5 minutes**, **WIP** = $(30/60) * (5/1) = (1/2) * 5 = 2,5$

↑
minutes



Solution (home study, C=Customer)

Proces	WIP	TH (Cust/hour)	CT
Buffer	8	30	
Service		30	5
Total		30	



Proces	WIP	TH (Cust/hour)	CT (min/Cust)	Time
Buffer	8	30	5	
Service		30	5	5
Total		30	5	

$$\text{WIP} = \text{TH} \times \text{CT}$$



Proces	WIP	TH (C/h)	CT (min/C)	Time
Buffer	8	30	5	
Service	2,5	30	5	5
Total	10,5	30	5	

$$\text{CT} = \text{WIP} / \text{TH} \text{ (third column is only se see the units)}$$



Proces	WIP	TH (C/h)	CT (/min/C)	Time
Buffer	8	30	5	16
Service	2,5	30	5	5
Total	10,5	30	5	21

Input (from previous slides)

30 customers/hour - (max Capacity of the facility) = Throughput = TH

8 customers are waiting in the queue = WIP = buffer

5 minutes per customer service = CT

$\text{WIP} = \text{TH} \times \text{CT} = ((30/60) * 5) = (3 * 5) / 6 = 2,5$, so how many customers can be served at the same time $10.5 = 8.0$ (queue) + 2.5 and then:

$\text{CT} = \text{WIP} / \text{TH} = 8 / (3/6) = (8 * 6) / 3 = 48/3 = 16$ (as long as the customer waits in the queue) and finally for control $\text{CT} = 5 = (2.5 / (3/6)) = 2.5 * 6/3 = 15/3$ is the service time (already entered)

Questions

- How long does the customer wait in the queue? -> **16 minutes**
- How many average people can be served at once? -> **2,5 customers**

- How many customers are in the facility just in time (both pending and those just serve by facility staff)? **10,5 customers**
- What is the average time of the "flow" of the customer by the facility (wait and service) -> **21 minutes**

Little's law-2nd part

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Based on resource : Factory Physics (Hopp and Spearman)

Little's law - definition (formula)

- Fundamental relationships among :
 - WIP (Work In Process)
 - Cycle Time (CT)
 - Throughput (T or sometimes TH)
- Formula

$$WIP = TH \times CT$$

- Can be applied to :
 - Single machine station
 - Complex production line
 - Entire plant

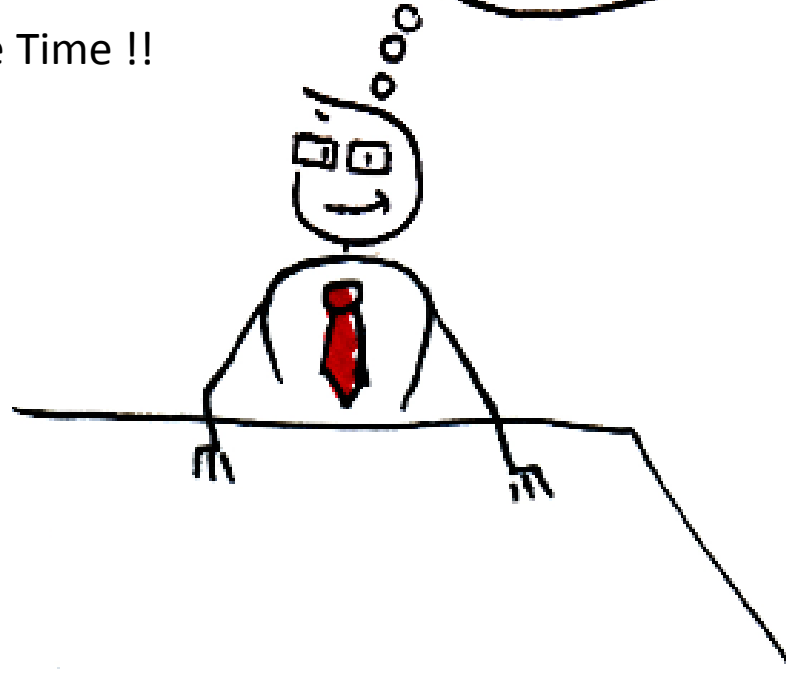
Relationships among these variables will serve to se clearly precise (quantitative) description of behaviour of the single production line . It helps user to use a given scale to benchmark actual production systems

I finally figured it out !!!!

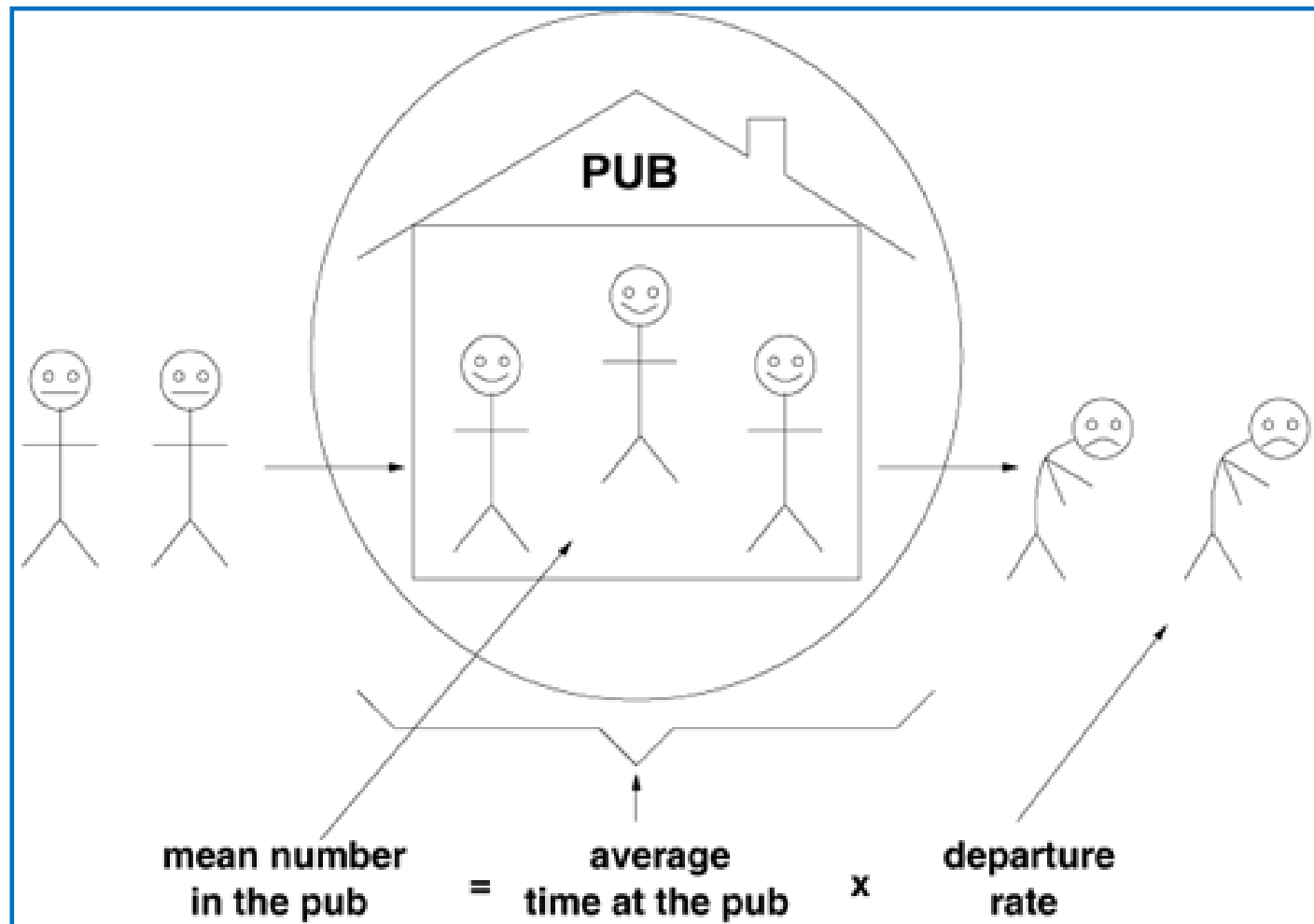
Little's Law

$$\text{avg. Lead Time} = \frac{\text{avg. Work in Progress}}{\text{avg. Throughput}}$$

Avg. Lead Time = Cycle Time !!



Daily application of the law....



Definition of basic parameters (supplements)

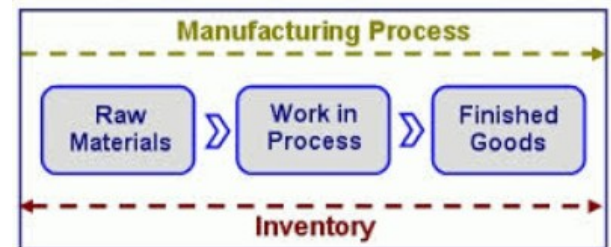
- **Throughput (Throughput rate, TH)** : production per unit time that is sold (see TOC definition)
- If **TH** is measured in cost dollars rather than in prices, it is typically called :

Cost of good sold (COGS)

- **The upper limit** of TH in the production process is the capacity
- If you release more raw material above the capacity of the line (machine), **the system becomes unstable** → WIP goes up !! See later ...

Definition of basic parameters (supplements)

- **WIP (Work In Process)** : inventory between start and end points of the product routing
- **WIP** can be used as one parameter to calculate (measure) an **efficiency**
- **Efficiency** can be defined as **Turnover Ratio** = TH / FGI for warehouses or $TH / (FGI + WIP)$ for production plants where **FGI**=Finished goods inventory
- **WIP** : inventory still in line
- **FGI** : inventory waiting for dispatch (shipping)

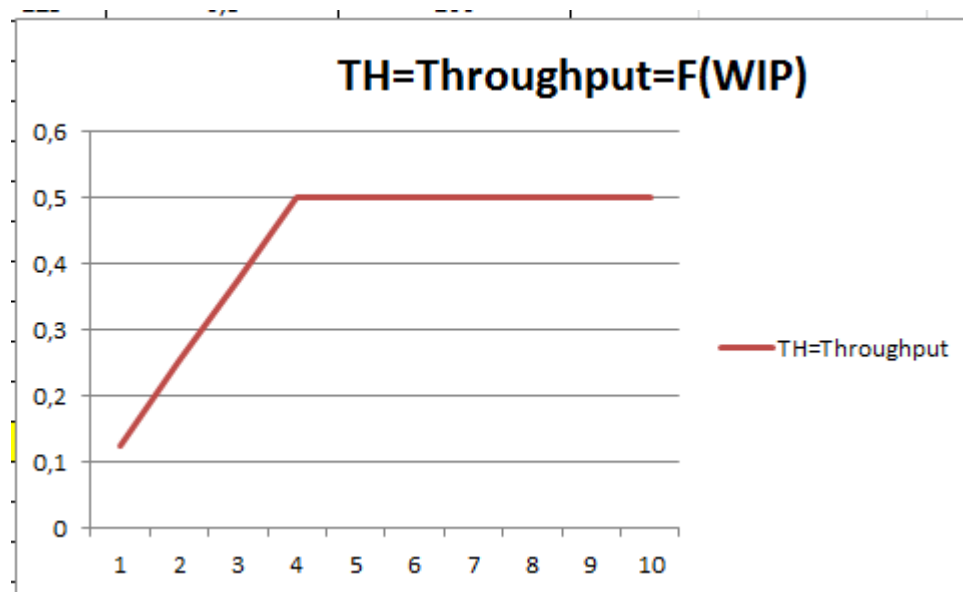


Definition of basic parameters (supplements-already mentioned)

- **CT (Cycle Time) or so called Throughput Rate** : average time from release of the job of the beginning of the routing until it reaches an inventory point at the end of the routing or time that part spends as a WIP.
- **LT (Lead Time)** : managerial constant used for planning of production
- **Service Level** (especially for M-T-O lines, where plant has to satisfy orders with specific due dates) :

Service level $P\{\text{Cycle time} \leq \text{Lead Time}\}$

Best case performance



Resources

• **WIP=TH * CT**

- *Source : Factory Physics, Wallace J Hopp and Mark L. Spearman ; ISBN 13: 978-1-57766-739-1 or ISBN 10 :1-57766-739-5*

<http://www.factoryphysics.com/principle/littleslaw.htm>

Example 1 (home study !!!)

- **Estimating Waiting Times:** If are in a grocery queue behind 10 persons and estimate that the clerk is taking around 5 minutes/per customer, we can calculate that it will take us 50 minutes (10 persons x 5 minutes/person) to start service.
- This is essentially **Little's law**. We take the number of persons in the **queue** (10) as the "**inventory**".
- The inverse of the average time per customer (1/5 customers/minute) provides us the rate of service or the Throughput.
- Finally, we obtain the waiting time as equal to number of persons in the queue divided by the processing rate $10/(1/5) = 50$ minutes).

Example 2 (home study)

- **Planned Inventory Time:** Suppose a product is scheduled so that we expect it to wait for 2 days in finished goods inventory before shipping to the customer. This two days is called **planned inventory time** and is sometimes used as protection against system variability to ensure high delivery service. Using Little's law the total amount of inventory in finished goods can be computed as :
- **FGI = throughput × planned inventory time**

Youtube examples (6 minutes)

- <http://www.youtube.com/watch?v=VU8TUSnQ-vw>
- <http://www.youtube.com/watch?v=rtGihR-bm-U>