SUPTECH WORKSHOP III

Tomáš Výrost

Network-based asset allocation strategies

Background and motivation

Prior research on stock market networks

- Dynamic **conditional correlation** networks (Lyócsa et al., 2012).
- **Granger causality** – temporal proximity and preferential attachment (Výrost et al., 2015).
- Return and volatility **spillovers** (Lyócsa et al., 2015).

Question: is network analysis purely academical?

- Networks capture the **structure** of relationships.
- Does the additional information have **practical meaning**?
- How to **incorporate** the information on structure into investment decisions?

Research outline

Objective: to explore the potential benefits the information on the interconnectedness of returns within the topological structure of a network brings to portfolio management.

Analysis has several steps:

1. construction of **return series** for each asset and calculation correlations among returns
2. construction of **time-varying correlation networks** and quantification of relative importance of assets within the network
3. construction of benchmark and network-information augmented **investment portfolios**
4. evaluation of **performance** of constructed portfolios by various measures

Literature review

- **Classical theory:** Markowitz (1952), Black and Litterman (1991)
- **Post-modern theory:** downside risk (Rom and Ferguson, 1994), higher moments (Kane, 1982; de Athayde and Flôres, 2004), improved covariance estimators (Tola et al., 2008; Pantaleo et al., 2011).
- **Network interconectedness studies:** Billio et al. (2012), Diebold and Yilmaz (2014, 2015), Baitinger and Papenbrock (2016), Kaya (2015), López de Prado (2016)
- **Portfolio weights vs. network centrality:** Peralta and Zareii (2016)

45 assets, weekly returns and yields, 12 month rolling windows, 1 week drift. Sample covers January 1999 – December 2015.

- **11 stock indices**
  BVSP, DAX, FTSE100, KOSPI, MERV, N225, SMI, SP500, SSE, TSE, TWII
- **7 commodities**
  Brent, Cocoa, Copper, Cotton, Gold, Natural gas, Silver
- **8 FX pairs**
  AUD/USD, CAD/USD, CHF/USD, CNY/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD
- **19 bond/money market instruments**
  bAAA, bBBB, bCPF-1M, bCPNF-1M, bEMEA-corp, bEMER-corp, bEMER-corp-high, bEMER-EURO-corp, bEUR-HY, bGER-1Y, bGER-5Y, bGER-20Y, bGER-corp, bJPN-1Y, bJPN-5Y, bJPN-20Y, bUS-1Y, bUS-5Y, bUS-20Y

The long-run correlation coefficient (Andrews, 1991; Panopoulou et al, 2010)

$$\hat{\rho}_{i,j} \equiv \frac{\hat{\omega}_{i,j}}{\sqrt{\hat{\omega}_{i,i}\hat{\omega}_{j,j}}}$$

is for a sample of length T obtained from

$$\hat{\mathbf{\Omega}}_{\mathrm{T}} = \left[\begin{array}{cc} \hat{\omega}_{i,i} & \hat{\omega}_{i,j} \\ \hat{\omega}_{j,i} & \hat{\omega}_{j,j} \end{array} \right] = \sum_{m=-T+1}^{T-1} k\left(\frac{m}{M}\right) \hat{\mathbf{\Gamma}}_T\left(m\right)$$

where

$$\mathbf{\Gamma}_{\mathrm{T}}\left(m\right) = \begin{cases} T^{-1} \sum_{t=m+1}^{T} \left[\mathrm{Z}_t \mathrm{Z}_{t-j}\right], j \geq 0 \\ T^{-1} \sum_{t=-m+1}^{\mathrm{T}} \left[\mathrm{Z}_{t+j} \mathrm{Z}_t\right], j < 0 \end{cases}$$

and quadratic spectral kernel weighting function

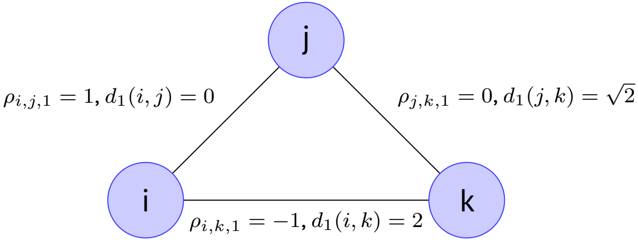
$$k\left(\frac{m}{M}\right) = \frac{25}{12\pi^2 x^2}\left(\frac{\sin\left(6\pi x/5\right)}{6\pi x/5} - \cos\left(6\pi x/5\right)\right)$$

$Z_t = [r_{i,t}, r_{j,t}]^T$ for returns $r_{i,t}, r_{j,t}$. Bandwidth parameter $M = 3$.

What are stock market networks?

- Proposed by Mantegna (1999).
- Network is a graph $G(V, E)$, describing individual stocks (V) and their relationships (E).
- Relationships are typically measured as correlation, transformed into distances:

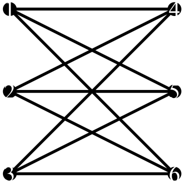
$$d_t(i,j) = \sqrt{2(1-\rho_{i,j,t})}, \qquad i,j \in V, t \in \mathbb{N}$$



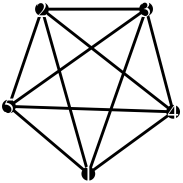
Complete graphs and complexity

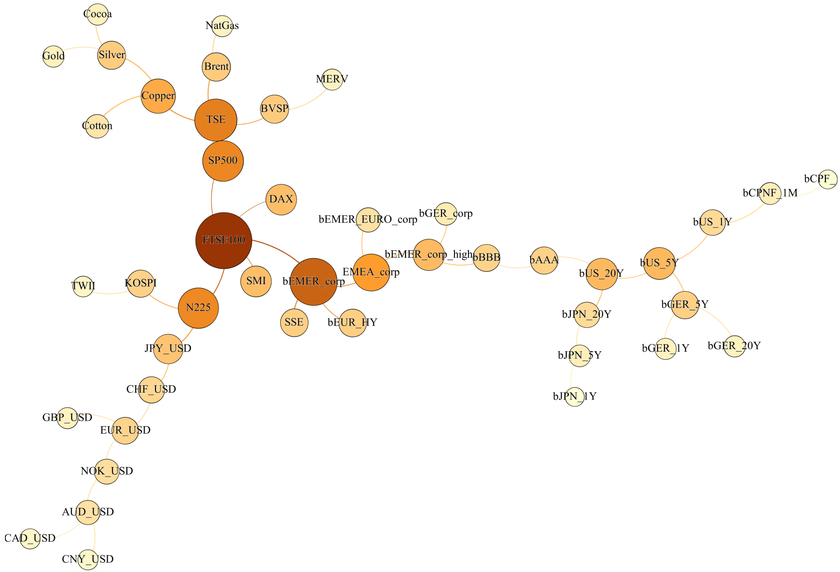
- **Minimum spanning tree (MST)** - Mantegna (1999). *Connected acyclic* undirected graph with a minimal sum of edge weights. The most common subgraph, $|V| - 1$ edges.
- **Planar maximally filtered graph (PMFG)** - Tumminello et al. (2005, 2010). *Maximal connected planar* undirected graph with a minimal sum of edge weights, $3|V| - 6$ edges.
- **Threshold graphs (THR)**. Keep all edges with weights above a selected threshold.

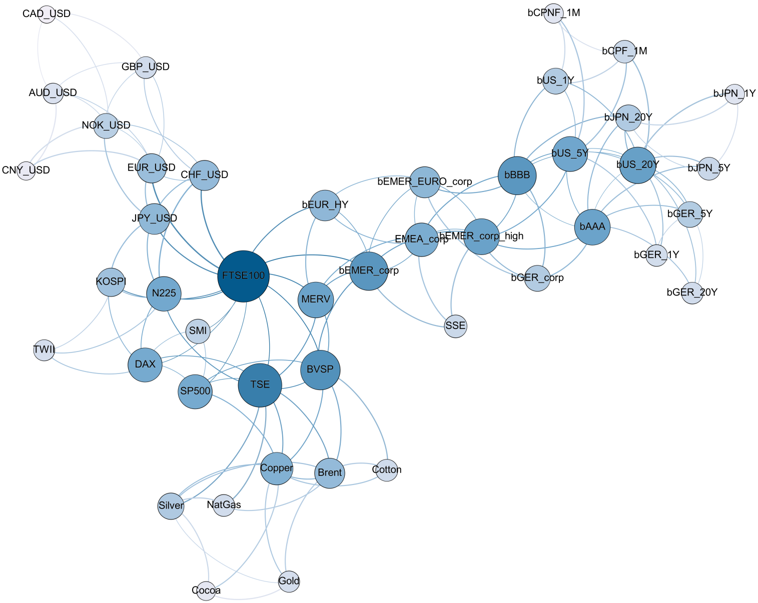
$K_{3,3}$



K_5




Cocoa
NatGas
Gold
Silver
Brent
MERV
Copper
BVSP
TSE
Cotton
SP500
bCPF_
bCPNF_1M
DAX
bEMER_EURO_corp
bGER_corp
bUS_1Y
FTSE100
bEMER_corp_high
bBBB
bAAA
bUS_5Y
bUS_20Y
EMEA_corp
TWII
KOSPI
SMI
bEMER_corp
N225
bGER_5Y
bJPN_20Y
SSE
bEUR_HY
JPY_USD
bGER_1Y
bGER_20Y
bJPN_5Y
CHF_USD
bJPN_1Y
GBP_USD
EUR_USD
NOK_USD
AUD_USD
CAD_USD
CNY_USD


CAD_USD
GBP_USD
AUD_USD
NOK_USD
CNY_USD
EUR_USD
CHF_USD
JPY_USD
KOSPI
N225
FTSE100
TWII
DAX
SMI
SP500
TSE
BVSP
MERV
bEUR_HY
bEMER_corp
bEMER_EURO_corp
EMEA_corp
bEMER_corp_high
SSE
bBBB
bUS_5Y
bUS_20Y
bAAA
bGER_corp
bCPNF_1M
bCPF_1M
bUS_1Y
bJPN_1Y
bJPN_20Y
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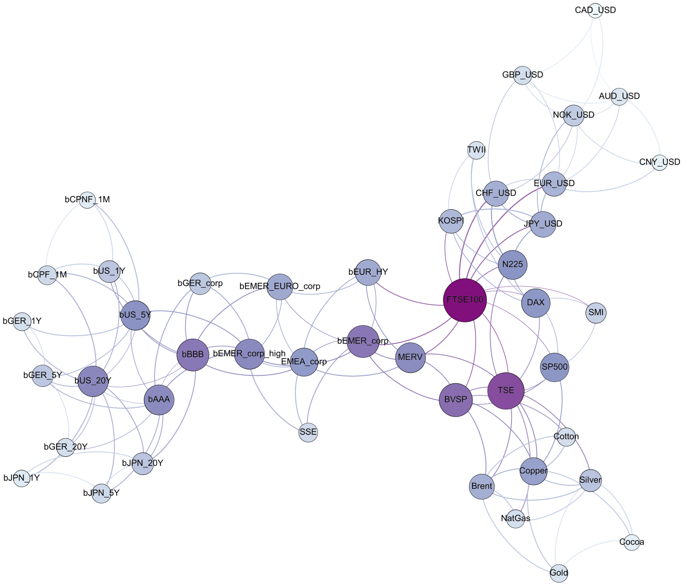
Copper
Brent
Cotton
Silver
NatGas
Cocoa
Gold


CAD_USD
GBP_USD
AUD_USD
NOK_USD
TWII
CNY_USD
CHF_USD
EUR_USD
KOSPI
JPY_USD
N225
bCPNF_1M
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SP500
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bGER_20Y
bJPN_20Y
Copper
Silver
Brent
bJPN_1Y
bJPN_5Y
NatGas
Cocoa
Gold

Centrality measures

- **Betweenness:** counts the number of times a vertex lies on the shortest path between other vertices in the network. Vertices with high betweenness mediate the interconnection between other vertices and act as spillover hubs.
- **Eigenvalue centrality:** not onlythe number of connections, their quality also is relevant. Few links to important vertices vs. higher number of connections with inconsequential vertices.
- **Expected force:** classical centrality measures are good for most important vertices, but not others. Evaluates spreading power of a node (either own interconectendess, or strong neighbors).

Portfolio strategies

Return maximization:

$$\arg\max_{\boldsymbol{\alpha}\in R^M} \boldsymbol{\alpha}^T E(\boldsymbol{r})$$
$$\boldsymbol{\alpha}^T D(\boldsymbol{r})\boldsymbol{\alpha} \leq \frac{tr(D(\boldsymbol{r}))}{M}$$
$$\boldsymbol{\alpha}^T \mathbf{1} = 1$$
$$\alpha_i \geq 0, i = 1, 2, ..., M$$

Risk minimization:

$$\arg\min_{\boldsymbol{\alpha}\in R^M} \boldsymbol{\alpha}^T D(\boldsymbol{r})\boldsymbol{\alpha}$$
$$\boldsymbol{\alpha}^T E(\boldsymbol{r}) \geq \frac{\mathbf{1}^T E(\boldsymbol{r})}{M}$$
$$\boldsymbol{\alpha}^T \mathbf{1} = 1$$
$$\alpha_i \geq 0, i = 1, 2, ..., M$$

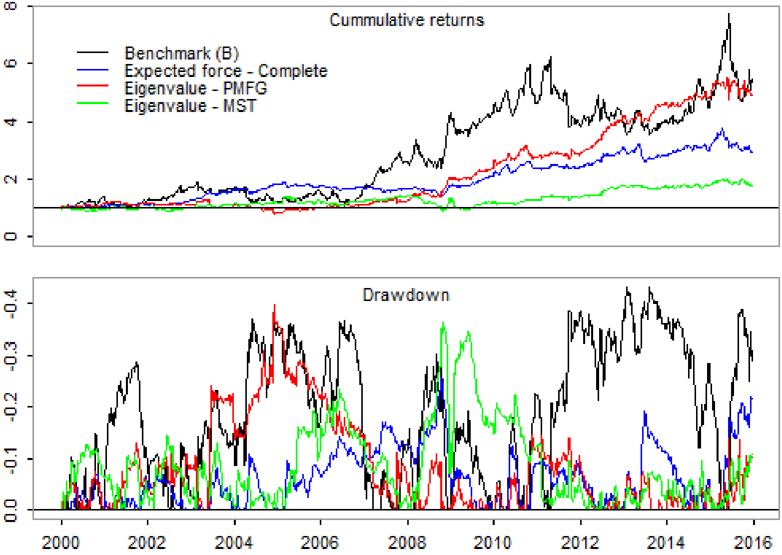
Alternative:

$$b_i \geq b_j \Rightarrow \alpha_i \leq \alpha_j, i, j = 1, 2, ..., M$$

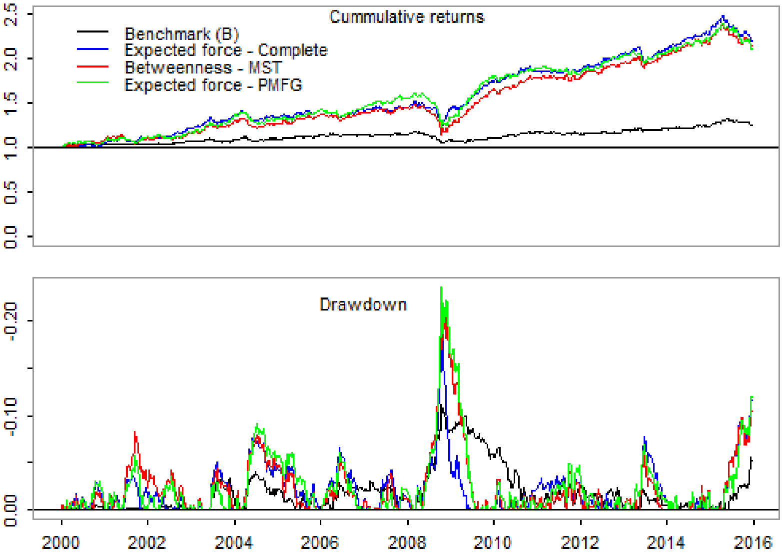
Evaluating portfolio performance

- **Descriptive statistics:**
  minimum, maximum, quartiles, standard deviation, average drawdown (DD), expected shortfall (ES), Burke ratio (BR) and Sharpe ratio (SR)
- **Model confidence set** of Hansen et al. (2011):
  block-bootstrap with 5,000 replications, test for a superior subset, run on portfolio returns and Sharpe ratios.
- **Break-even transaction cost** as in Peralta and Zareei (2016):

$$\sum_{w=1}^{N}\left[(1+r_{i,w})\left(1-BETC_i\sum_{j=1}^{M}\left|\alpha_{i,j,w+1}-\frac{\alpha_{i,j,w}(1+r_{i,j,w+1})}{\sum_{k=1}^{M}\alpha_{i,k,w}(1+r_{i,k,w+1})}\right|\right)-1\right]=0$$


Cummulative returns
Benchmark (B)
Expected force - Complete
Eigenvalue - PMFG
Eigenvalue - MST
0
2
4
6
8
Drawdown
0.0
-0.1
-0.2
-0.3
-0.4
2000
2002
2004
2006
2008
2010
2012
2014
2016

	SD	**DD**	**ES10**	**ES5**	**ES1**	**BR**	**Turnover**	**BETC**	**M**	**SR**	**SR10**	**SR5**	**SR1**
V00: Benchmark (B)	3.630	11.462	-6.405	-10.186	-11.876	0.118	0.364	0.0076	0.278	0.077	0.043	0.027	0.023
Individual strategies													
Betweenness													
V01: MST	2.106	4.748	-2.505	-8.669	-10.311	0.132	1.177	0.0013	0.154	0.073	0.061	0.018	0.015
V02: PMFG	2.289	6.748	-2.675	-6.385	-10.731	0.066	1.363	0.0007	0.103	0.045	0.039	0.016	0.010
V03: Threshold	2.222	4.448	-2.859	-5.407	-8.697	0.198	1.534	0.0013	0.217	0.098	0.076	0.040	0.025
Eigenvalue													
V04: Complete	1.966	5.119	-2.028	-8.123	-10.591	0.100	1.126	0.0010	0.128	0.065	0.063	0.016	0.012
V05: MST	2.351	6.164	-17.414	-0.128	-18.118	0.078	1.459	0.0005	0.100	0.043	0.006	**0.781**	0.006
V06: PMFG	2.251	4.375	-2.338	-7.730	-11.434	0.197	1.517	0.0013	0.224	0.100	0.096	0.029	0.020
V07: Threshold	2.137	7.841	-3.235	-9.094	-9.887	0.021	1.510	0.0001	0.047	0.022	0.015	0.005	0.005
Expected force													
V08: Complete	1.386	4.615	-2.124	-4.304	-5.220	0.179	0.151	0.0089	0.144	**0.104**	**0.068**	0.033	0.028
V09: MST	2.916	7.819	-2.116	-3.883	-20.577	0.028	1.521	0.0005	0.086	0.029	0.041	0.022	0.004
V10: PMFG	2.349	6.143	-3.343	-5.166	-7.875	0.199	1.504	0.0016	0.255	0.109	0.076	0.049	0.032
V11: Threshold	2.597	9.235	-2.646	-9.531	-17.262	0.062	1.463	0.0007	0.112	0.043	0.042	0.012	0.006
Combination strategies													
V12: Betweenness + B	2.336	6.265	-4.157	-6.641	-7.650	0.167	0.701	0.0030	0.218	**0.093**	**0.052**	**0.033**	**0.028**
V13: Eigenvalue + B	2.211	6.132	-3.879	-5.687	-7.097	0.162	0.704	0.0027	0.201	**0.091**	**0.052**	**0.035**	**0.028**
V14: Exp. force + B	2.305	6.526	-4.118	-6.637	-7.617	0.162	0.628	0.0030	0.214	**0.093**	**0.052**	**0.032**	**0.028**
V15: MST + B	2.377	6.942	-4.027	-6.925	-8.180	0.145	0.739	0.0025	0.196	**0.082**	**0.049**	**0.028**	**0.024**
V16: PMFG + B	2.316	6.154	-3.968	-5.919	-7.248	0.187	0.757	0.0029	0.236	**0.102**	**0.059**	**0.040**	**0.033**
V17: Threshold + B	2.274	7.352	-4.157	-6.265	-7.171	0.154	0.764	0.0025	0.202	**0.089**	**0.049**	**0.032**	**0.028**
V18: Complete + B	2.177	6.056	-3.867	-5.897	-6.843	0.165	0.392	0.0051	0.207	**0.095**	**0.054**	**0.035**	**0.030**


Cummulative returns
Benchmark (B)
Expected force - Complete
Betweenness - MST
Expected force - PMFG
0.0
0.5
1.0
1.5
2.0
2.5
Drawdown
0.00
-0.10
-0.20
2000
2002
2004
2006
2008
2010
2012
2014
2016

	SD	DD	ES10	ES5	ES1	BR	Turn over	BETC	M	SR	SR10	SR5	SR1
R00: Benchmark (B)	0.431	0.902	-0.708	-1.332	-1.581	0.131	0.352	0.0007	0.029	0.067	0.041	0.022	0.018
Individual strategies													
Betweenness													
R01: MST	0.842	1.884	-0.864	-2.810	-4.240	0.214	0.499	0.0018	**0.098**	**0.117**	**0.113**	**0.035**	**0.023**
R02: PMFG	0.875	2.118	-0.855	-3.022	-5.647	0.188	0.502	0.0018	**0.091**	**0.105**	**0.106**	**0.030**	**0.016**
R03: Threshold	0.826	2.029	-1.262	-1.973	-2.679	0.179	0.754	0.0009	**0.083**	**0.100**	0.066	0.042	0.031
Eigenvalue													
R04: Complete	0.762	1.741	-0.878	-1.904	-3.249	0.249	0.580	0.0015	**0.098**	**0.128**	0.112	0.051	**0.030**
R05: MST	0.769	2.021	-0.962	-2.614	-3.419	0.148	0.747	0.0008	0.064	0.083	0.067	0.024	0.019
R06: PMFG	0.784	1.896	-1.156	-2.033	-2.726	0.237	0.727	0.0012	**0.094**	**0.120**	0.081	0.046	**0.034**
R07: Threshold	0.744	1.901	-0.887	-2.465	-3.354	0.178	0.824	0.0008	**0.073**	0.098	**0.082**	**0.030**	**0.022**
Expected force													
R08: Complete	0.757	1.811	-1.012	-2.226	-2.975	0.232	0.054	0.0170	**0.101**	**0.133**	**0.100**	**0.045**	**0.034**
R09: MST	0.844	2.178	-0.800	-3.579	-5.523	0.158	0.742	0.0008	**0.072**	**0.085**	**0.090**	**0.020**	**0.013**
R10: PMFG	0.855	2.030	-0.947	-1.584	-4.132	0.202	0.759	0.0012	**0.097**	**0.113**	0.102	0.061	**0.023**
R11: Threshold	0.819	1.705	-0.902	-1.840	-3.715	0.233	0.770	0.0012	**0.102**	**0.125**	0.113	0.055	**0.027**
Combination strategies													
R12: Betweenness + B	0.563	1.404	-0.656	-2.080	-2.638	0.196	0.335	0.0016	**0.060**	0.107	**0.091**	**0.029**	**0.023**
R13: Eigenvalue + B	0.509	1.197	-0.665	-1.864	-2.244	0.198	0.347	0.0015	**0.056**	0.109	0.084	0.030	**0.025**
R14: Exp. force + B	0.541	1.178	-0.591	-2.043	-2.637	0.205	0.313	0.0018	**0.061**	0.113	**0.103**	**0.030**	**0.023**
R15: MST + B	0.541	1.417	-0.560	-2.262	-2.827	0.180	0.357	0.0014	**0.053**	0.099	**0.095**	**0.023**	**0.019**
R16: PMFG + B	0.551	1.286	-0.604	-2.067	-2.679	0.219	0.354	0.0016	**0.062**	0.112	**0.103**	**0.030**	**0.023**
R17: Threshold + B	0.527	1.253	-0.734	-1.757	-2.163	0.201	0.384	0.0013	**0.057**	0.109	0.078	0.032	0.026
R18: Complete + B	0.526	1.195	-0.686	-1.767	-2.227	0.229	0.248	0.0024	**0.064**	0.122	0.093	0.036	**0.029**

Conclusion

- We propose **simple network-based asset allocation extensions** of standard Markowitz portfolio strategies.
- We consider four **types** of network topologies and three centrality measures.
- Information on the topological structure of a network **improves risk-return characteristics** of standard benchmark portfolios.

Conclusion (cont.)

- With **no transaction costs**, simple extensions generally improve risk-return characteristics.
- In **return maximization**, improvements are costly (increased turnover and transaction costs).
- In **risk minimization**, improvements are retained (cheaper than pure Markowitz strategy).
- Improvements **irrespective** of the employed network model or centrality measure used.
- Best improvements in **left-tail risk-adjusted returns**.
- **Combination** portfolios (50% benchmark + 50% network) are recomended.

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SUPTECH WORKSHOP III

Tomáš Výrost

Network-based asset allocation strategies