# Linear programming-introduction

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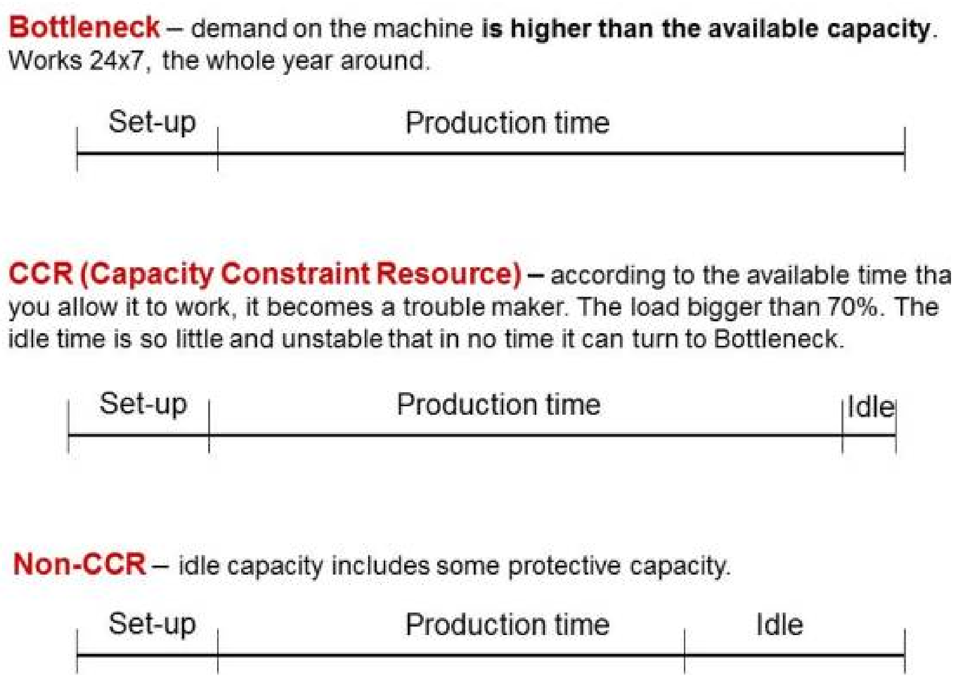
# USE

- **Slitting and Levelling of material (coils, bars, sheets)-**Cutting material, trimming,…
- **Blending -** blending, diet, feeding rations for animals, ..
- **Transport problems -** material flow from stock to the destination and route planning - shortest route
- **Assignment of resources with limited capacities -** CCR
- **Sources :** Operation Management, Quality and Competitiveness in a global environment, Russel and Taylor (can be found easily in ESF library)

CCR=Capacity Constraint Resource

# CCR –additional information

- There are 3 categories of resources from the point of view of capacity:
- Bottleneck
- CCR – Capacity Constraint Resource
- Non-CCR



# Formulation of the simple model

| | **Product** | **Description** | **Work /hour** | **Material/pcs** | **Return/pcs** |
|---|---|---|---|---|---|
|  | **Dish** | x1 | 1 | 4 | 40 |
|  | **Mug** | x2 | 2 | 3 | 50 |

Which combination of products will have the greatest return at the limits of maximum production capacity type = **40** hours moreover, the amount of material that is limited to **120** kg of clay?

**Note:** A similar task in terms of flow was solved in the P&Q example (only valid for Czech student), where the limitation in resource B and with a maximum capacity of 2400 minutes)

Description x1 and x2 stands for variables, Material means e.g. 4 kg for one piece

# Basic structures and used terminology

Target function
Z=Cx

- We minimize our target function in the form of:

**Z = c1*x1+c2*x2+…..+cn*xn** with respect to the matrix of restrictive conditions:
(in our case **c1=40** and **c2=50 which means return/pc**)

A11*x1 + A12*x2+ …+ A1n*xn (<>=) B1
A22*x1 + A22*x2+ …+ A2n*xn (<>=) B2
⋮
Am1*x1 + Am2*x2+ …+ Amn*xn (<>=)Bm

- It is a classical system of linear equations is **Ax=B**
- The solving of such a linear equation system, e.g. By use of GAUSS-JORDAN algorithm is not required if we will use **Excel Solver**.
- **xij** : decision variable= level of operation activity specified by this variable
- Bi : restrictive conditions , allowed deviations from the norm (in time and material)
- **cj** : coefficient of the target function (in our case returns, meaning return 40 and 50 )
- **Aij** : restrictive coefficients: work and material for one unit (pcs) of the product

# Example I (introduction to the problem – practical demonstration )

| Product | Description | Work /hour | Material/pcs | Return/pcs |
|---|---|---|---|---|
| Dish | x1 | 1 | 4 | 40 |
| Mug | x2 | 2 | 3 | 50 |

**Z = c1*x1+c2*x2+…..+cn*xn** (classical equation from)

**Target function**: **Z**= 40*x1+50*x2, which we must maximize

Maximal production capacity = 40 hours and Maximal quantity of material =120 kg
(B1 and B2 in our mathematical expression)

Specifications of task restrictions by use of 2x2 matrix:

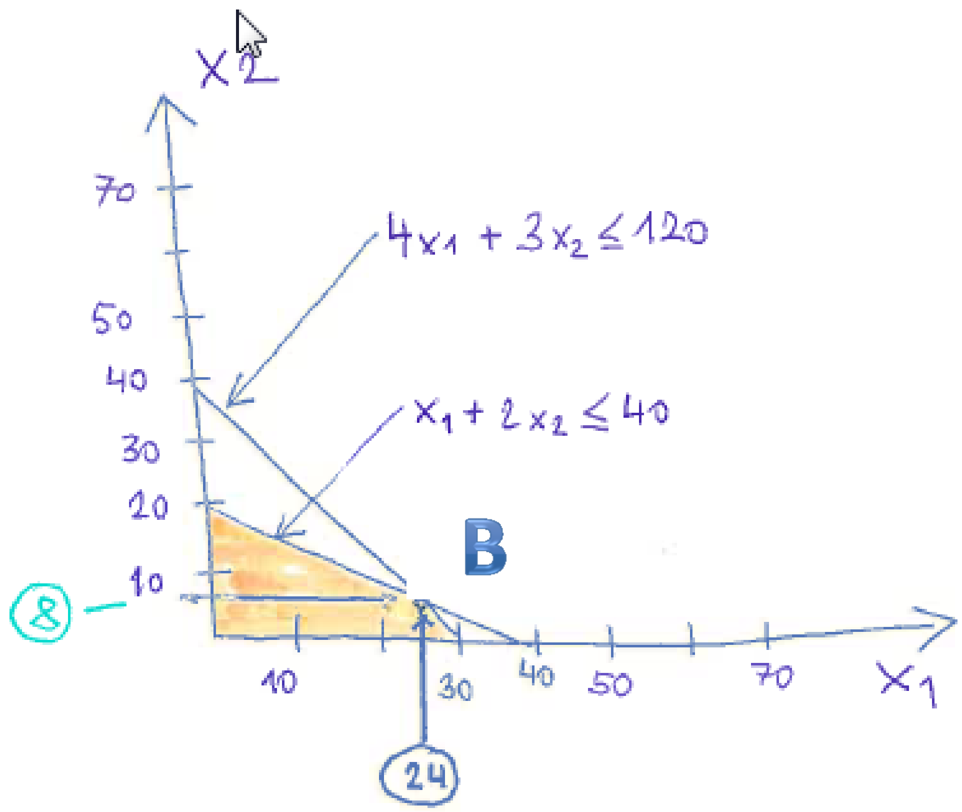
1*x1 + 2*x2 =40 (work- no more than 40 hours)
4*x1 + 3*x2 =120 (material=kg of clay in our case)->x1=(40-2x2)+3x2=120….

**Manual solving :** -> x1=24 a x2=8 and after substitution od variables (24 pcs of Dish and 8 pcs of Mug) in target function we will get

Z=40*24+50*8=**1360**

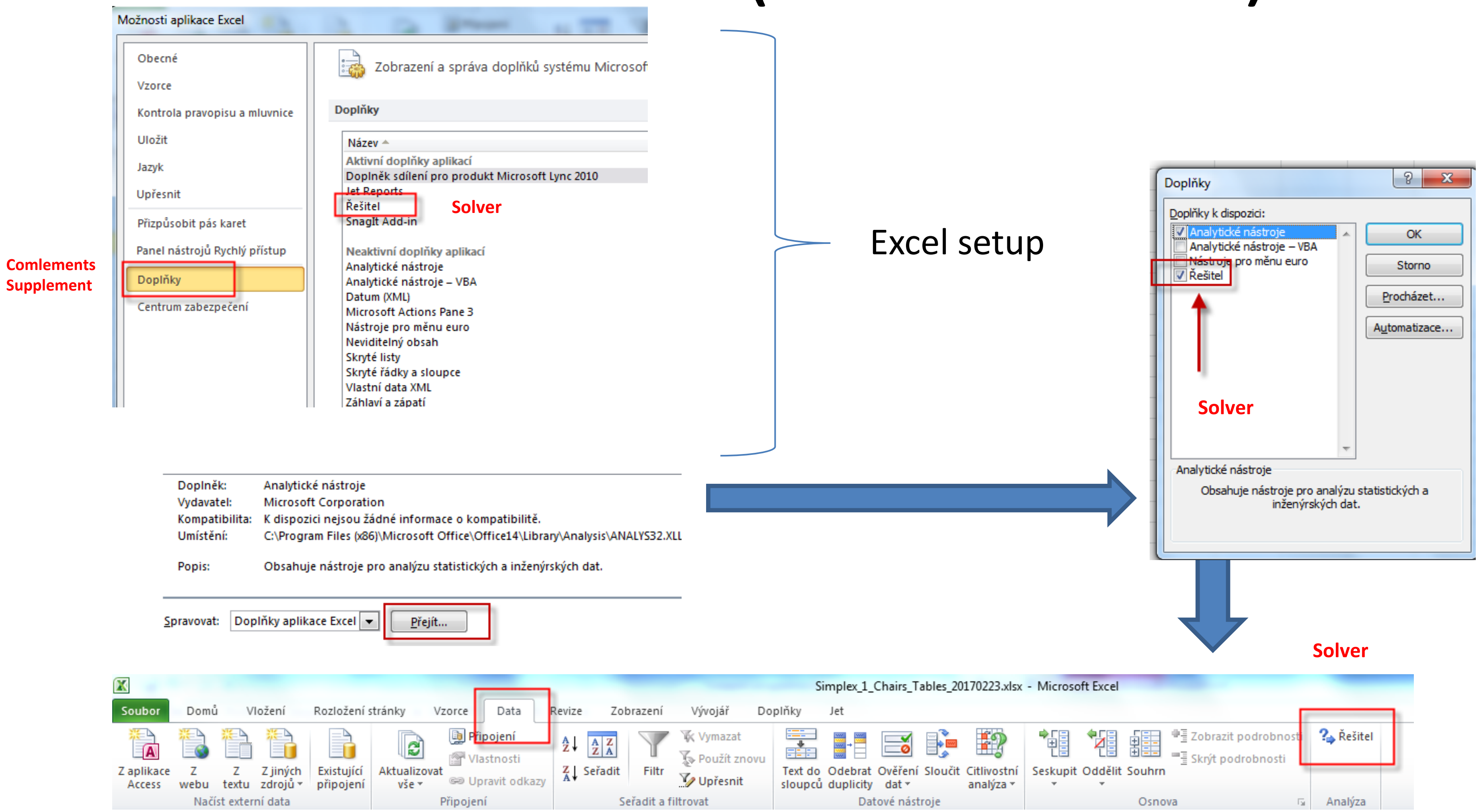
(optimal Return meets the point B – see next slide)

# Graphical solution




I apologize for the inappropriate graphic expression….

# Use of Solver (Czech EXCEL)



# Use o solver (see actual Excel formulas on one of the next slides)

x1 x2

| | Dish | Mug | Total | Capacity |
|---|---|---|---|---|
| Variables (x1, x2) | 0 | 0 | | |
| Return | 40 | 50 | 0 | |
| | | | | |
| Material | 4 | 3 | 0 | 120 |
| Work | 1 | 2 | 0 | 40 |

=D7*D6+E7*E6

Z

=D10*D6+E10*E6

=D11*D6+E11*E6

Assignment entered in table

x1=Dish , x2=Mug, max 40 hour (B1), max 120 kg (B2)

Assignment

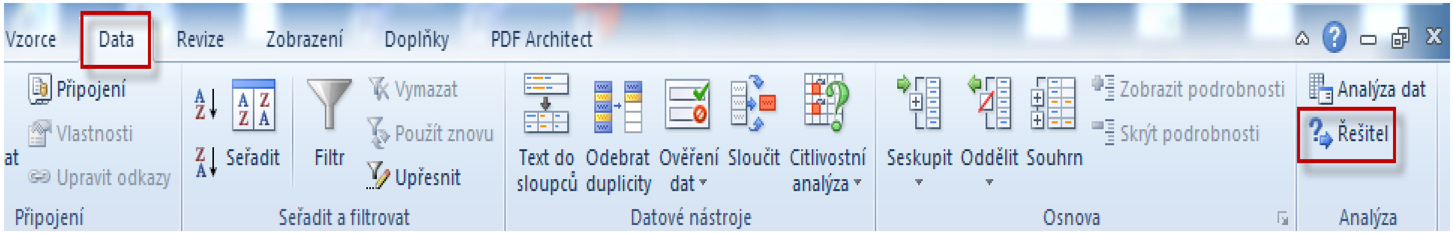
**Target function** **Z** = x1*c1 + x2*c2 = 40*x1+50*x2

4 * x1 + 3 *x2 =120 - capacity restrictions= max quantity of material =B1

1 * x1 + 2 *x2 = 40 -capacity restrictions by max work capacity=B2

| **Product** | **Description** | **Work /hour** | **Material/pcs** | **Return/pcs** |
|---|---|---|---|---|
| Dish | x1 | 1 | 4 | **40** |
| Mug | x2 | 2 | 3 | **50** |

# Solver start



# Use of Solver (Czech- not for MHP_AOPR )

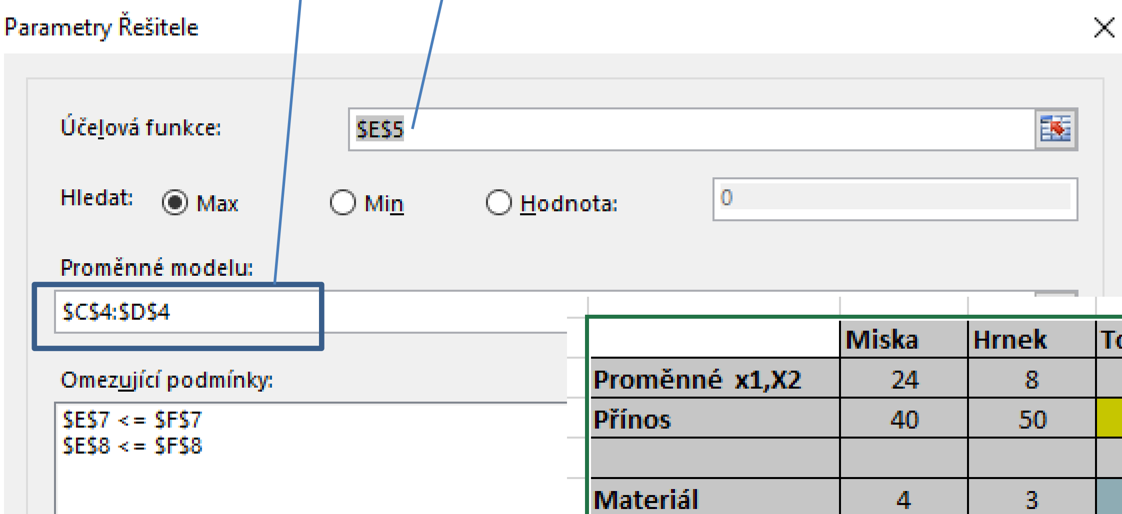
| | Miska | Hrnek | Total | Kapacita |
|---|---|---|---|---|
| Proměnné x1,X2 | 0 | 0 | | |
| Přínos | 40 | 50 | 0 | |
| | | | | |
| Materiál | 4 | 3 | 0 | 120 |
| Práce | 1 | 2 | 0 | 40 |

E5 = =D7*D6+E7*E6

$Z = x1*c1 + x2*c2 = 40*x1+30*x2$

$E7=C7*C4+D7*D4=4*x1+3*x2=120$

$E8=C8*C4+D8*D4=x1+2*x2=40$




| | Miska | Hrnek | Total | Kapacita |
|---|---|---|---|---|
| Proměnné x1,X2 | 24 | 8 | | |
| Přínos | 40 | 50 | 1360 | |
| | | | | |
| Materiál | 4 | 3 | 120 | 120 |
| Práce | 1 | 2 | 40 | 40 |

# Use of solver (for MPH_AOPR)

F7 = =D7*D6+E7*E6

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | Dish | Mug | Total | Capacity | |
| 6 | | | Varaibles X1, X2 | 0 | 0 | | | |
| 7 | | | Return | 40 | 50 | 0 | | |
| 8 | | | | | | | | |
| 9 | | | | | | | | |
| 10 | | | Material | 4 | 3 | 0 | 120 | |
| 11 | | | Work | 1 | 2 | 0 | 40 | |
| 12 | | | | | | | | |

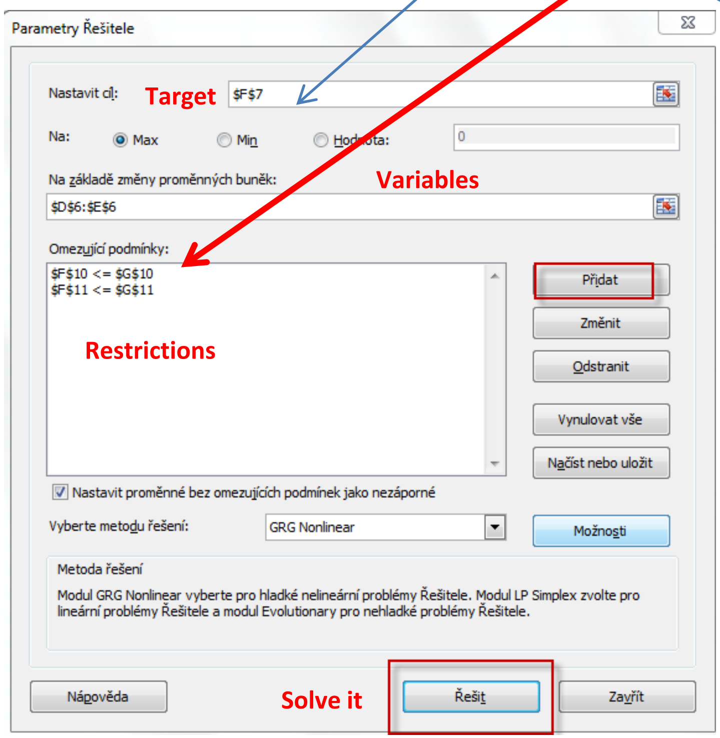
Z = x1*c1 + x2*c2 = 40*x1+30*x2

F10=D10*D6+E10*E6=4*x1+3*x2=120

F11=D11*D6+E11*D6=x1+2*x2=40

=D10*D6+E10*E6

=D11*D6+E11*E6




| | Dish | Mug | Total | Capacity |
|---|---|---|---|---|
| Varaibles X1, X2 | 24 | 8 | | |
| Return | 40 | 50 | 1360 | |
| | | | | |
| | | | | |
| Material | 4 | 3 | 120 | 120 |
| Work | 1 | 2 | 40 | 40 |

# Využití Řešitele (use of Solver)

**Microsoft Excel 15.0 Citlivostní sestava**
**List: [Simplex_1_Misky_Hrnky_Chairs_Tables_20170228.xlsx]List1**
**Sestava vytvořena: 9. 3. 2017 16:19:56**

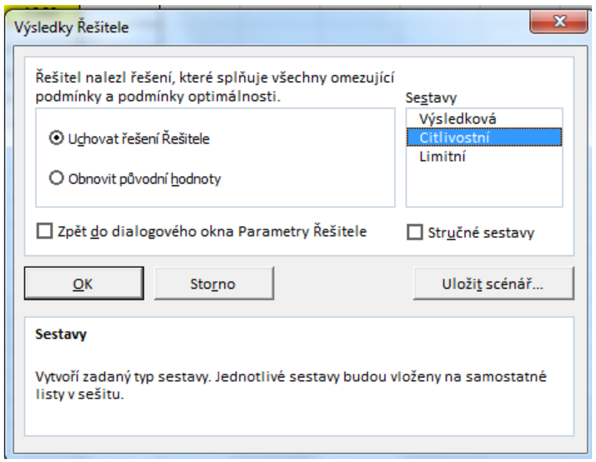
Proměnné

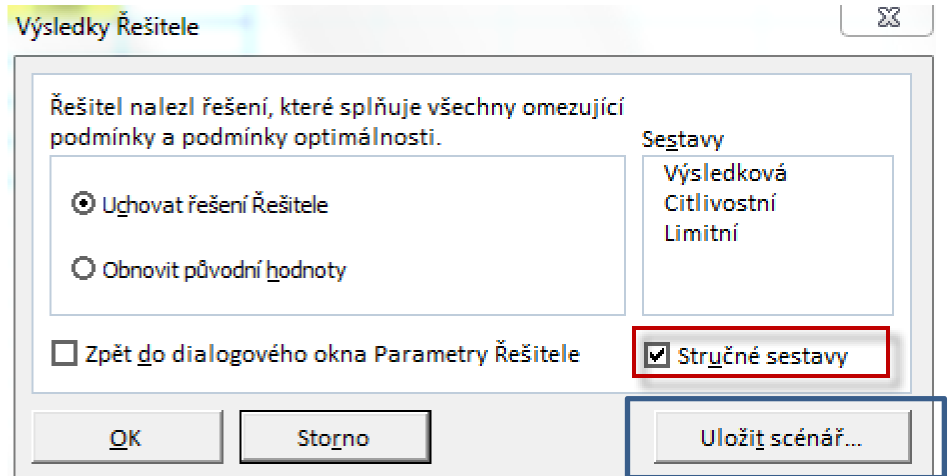
| Levá strana omezující podmínky | Název | Konečná Hodnota | Redukovaná náklady | Účelová funkce koeficient | Povolený nárůst | Povolený pokles |
|---|---|---|---|---|---|---|
| $C$4 | Proměnné x1,X2 Miska | 24 | 0 | 40 | 26,66666667 | 15 |
| $D$4 | Proměnné x1,X2 Hrnek | 8 | 0 | 50 | 30 | 20 |

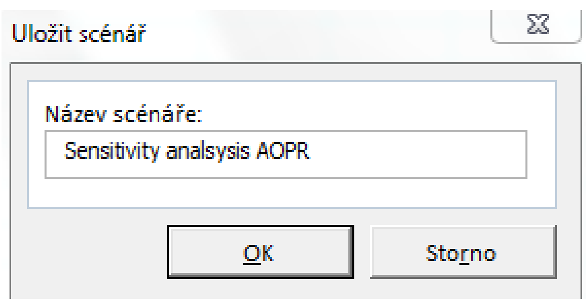
Omezující podmínky

| Levá strana omezující podmínky | Název | Konečná Hodnota | Stínová cena | Pravá strana omezující podmínky | Povolený nárůst | Povolený pokles |
|---|---|---|---|---|---|---|
| $E$7 | Materiál Total | 120 | 6 | 120 | 40 | 60 |
| $E$8 | Práce Total | 40 | 16 | 40 | 40 | 10 |

# Use of Solver (English)




**Microsoft Excel 14.0 Citlivostní sestava**
**List: [LP_EXCEL_SOLVER USE_20171101.xlsx]List1**
**Sestava vytvořena: 2.11.2017 8:49:10**

Proměnné buňky

| Buňka | Název | Konečná Hodnota | Snížené Gradient |
|---|---|---|---|
| $D$6 | Varaibles X1, X2 Dish | 24 | 0 |
| $E$6 | Varaibles X1, X2 Mug | 8 | 0 |

Omezující podmínky

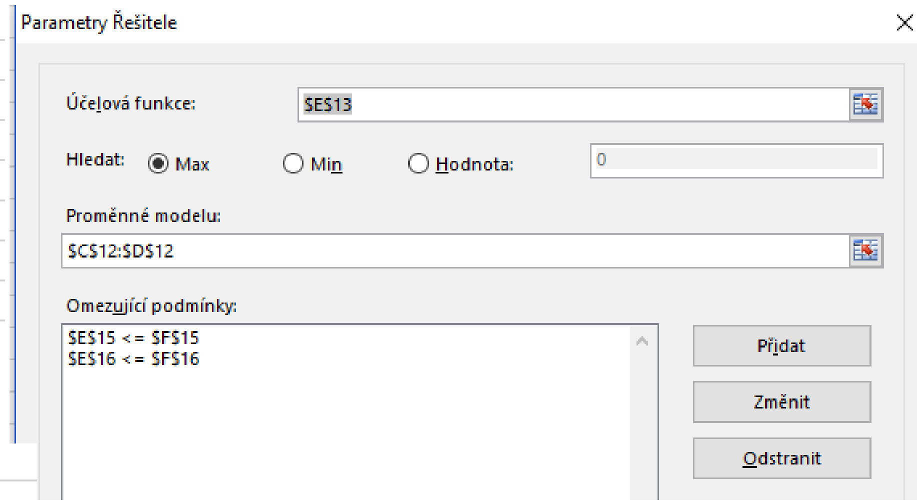
| Buňka | Název | Konečná Hodnota | Lagrangeův multiplikátor |
|---|---|---|---|
| $F$10 | Material Total | 120 | 6 |
| $F$11 | Work Total | 40 | 16 |

New Excel List

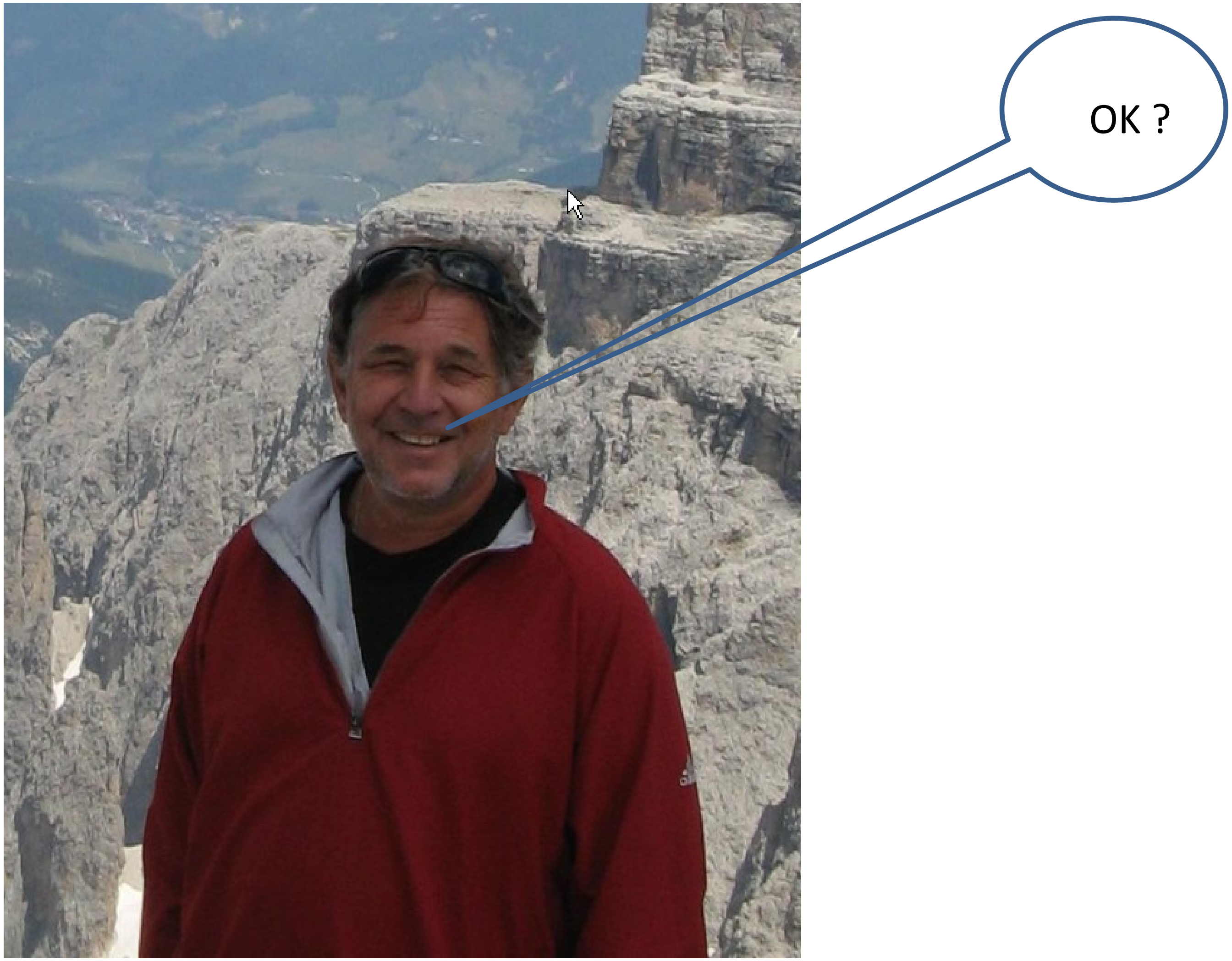
# Změna úlohy- jiné výnosy jiná omezení typu práce na dvou strojích a jejich kapacitní omezení

(**Change of parameters- not necessary for MPH_AOPR !!!!!**)

| | Miska | Hrnek | Total | Kapacita |
|---|---|---|---|---|
| Proměnné x1,x2 | 0 | 0 | | |
| Přínos | 40 | 50 | 0 | |
| | | | | |
| Stroj 1 | 7 | 5 | 0 | 200 |
| Stroj 1 | 5 | 5 | 0 | 400 |




| | Miska | Hrnek | Total | Kapacita |
|---|---|---|---|---|
| Proměnné x1,x2 | 0 | 40 | | |
| Přínos | 40 | 50 | 2000 | |
| | | | | |
| Stroj 1 | 7 | 5 | 200 | 200 |
| Stroj 1 | 5 | 5 | 200 | 400 |


OK ?