Exercise session #1 Dali Laxton

1) # of professors = 100
values of wealth 400 - 400 000$
mean = 40000$
median = 25 000$
mistake 400 000$ ⟶ 4 000 000$

answer: median will not change since only the number of the income standing last in the order changed median is just the amount of income standing in the middle of the ordered data sequence (or if the dataset is even, the average of middle two)

mean will definitely change:

$$E(\text{wealth}) = \frac{\underbrace{400\$ + X_2 + X_3 + X_4 \cdots}_{=y} + 400\,000\$}{100} = 40\,000\$$$

where $(X_1 \cdots X_{100})$ is the wealth of each professor in the set.

The first part of the equation circled with red pen will not change after the mistake, so we can denote it with y. Therefore, previously, the last professors expected wealth part was 4000$ $\left(= \frac{400\,000}{100}\right)$ and other professors:

$$y + 4000\$ = 40\,000\$$$
$$y = 36\,000\$$$

after the mistake

$$E(\text{wealth}) = y + \frac{4\,000\,000}{100} = 36\,000 + 40000 = 76\,000\$$$

this means that the mistake almost doubled the mean value

2) compare expected value and variance of six-sided vs four-sided die

six sided:

$$E(X) = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3{,}5$$

$>$

four sided

$$E(X) = \frac{1}{4}(1+2+3+4) = \frac{1}{4} \cdot 10 = 2{,}5$$

six sided:

$$V(X) = E(X^2) - (E(X))^2 = \frac{1}{6}(1+4+9+16+25+36) - 3{,}5^2 =$$

$$= \frac{91}{6} - 12.25 = 2{,}91$$

$>$

four sided:

$$V(X) = \frac{1}{4}(1+4+9+16) - 2{,}5^2 = \frac{30}{4} - 6{,}25 = 1{,}25$$

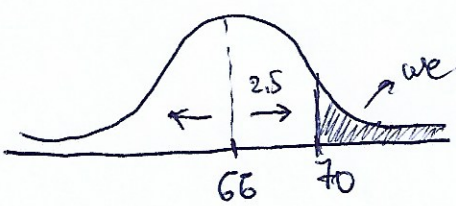
both variance and expected value of six-sided die is higher

3) normally distributed female heights

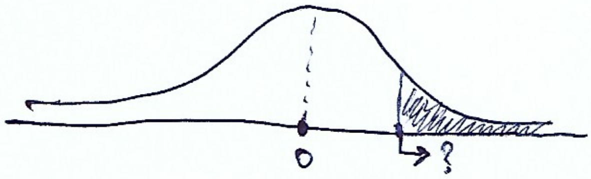
$$N \sim (66, 2{,}5^2)$$

what fraction $\geq$ 70 inches?

first let's standardize the heights of females and then look it up in the normal distribution table



we are looking for this area



we need to find the value of the point marked with question mark

$$P(X \geq 70) = P\left(Z \geq \frac{70-66}{2.5}\right) = P(Z \geq 1.6) =$$
$$= 1 - P(Z < 1,6)$$

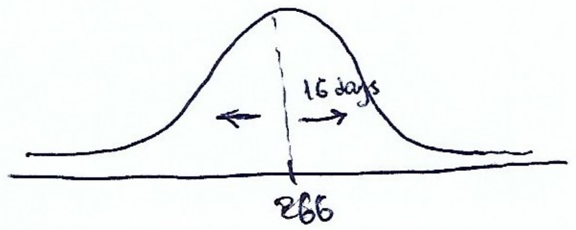
in the normal distribution table we need to look up value 1,6. in the table the area is given in the left side of the distribution function, that is why we subtract the given area from 1

$P(Z < 1,6) = 0,94520$

$P(X \geq 70) = 1 - 0,94520 = 0,0548 = 5,48\%$ of females are taller than 70inch

during lecture we did not do precise calculation this is a better approach written out here

4) distribution of pregnancies:



a) completed pregnancy lasts at least 270 days

$$p(X \geq 270) = p\left(z \geq \frac{270-266}{16}\right) = 1 - \underbrace{p(z < 0{,}25)}_{\text{look in the table}} =$$

$$= 1 - 0{,}59871 = 0{,}40129$$

answer: ≈ 40% of pregnancies last at least 270 days or probability of pregnancies lasting more than 270 days is ≈ 0,4

b) completed pregnancy lasts at least 310 days

$$p(X \geq 310) = p\left(z \geq \frac{310-266}{16}\right) = 1 - \underbrace{p(z \leq 2{,}75)}_{\text{look up}} =$$

$$= 1 - 0{,}99702 = 0{,}00298$$

answer: only 0,2% of completed pregnancies last for 310 days, meaning that Abby is a big exception (outlier)