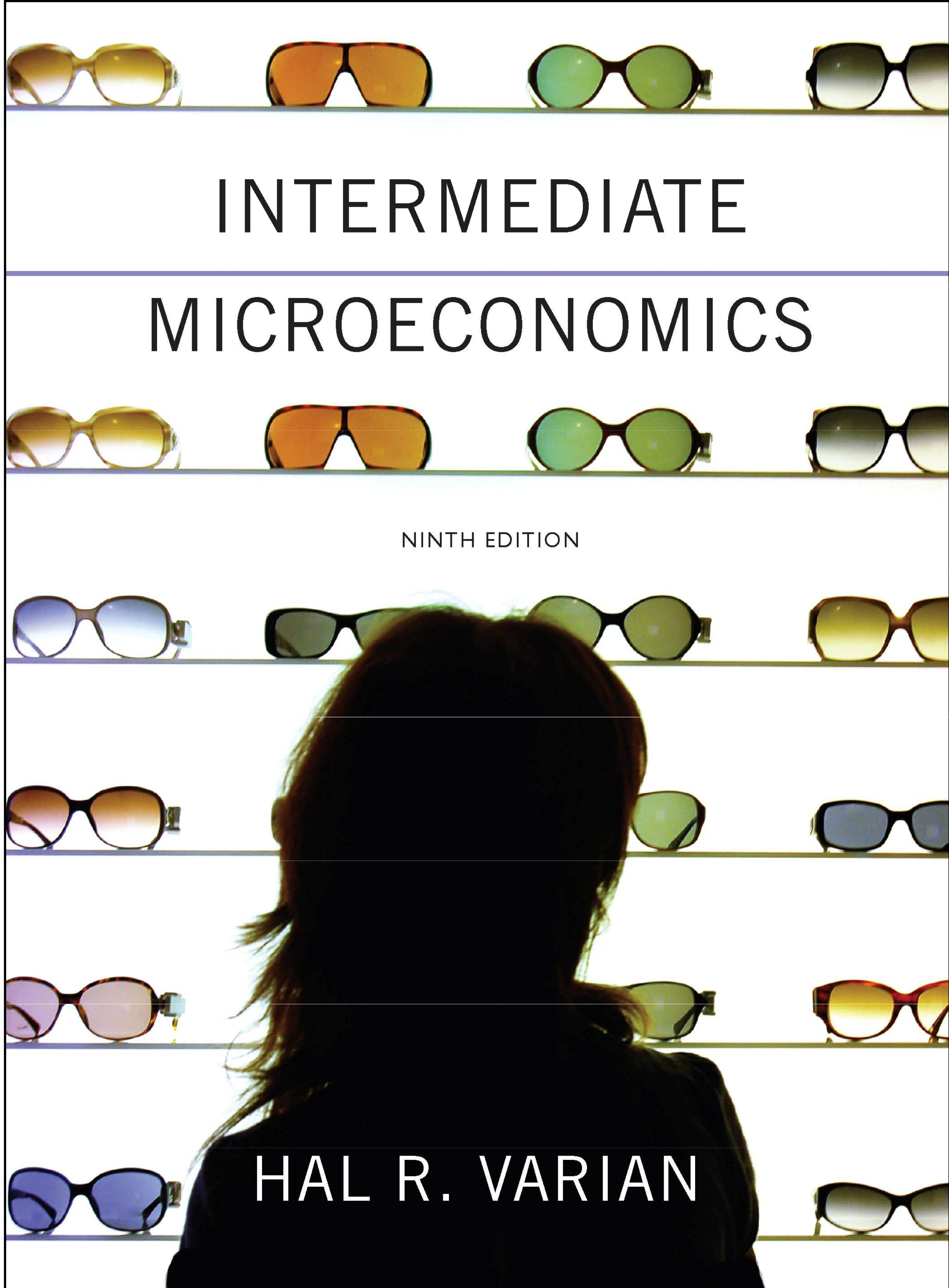


Chapter 15

Market Demand

From Individual to Market Demand Functions

- **Think of an economy containing n consumers, denoted by i = 1, … ,n.**
- **Consumer i's ordinary demand function for commodity j is**

$$x_j^{*i}(p_1, p_2, m^i)$$

From Individual to Market Demand Functions

- **When all consumers are price-takers, the market demand function for commodity j is**

$$X_j(p_1, p_2, m^1, \cdots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i).$$

- **If all consumers are identical then**

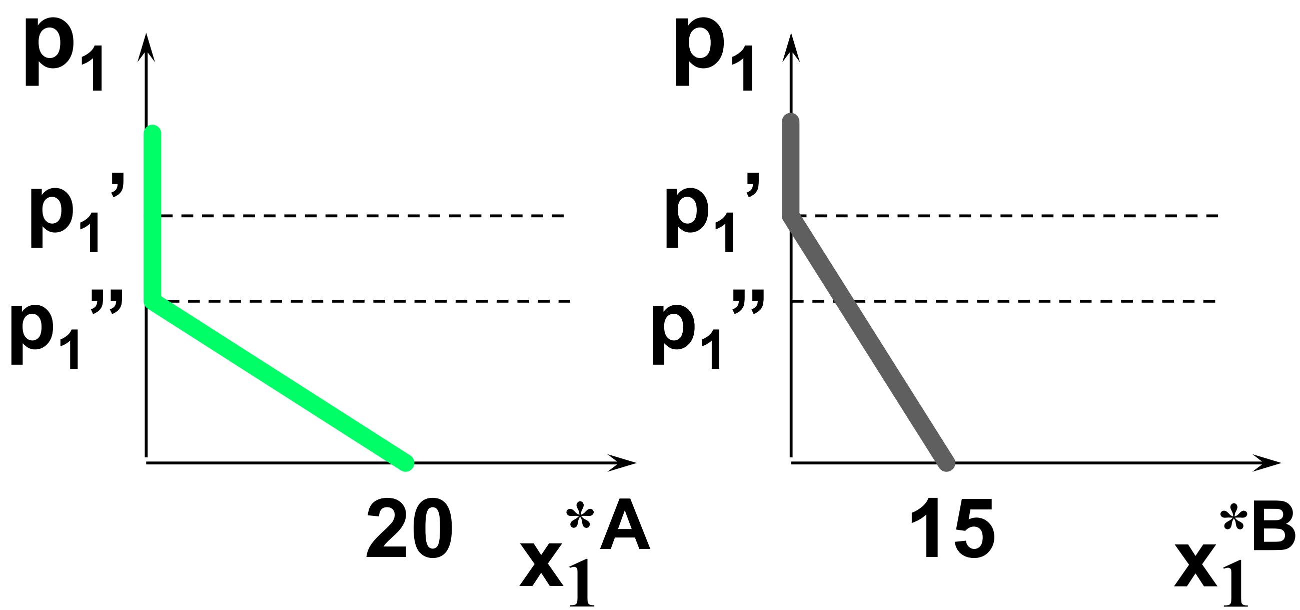
$$X_j(p_1, p_2, M) = n \times x_j^*(p_1, p_2, m)$$

where M = nm.

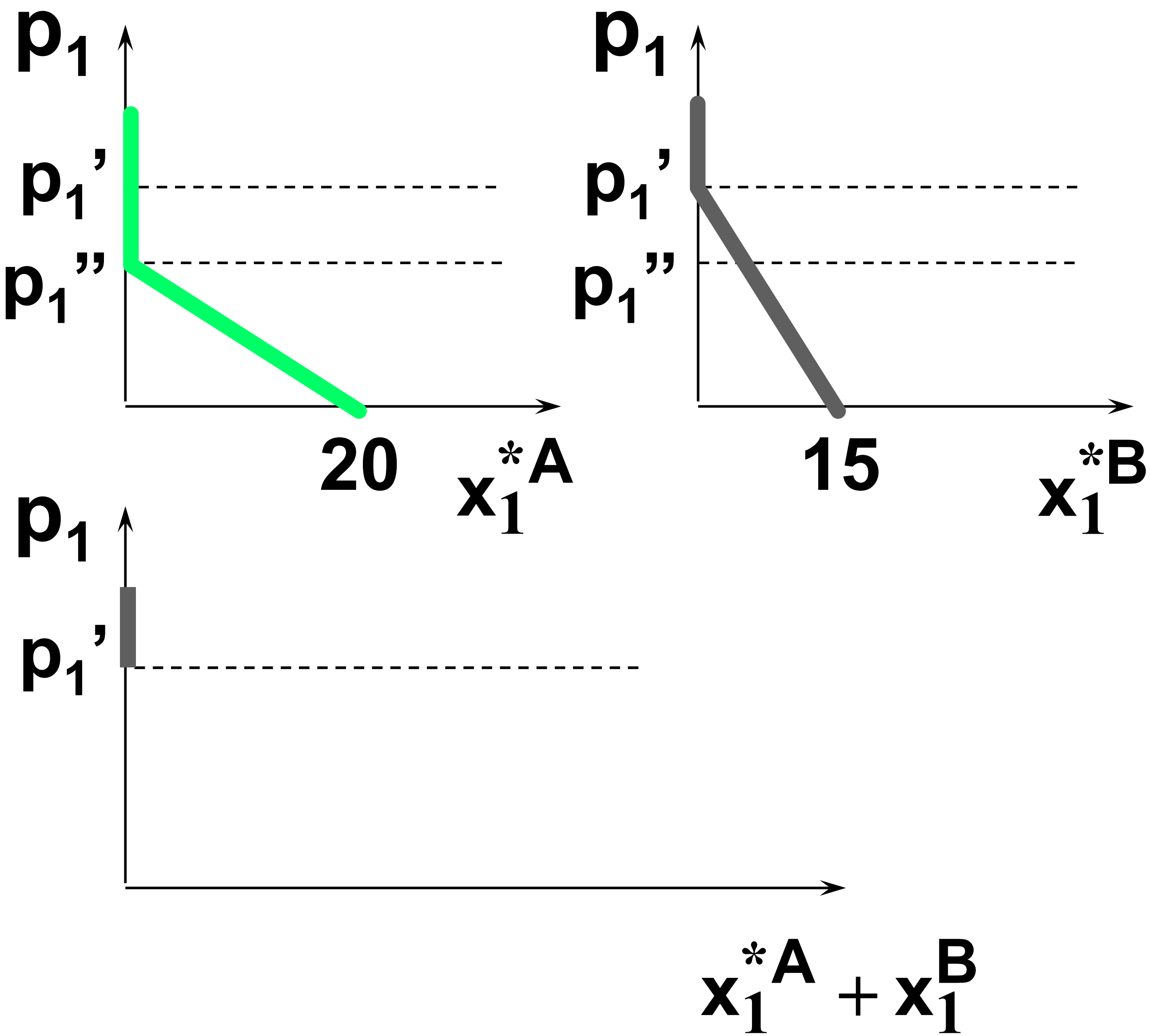
From Individual to Market Demand Functions

- **The market demand curve is the "horizontal sum" of the individual consumers' demand curves.**
- **E.g. suppose there are only two consumers; i = A,B.**

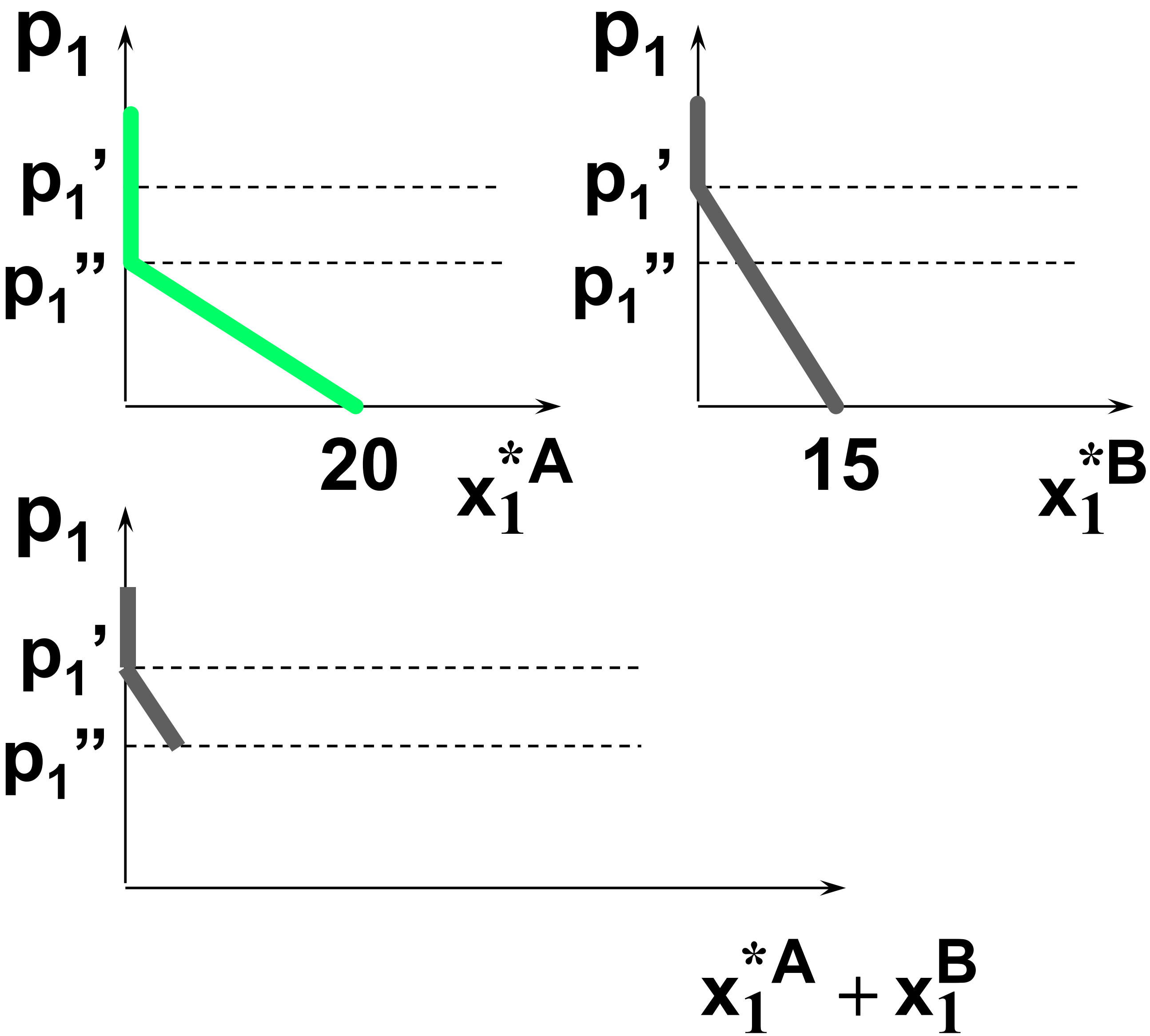
From Individual to Market Demand Functions



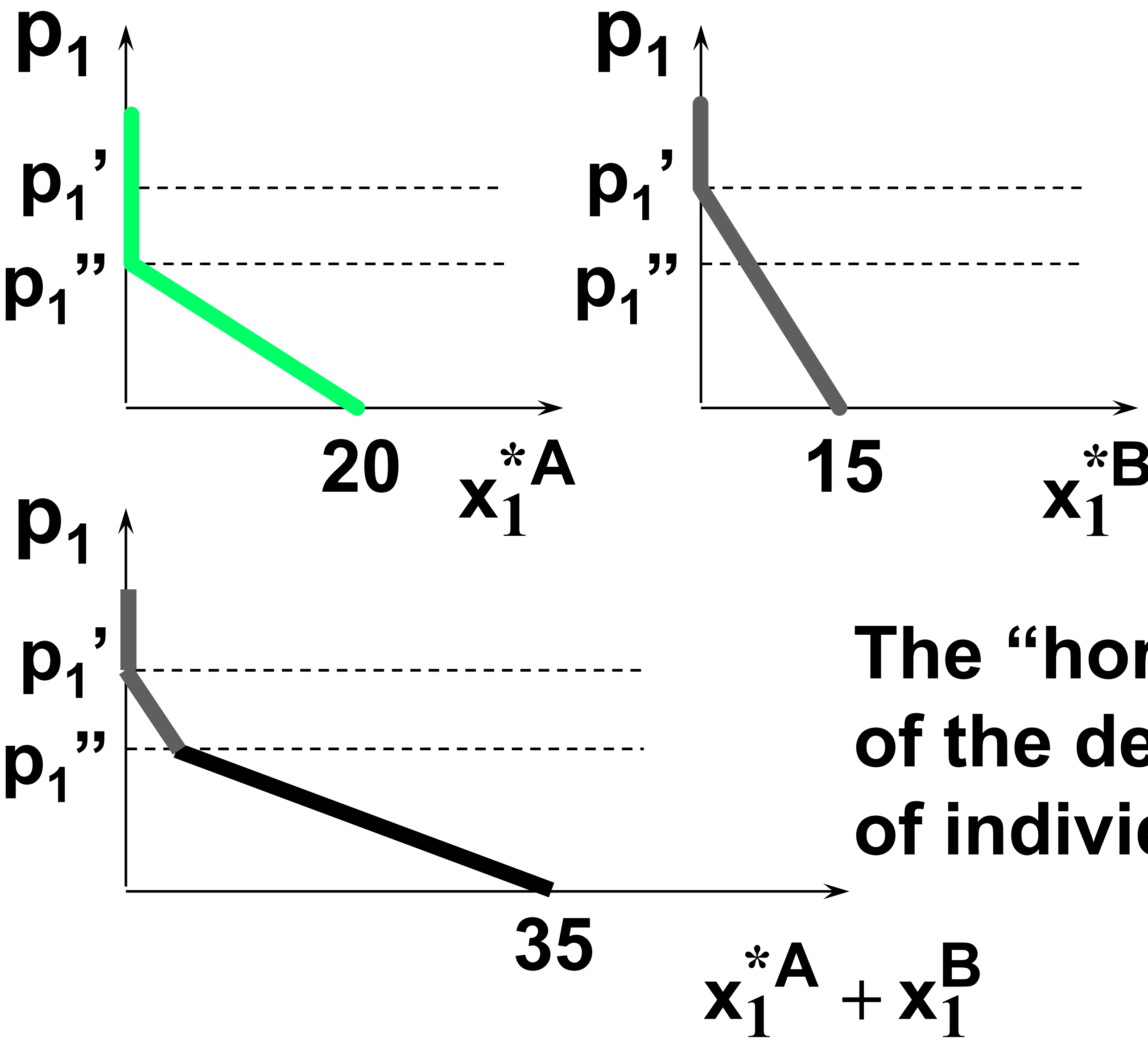
From Individual to Market Demand Functions



From Individual to Market Demand Functions



From Individual to Market Demand Functions



The "horizontal sum" of the demand curves of individuals A and B.

Elasticities

- **Elasticity measures the "sensitivity" of one variable with respect to another.**
- **The elasticity of variable X with respect to variable Y is**

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

Economic Applications of Elasticity

- **Economists use elasticities to measure the sensitivity of**
  - **quantity demanded of commodity i with respect to the price of commodity i (own-price elasticity of demand)**
  - **demand for commodity i with respect to the price of commodity j (cross-price elasticity of demand).**

Economic Applications of Elasticity

- demand for commodity i with respect to income (income elasticity of demand)
- quantity supplied of commodity i with respect to the price of commodity i (own-price elasticity of supply)

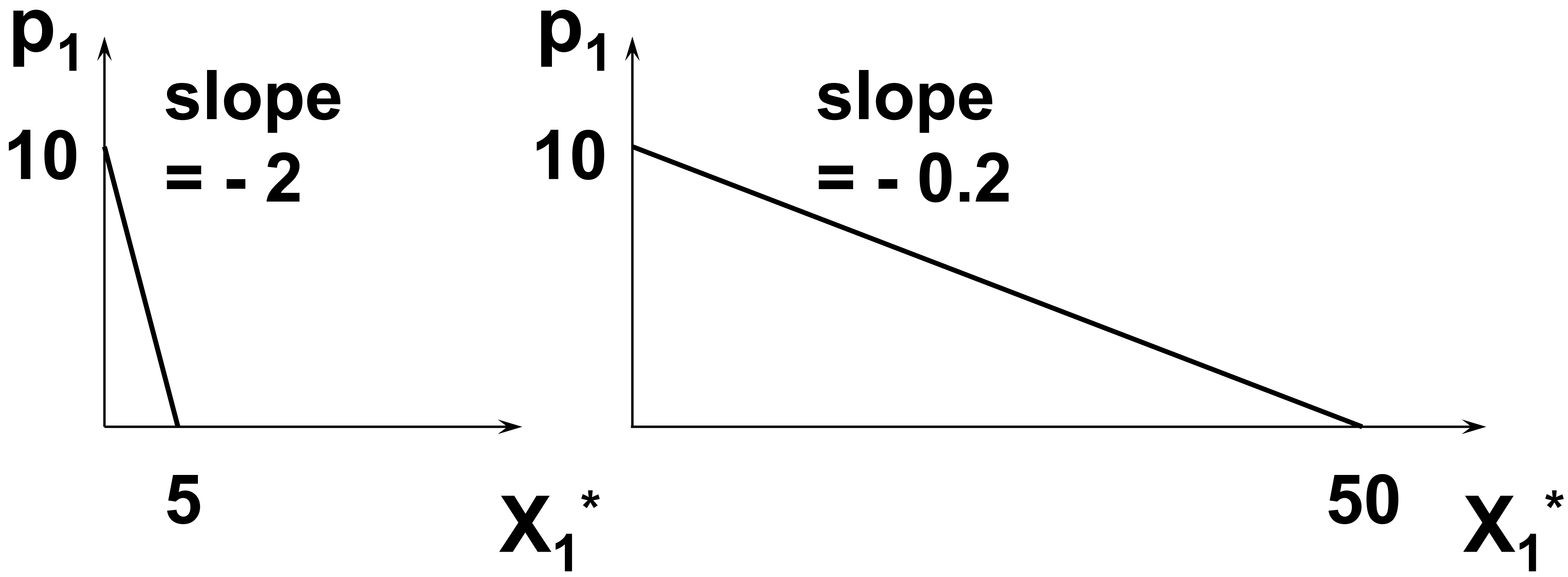
Economic Applications of Elasticity

- quantity supplied of commodity i with respect to the wage rate (elasticity of supply with respect to the price of labor)
- and many, many others.

Own-Price Elasticity of Demand

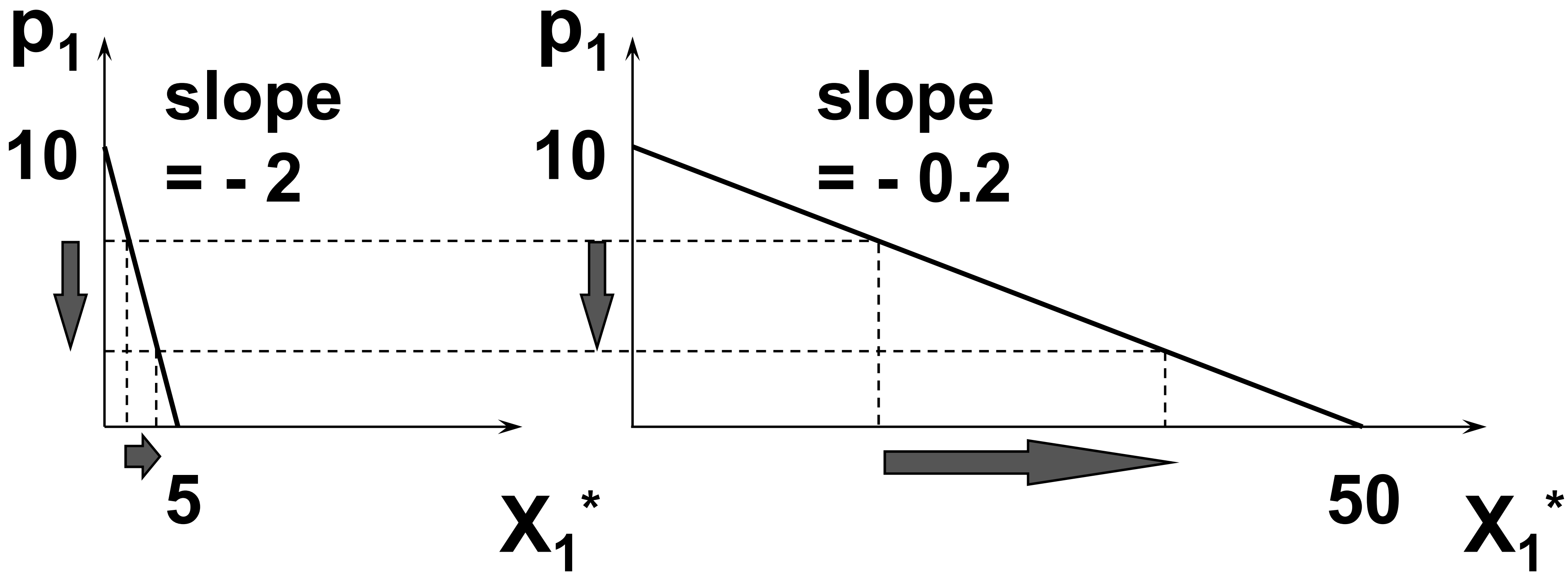
- **Q: Why not use a demand curve's slope to measure the sensitivity of quantity demanded to a change in a commodity's own price?**

Own-Price Elasticity of Demand



In which case is the quantity demanded X_1^* more sensitive to changes to p_1?

Own-Price Elasticity of Demand



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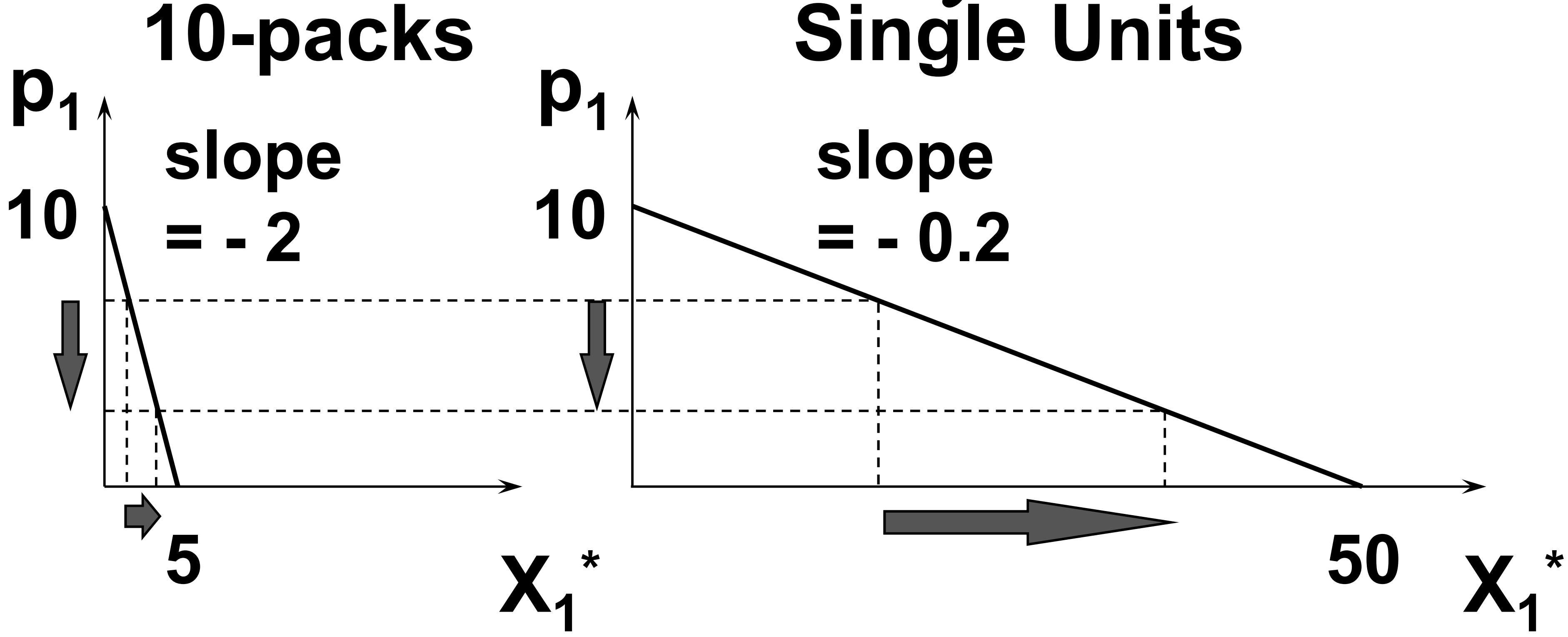
Own-Price Elasticity of Demand



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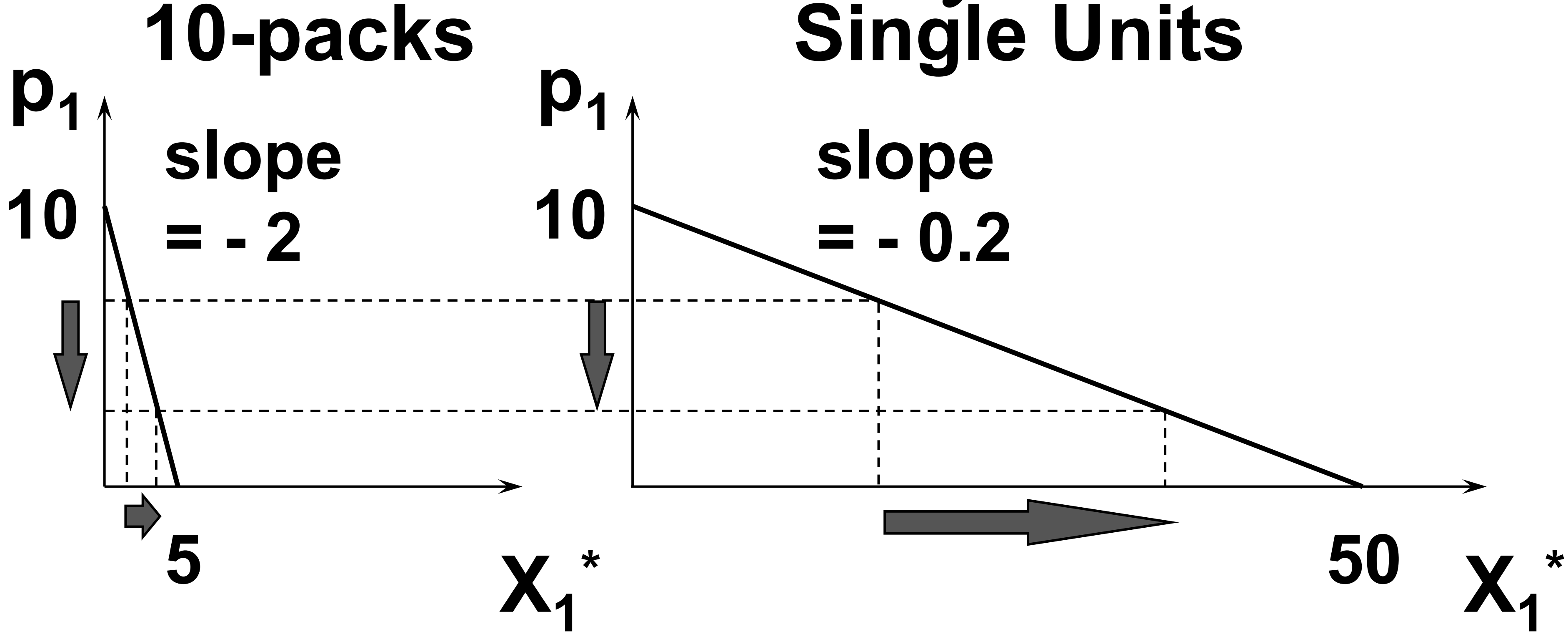
Own-Price Elasticity of Demand



In which case is the quantity demanded X_1^* more sensitive to changes to p_1?
It is the same in both cases.

Own-Price Elasticity of Demand

- **Q: Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity's own price?**
- **A: Because the value of sensitivity then depends upon the (arbitrary) units of measurement used for quantity demanded.**

Own-Price Elasticity of Demand

$$\varepsilon_{x_1^*, p_1} = \frac{\%\Delta x_1^*}{\%\Delta p_1}$$

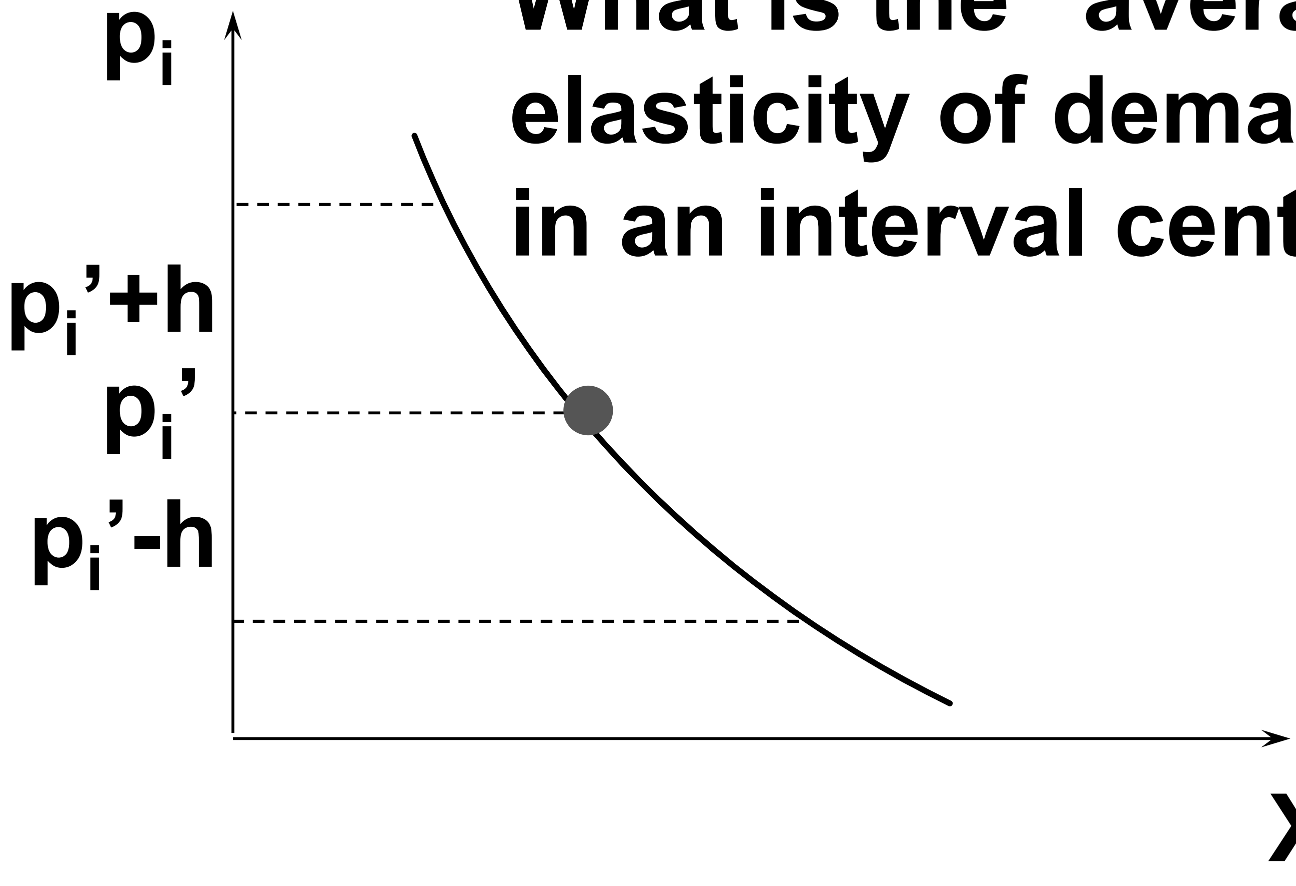
is a ratio of percentages and so has no units of measurement.
Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.

Arc and Point Elasticities

- **An "average" own-price elasticity of demand for commodity i over an interval of values for p_i is an arc-elasticity, usually computed by a mid-point formula.**
- **Elasticity computed for a single value of p_i is a point elasticity.**

Arc Own-Price Elasticity

What is the "average" own-price elasticity of demand for prices in an interval centered on p_i'?



Arc Own-Price Elasticity

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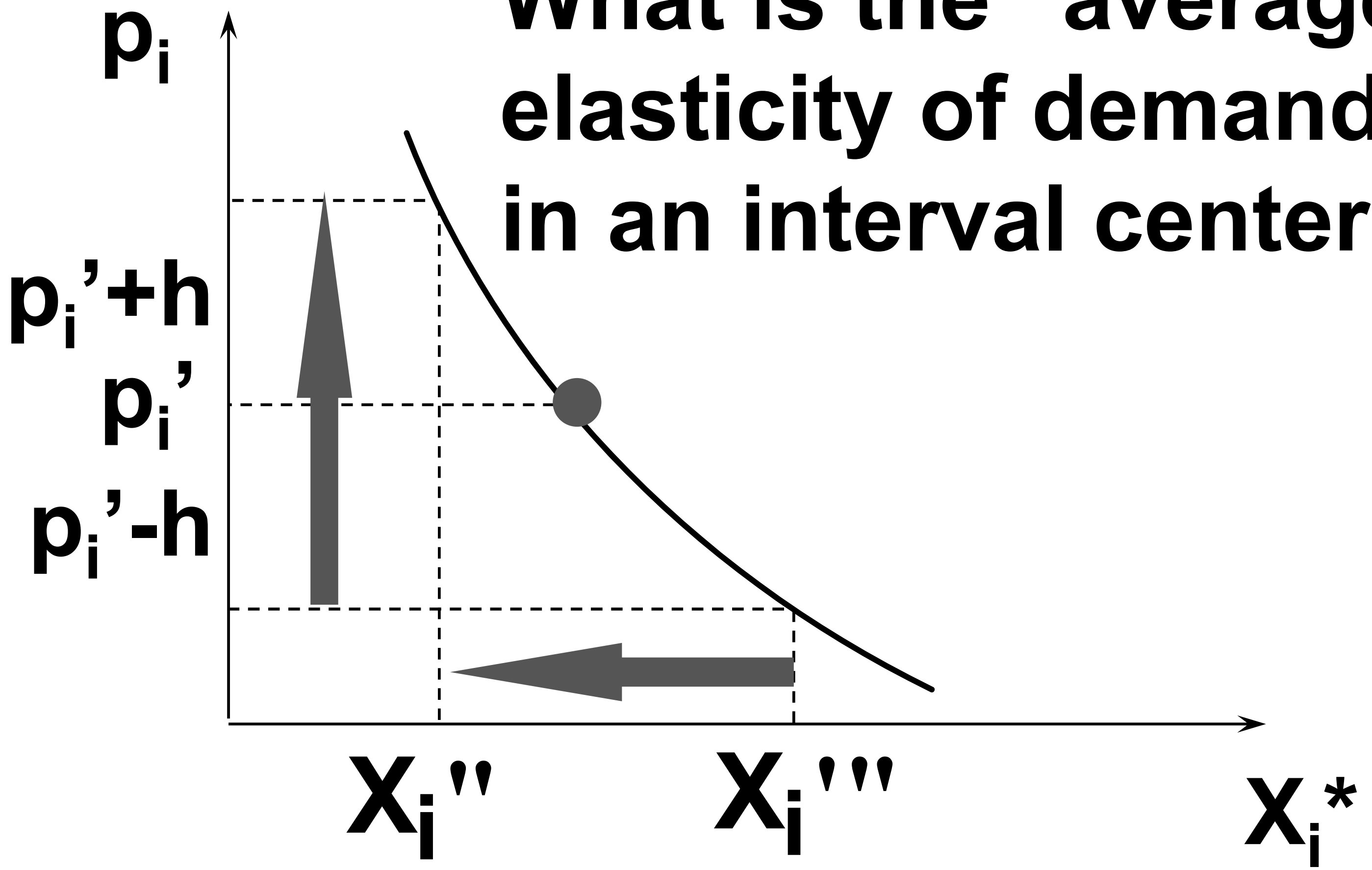

Arc Own-Price Elasticity

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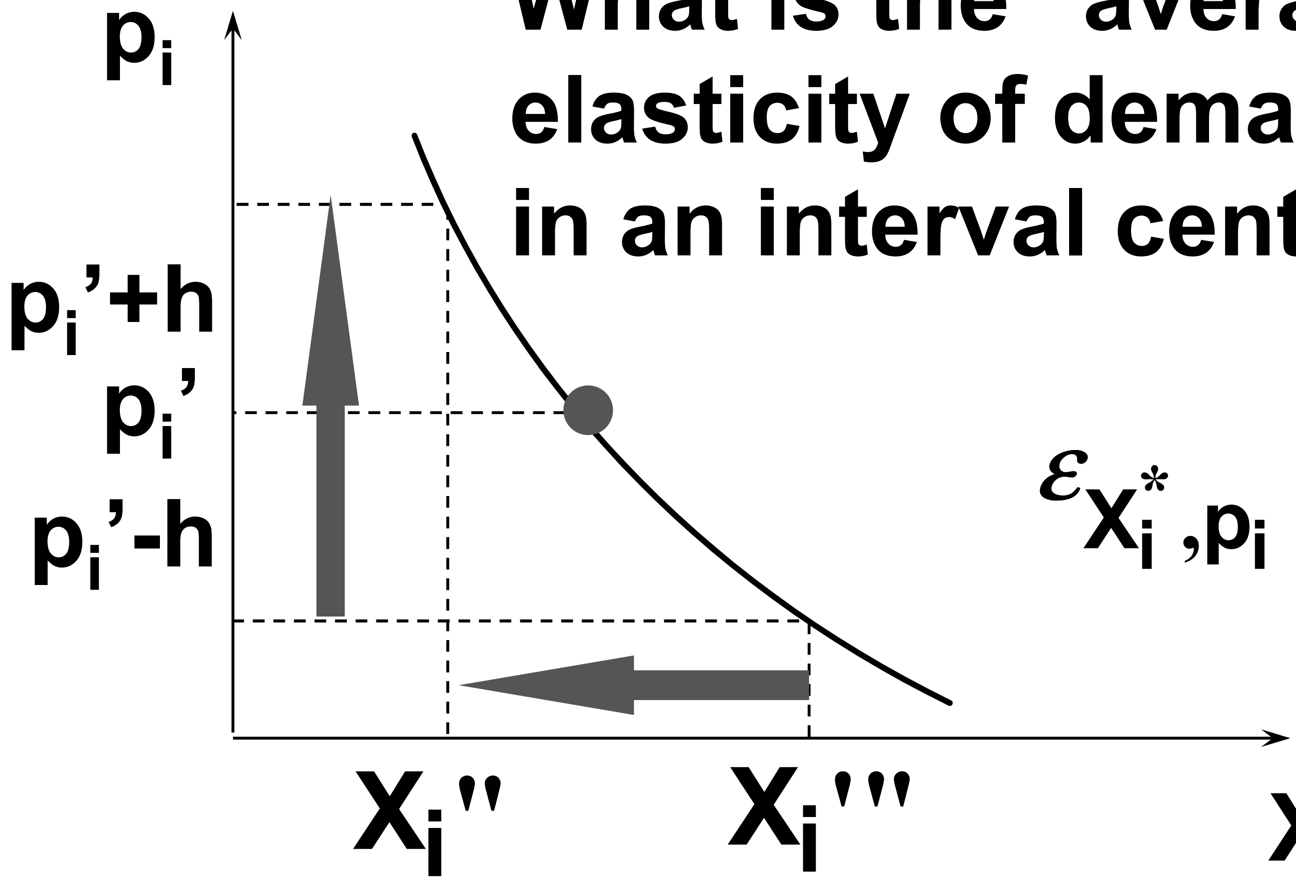
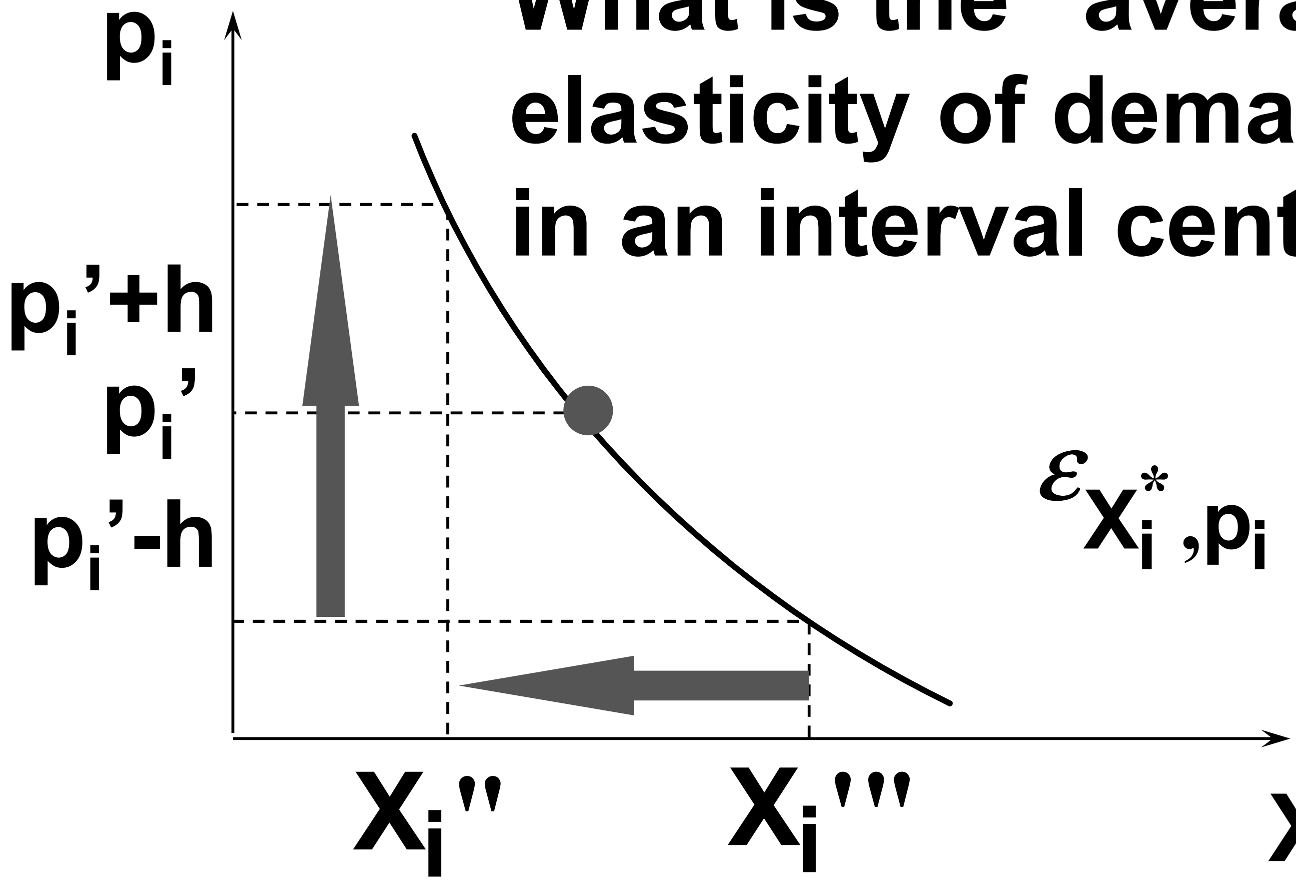


$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

Arc Own-Price Elasticity

What is the "average" own-price elasticity of demand for prices in an interval centered on p_i'?

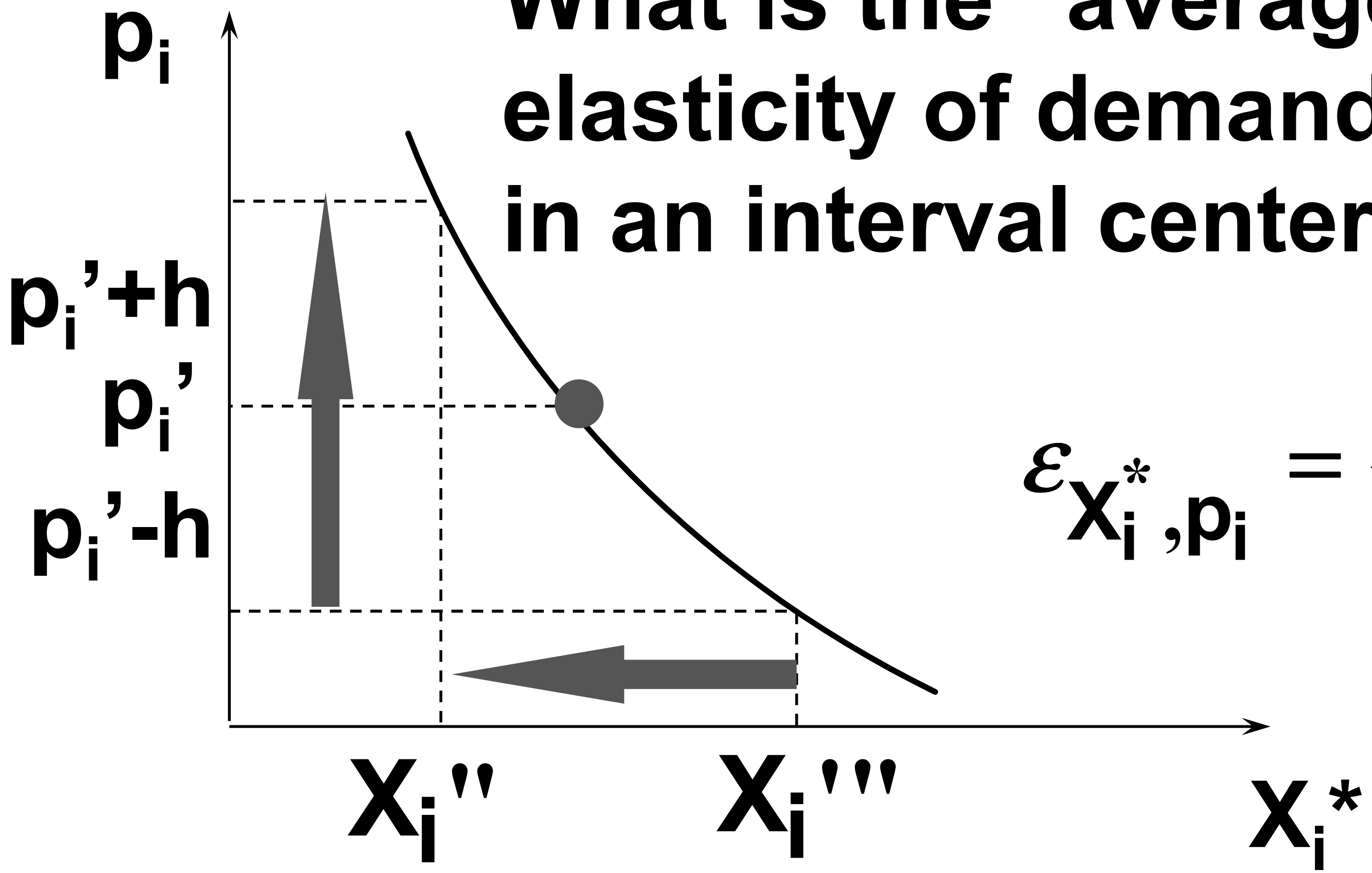


$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

Arc Own-Price Elasticity

What is the “average” own-price elasticity of demand for prices in an interval centered on p_i’?



$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'} \qquad \%\Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

Arc Own-Price Elasticity

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

Arc Own-Price Elasticity

$$\%\Delta p_i = 100 \times \frac{2h}{p_i{}'}$$

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta X_i^* = 100 \times \frac{(X_i{}'' - X_i{}''')}{(X_i{}'' + X_i{}''')/2}$$

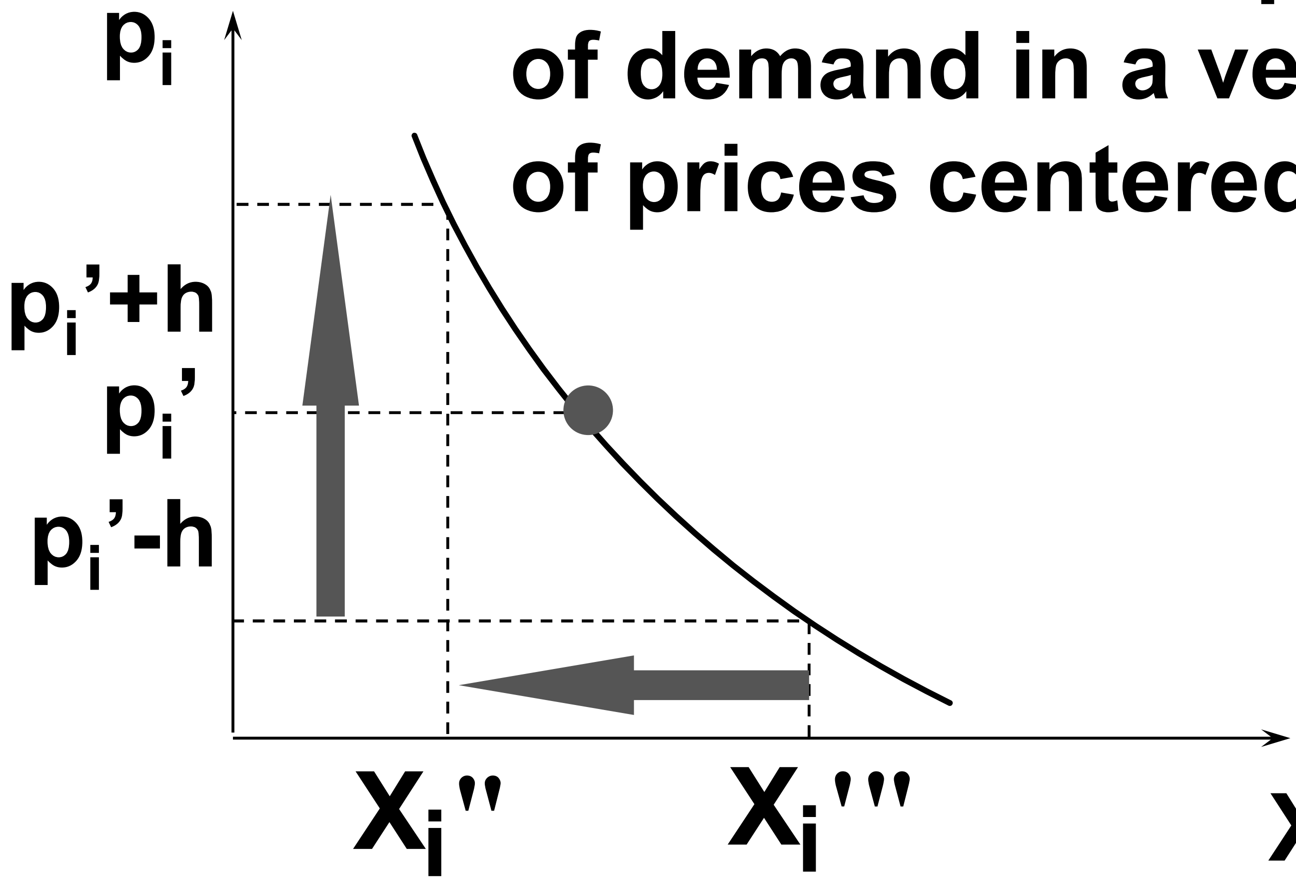
So

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i{}'}{(X_i{}'' + X_i{}''')/2} \times \frac{(X_i{}'' - X_i{}''')}{2h}.$$

is the arc own-price elasticity of demand.

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on p_i'?

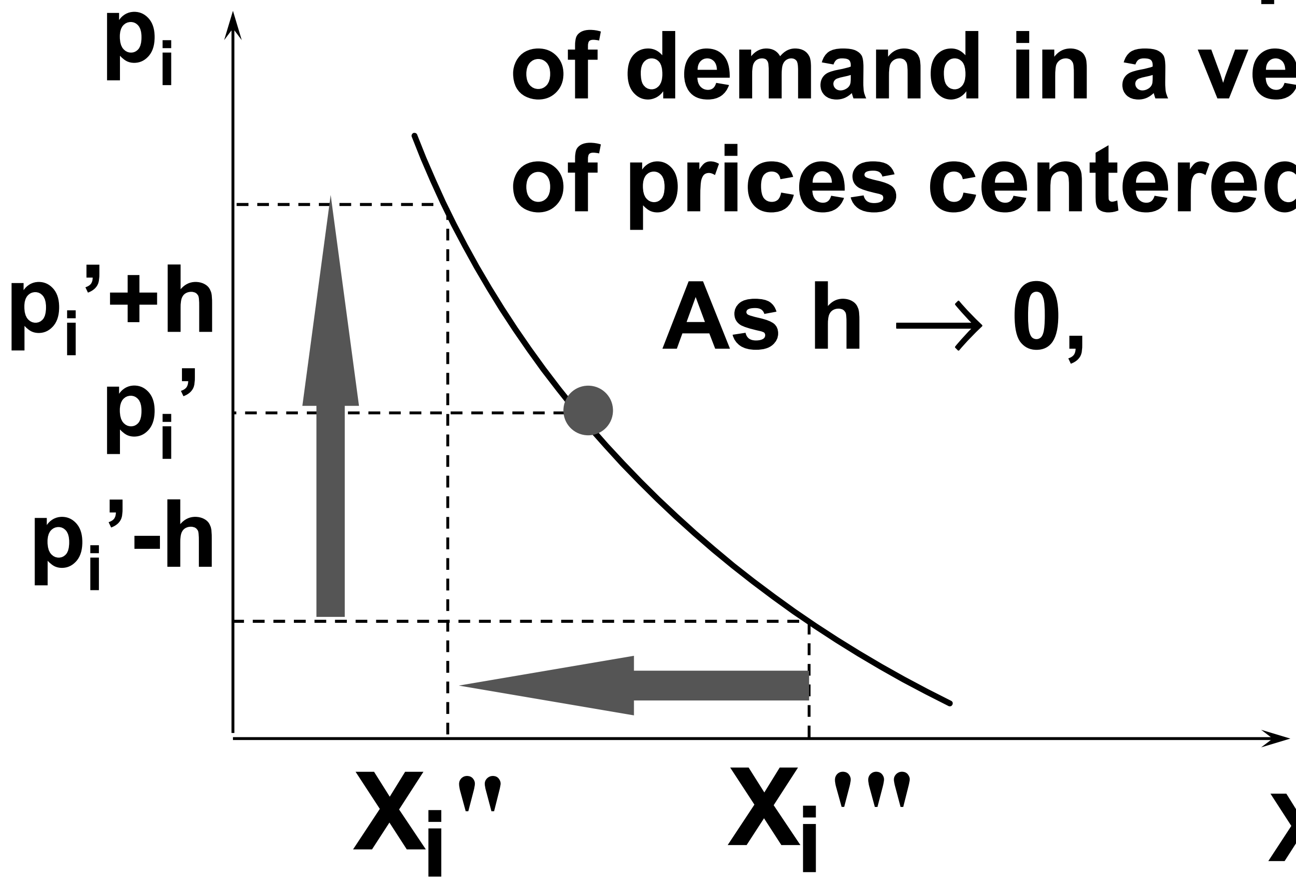


$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on p_i'?

As $h \to 0$,

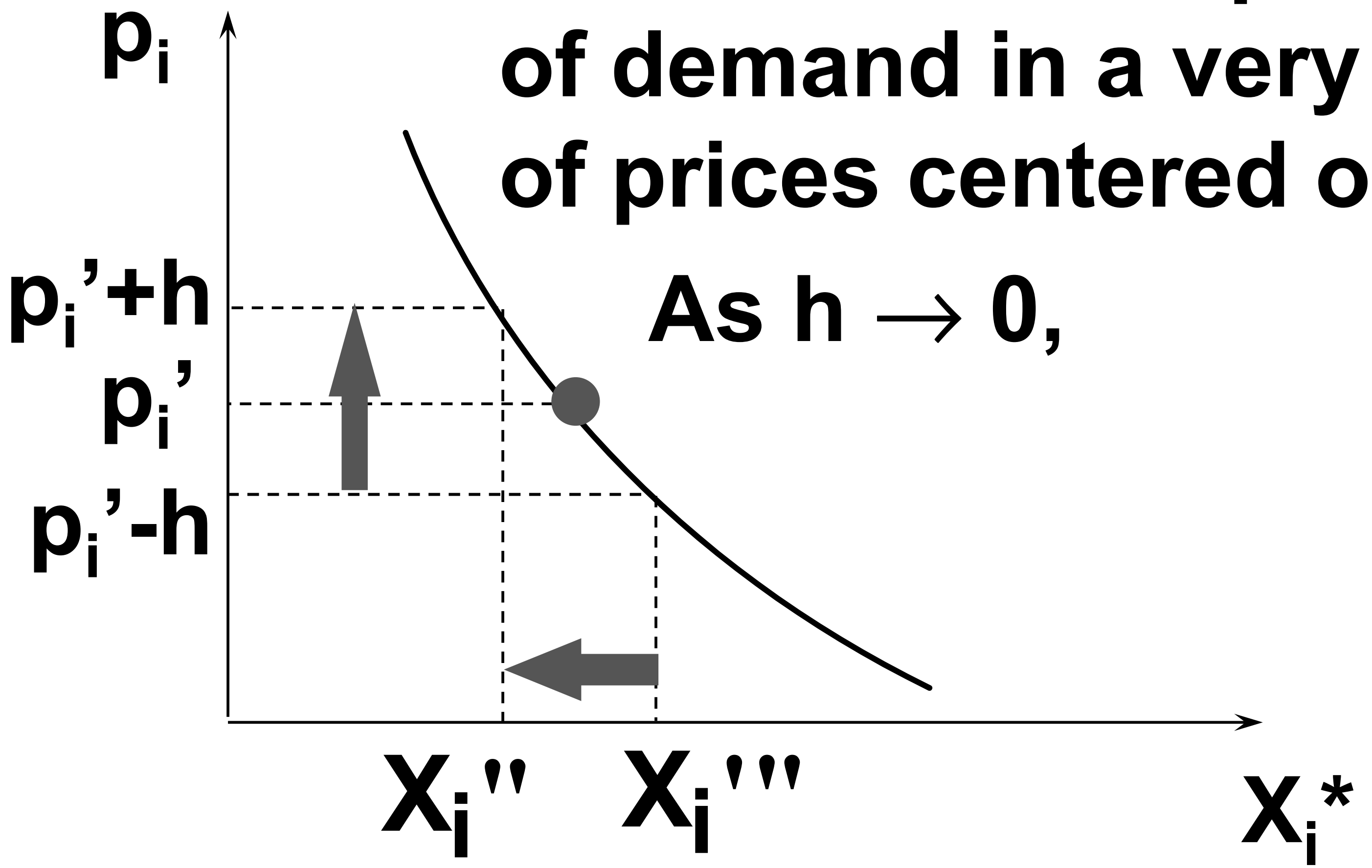


$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on p_i'?

As h → 0,



$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

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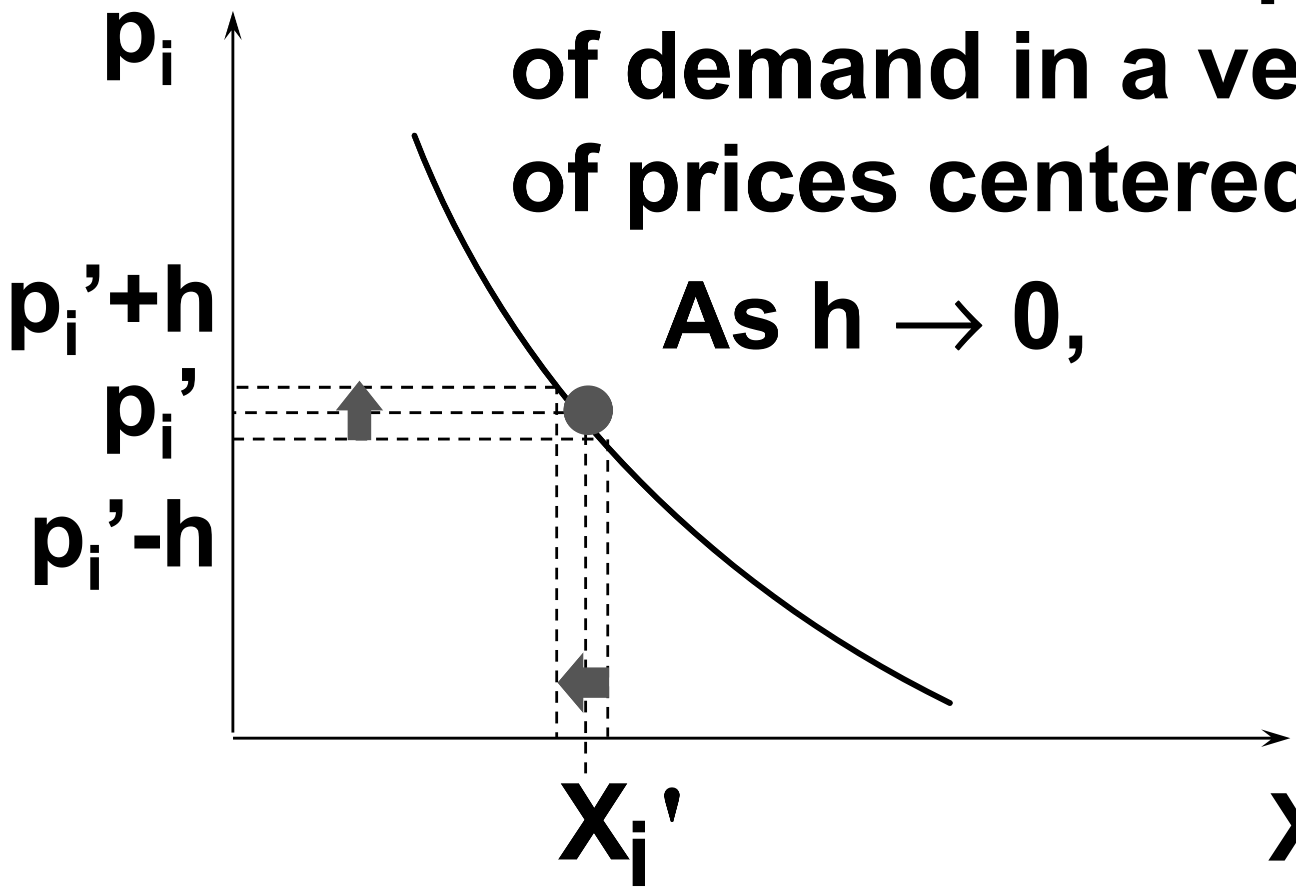
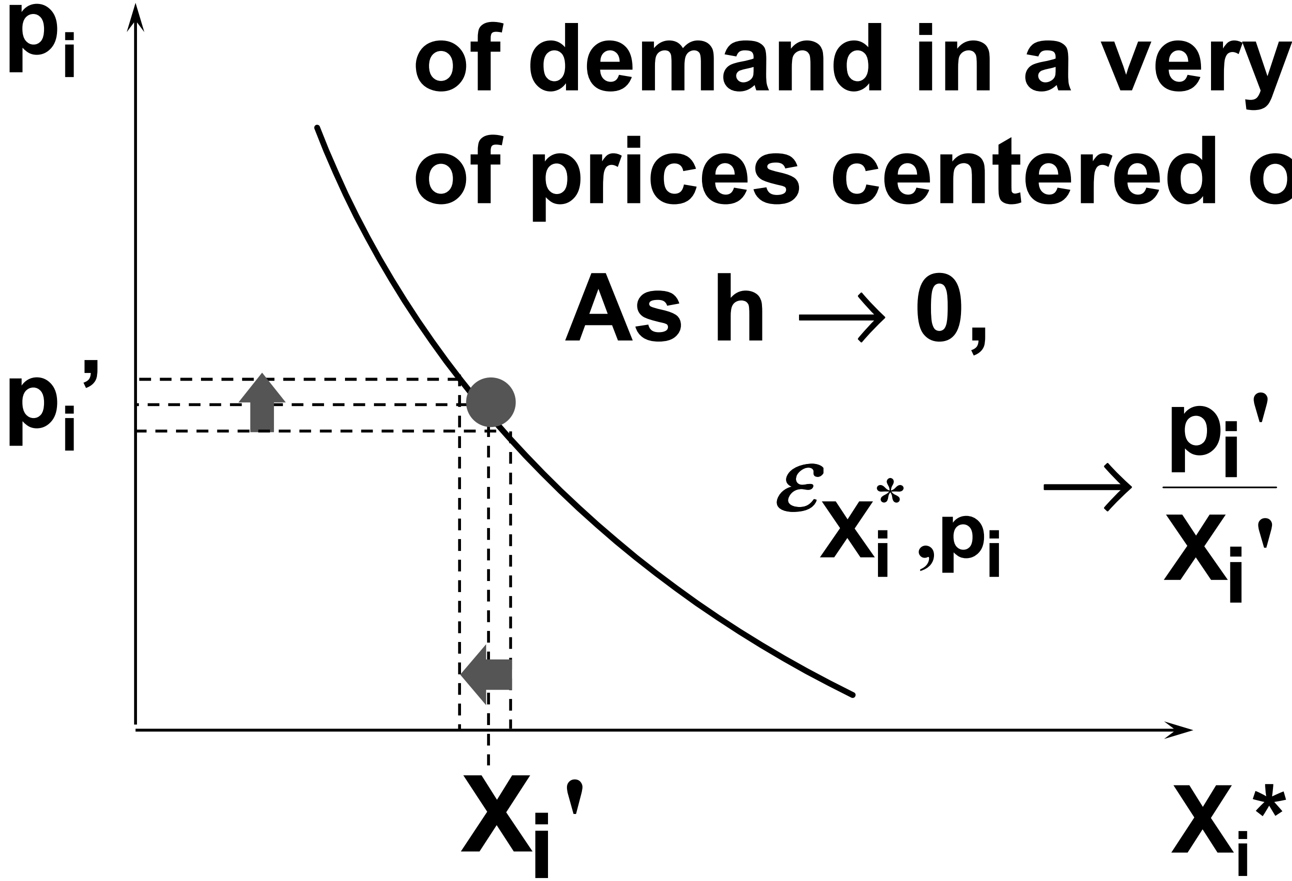


$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on p_i'?



As $h \to 0$,

$$\varepsilon_{X_i^*,p_i} \to \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

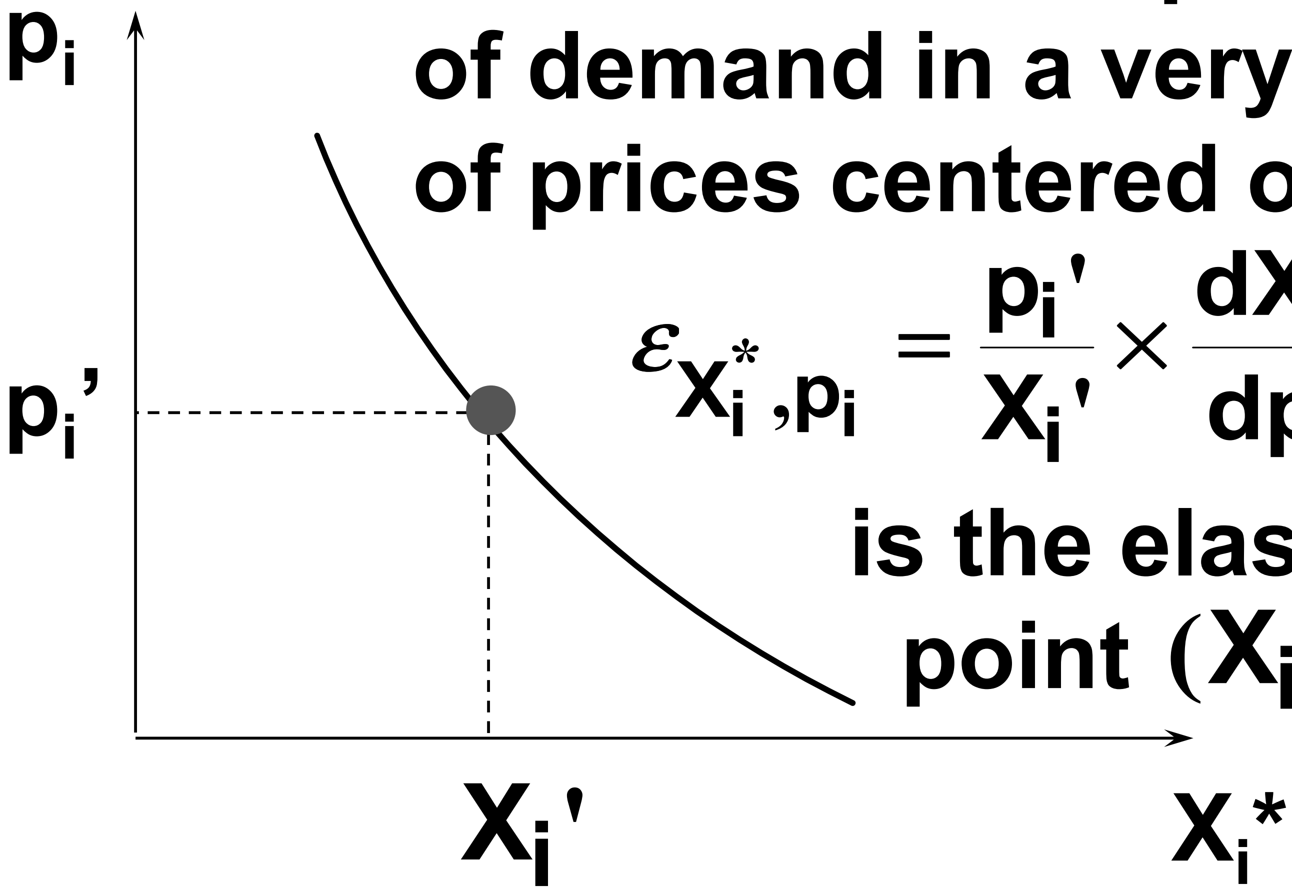
$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on p_i'?

$$\varepsilon_{X_i^*, p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

is the elasticity at the point (X_i', p_i').



Point Own-Price Elasticity

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

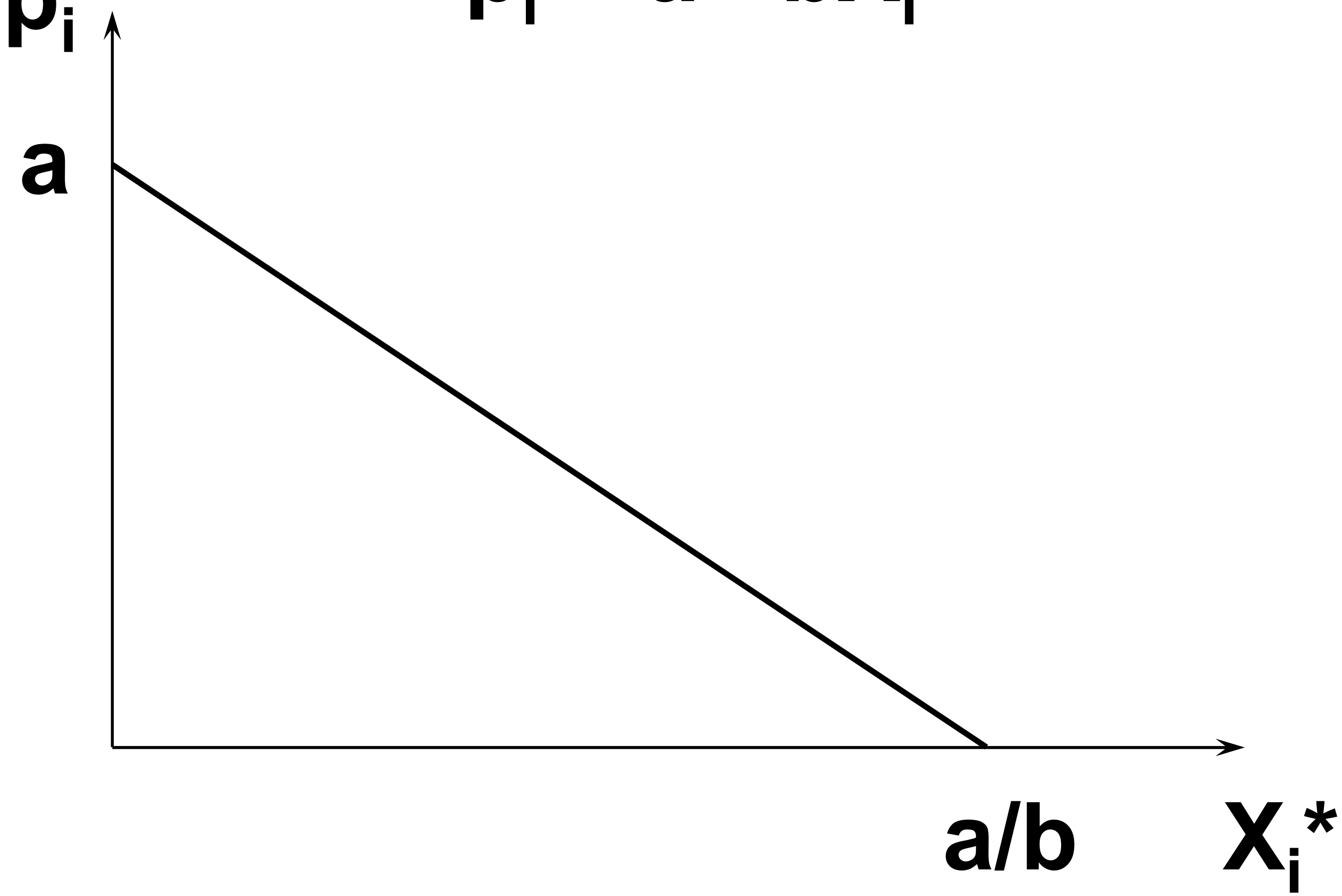
E.g. Suppose $p_i = a - bX_i$.
Then $X_i = (a-p_i)/b$ and

$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ **Therefore,**

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{(a - p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a - p_i}.$$

Point Own-Price Elasticity

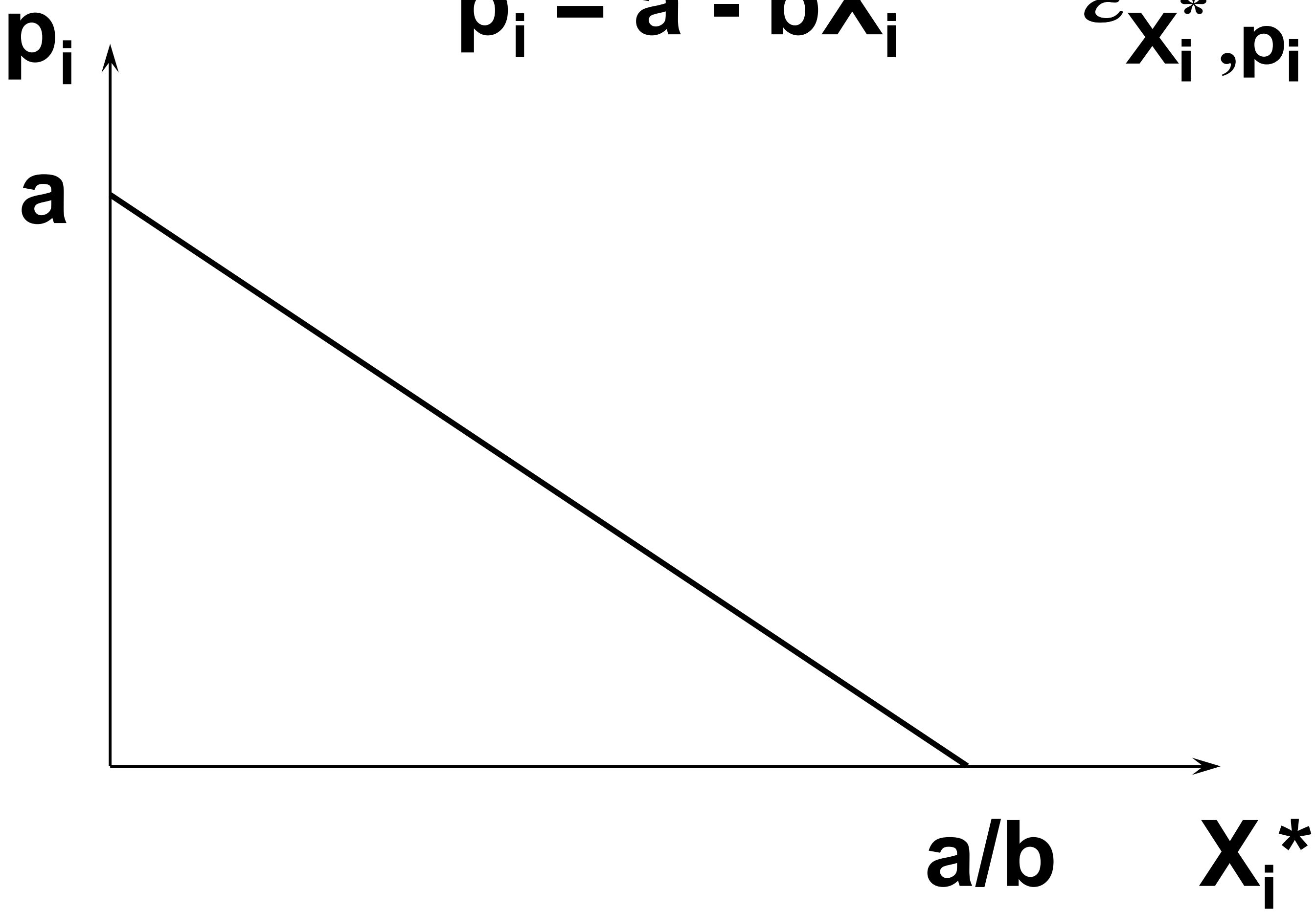
$p_i = a - bX_i^*$



Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*,p_i} = -\frac{p_i}{a - p_i}$$

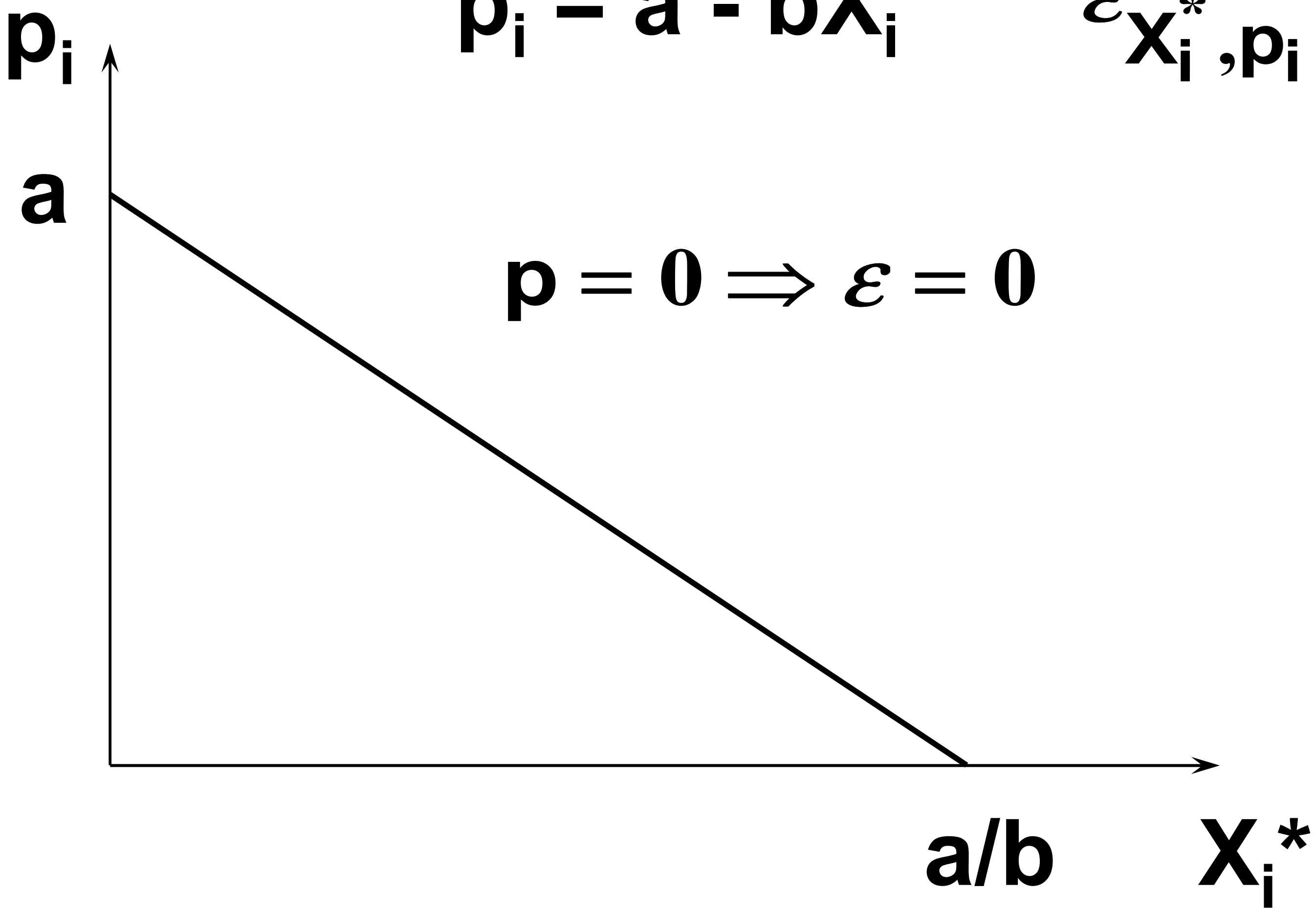


Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*,p_i} = -\frac{p_i}{a - p_i}$$

$$p = 0 \Rightarrow \varepsilon = 0$$



Point Own-Price Elasticity

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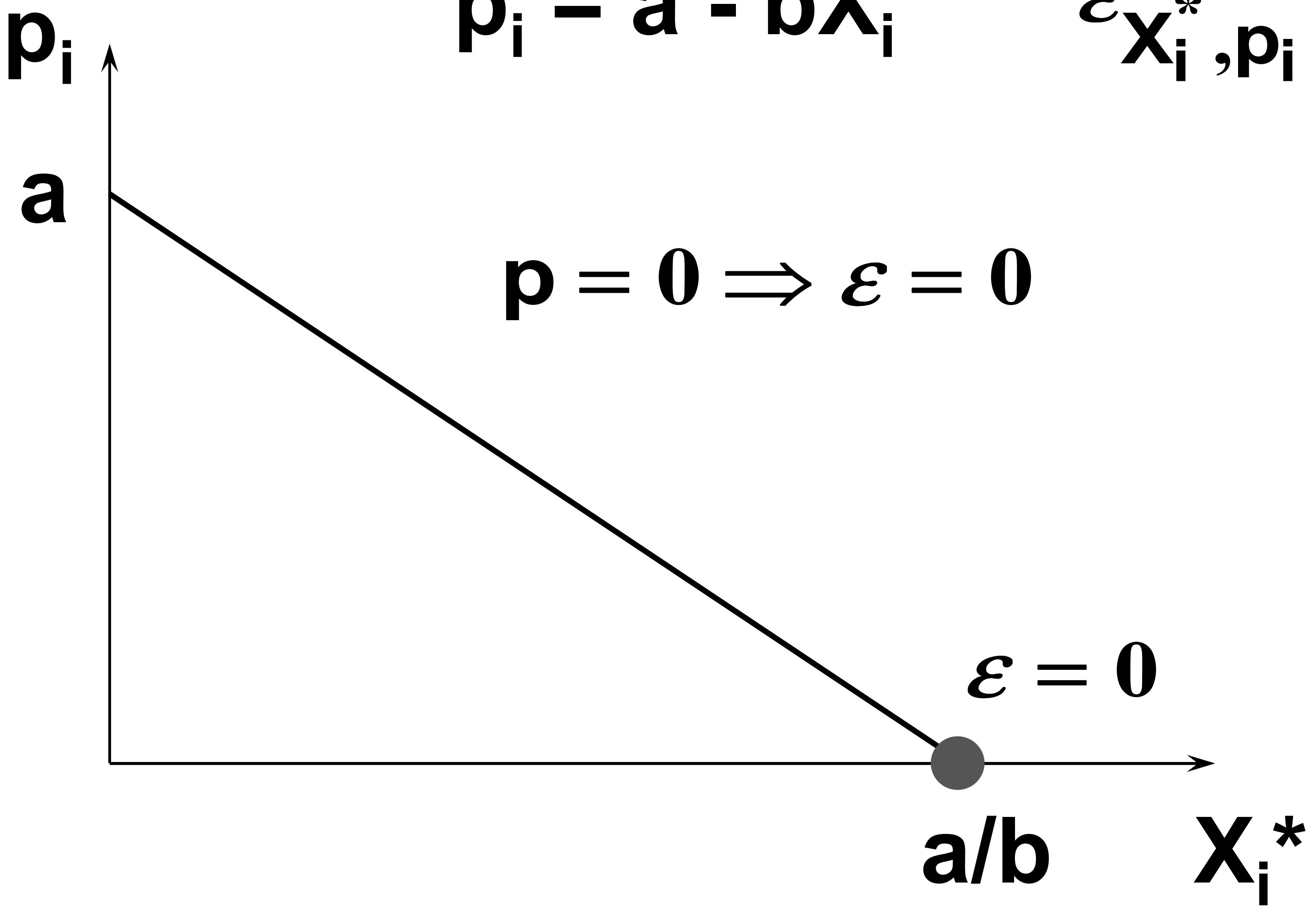
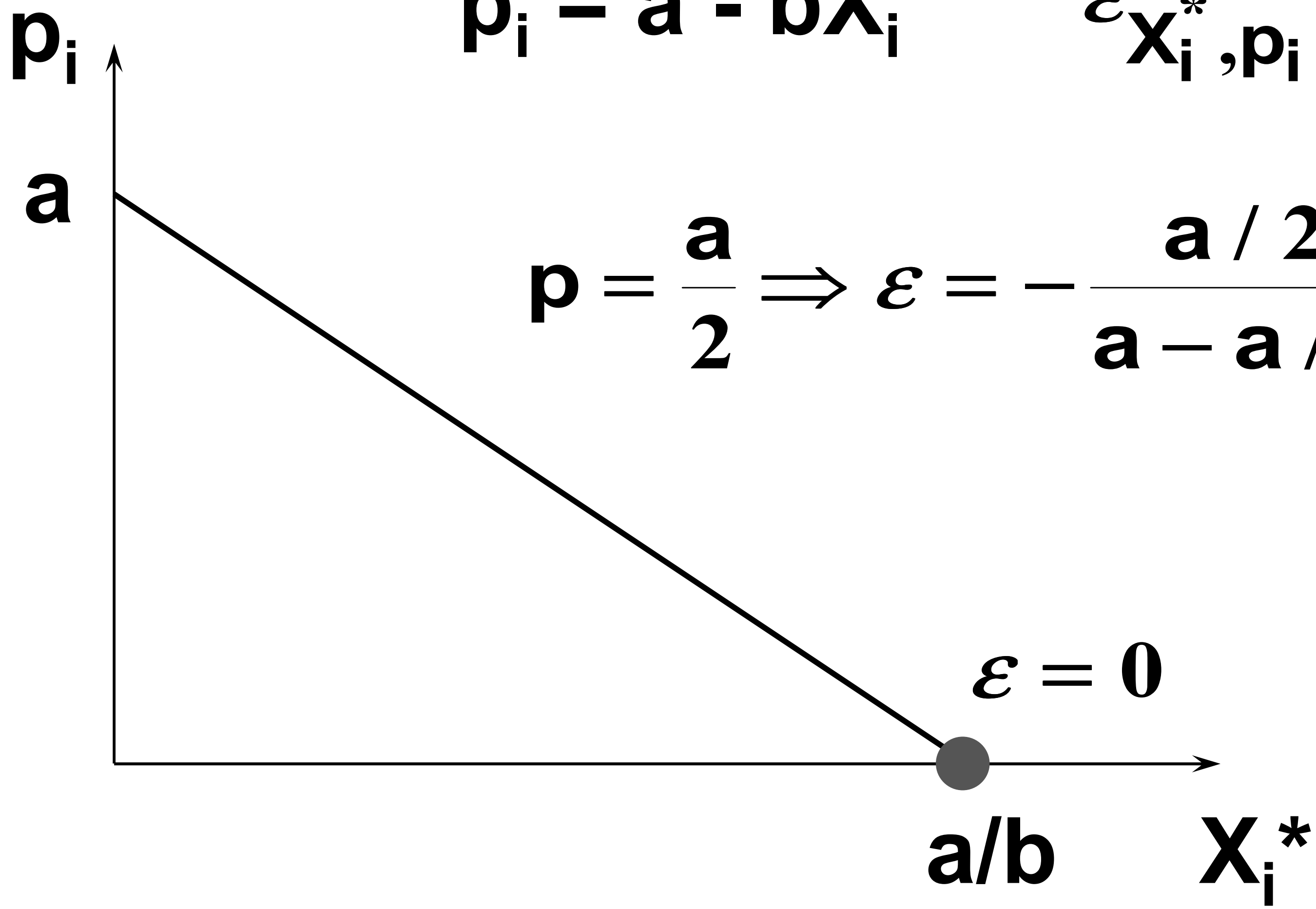


Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*,p_i} = -\frac{p_i}{a - p_i}$$

$$p = \frac{a}{2} \Rightarrow \varepsilon = -\frac{a/2}{a - a/2} = -1$$



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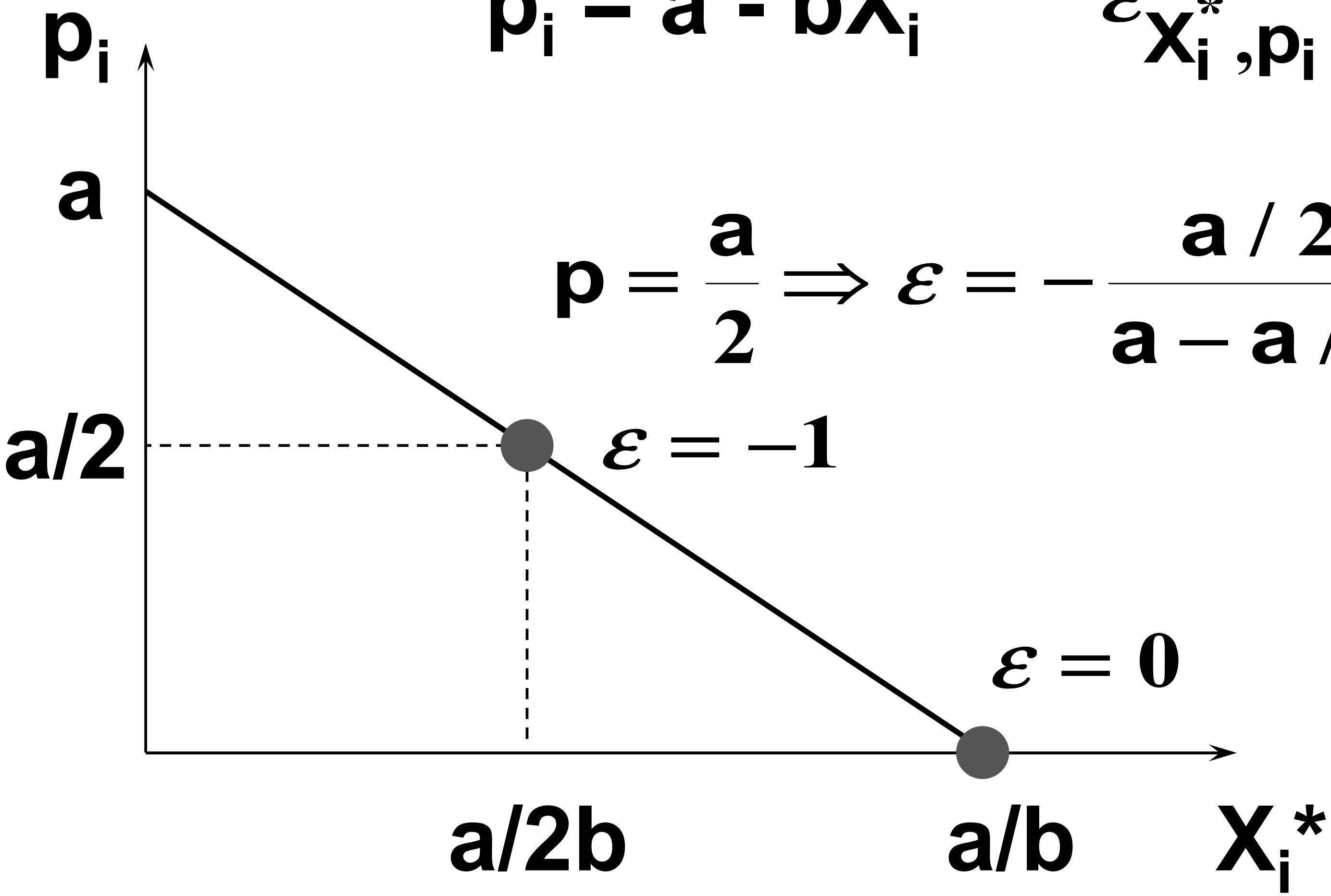
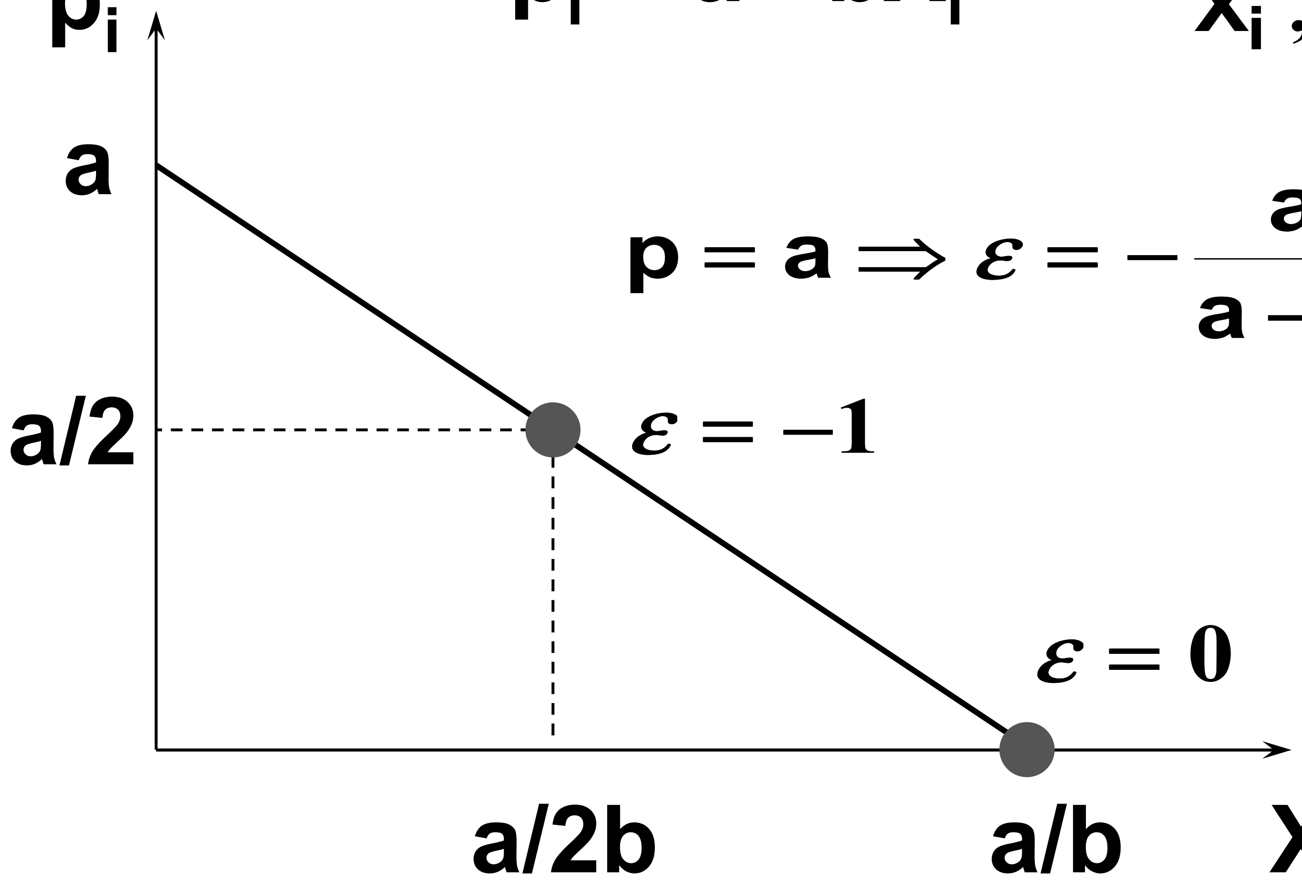


Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = a \Rightarrow \varepsilon = -\frac{a}{a - a} = -\infty$$



Point Own-Price Elasticity

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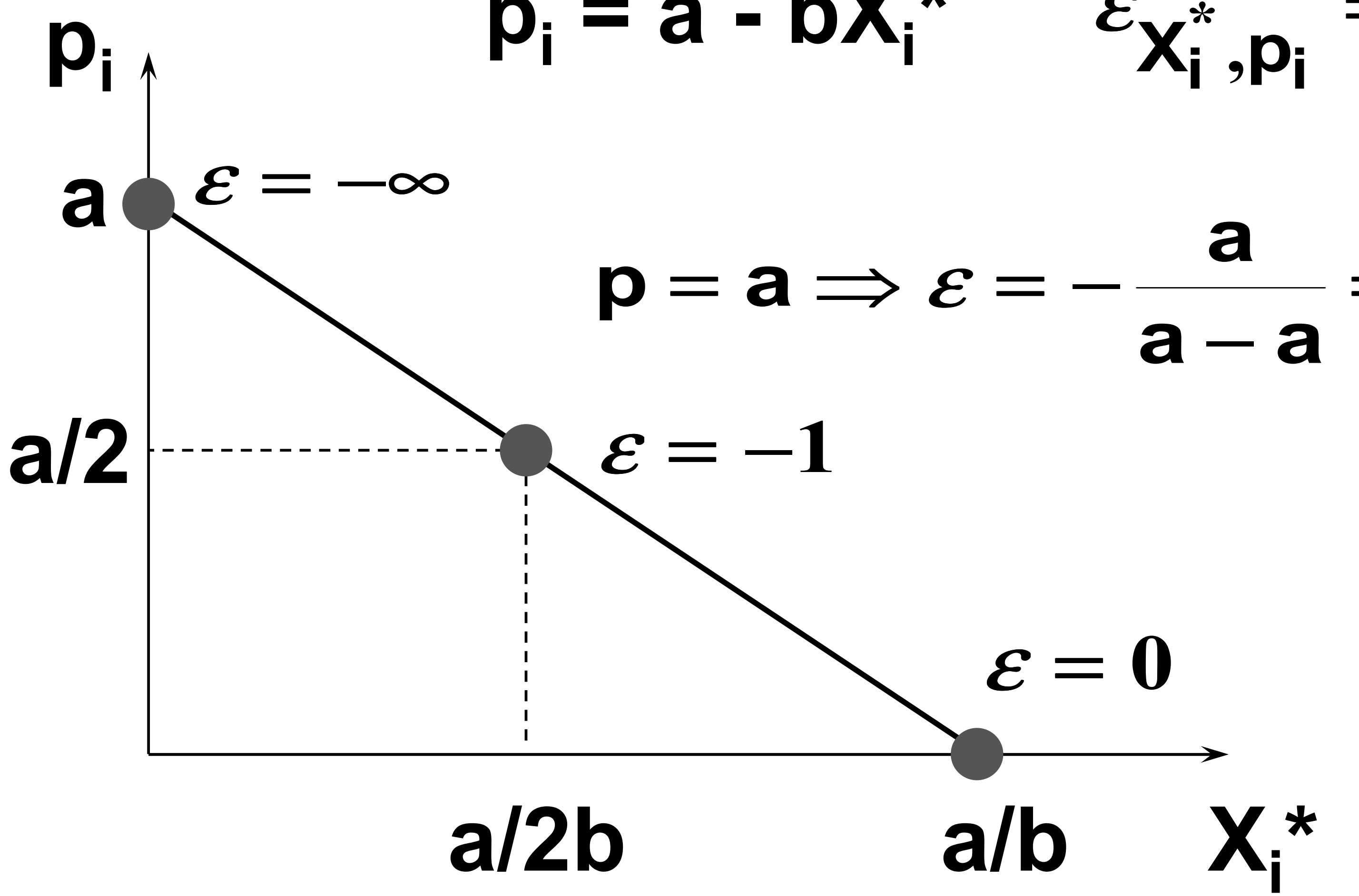

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Point Own-Price Elasticity

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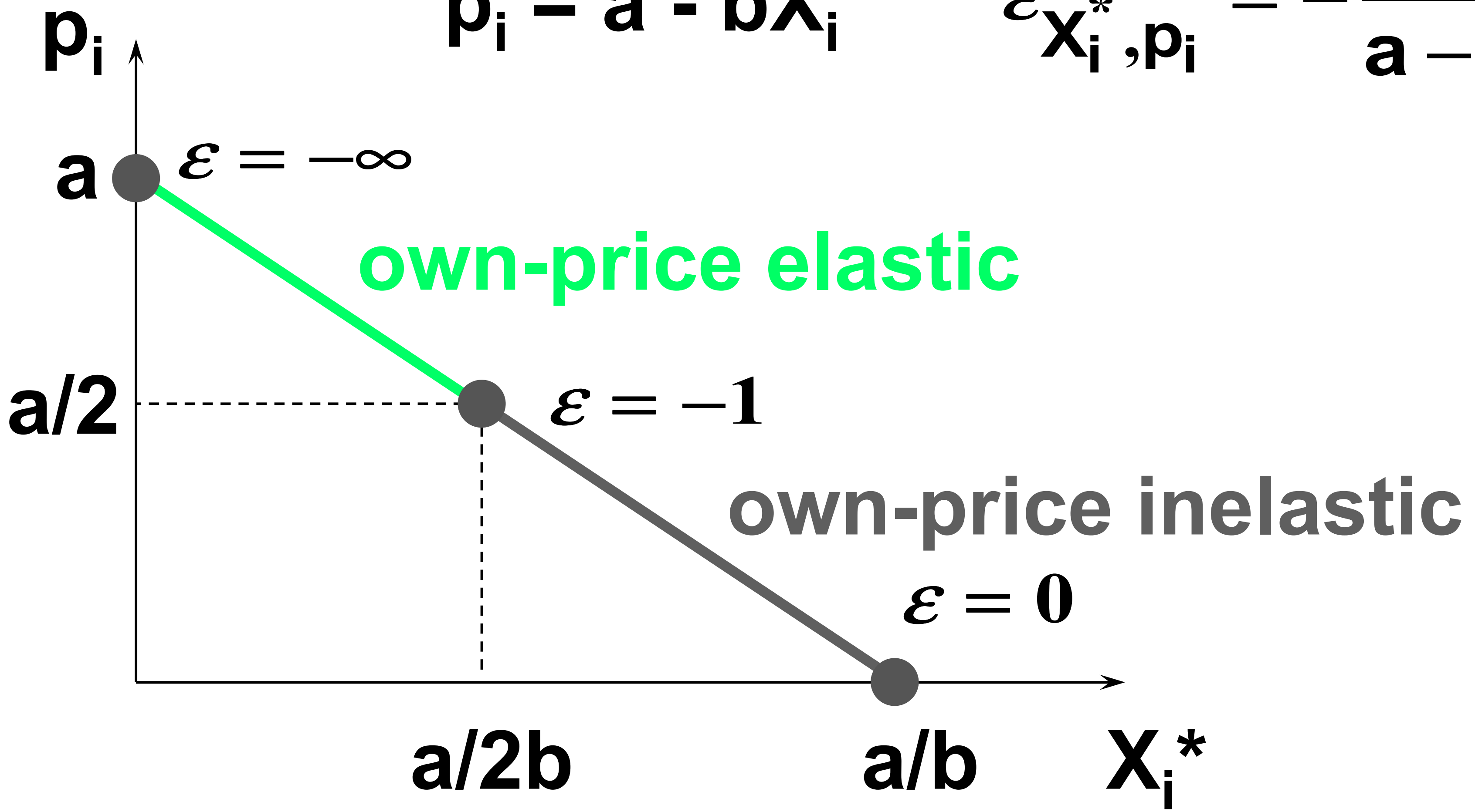

Point Own-Price Elasticity

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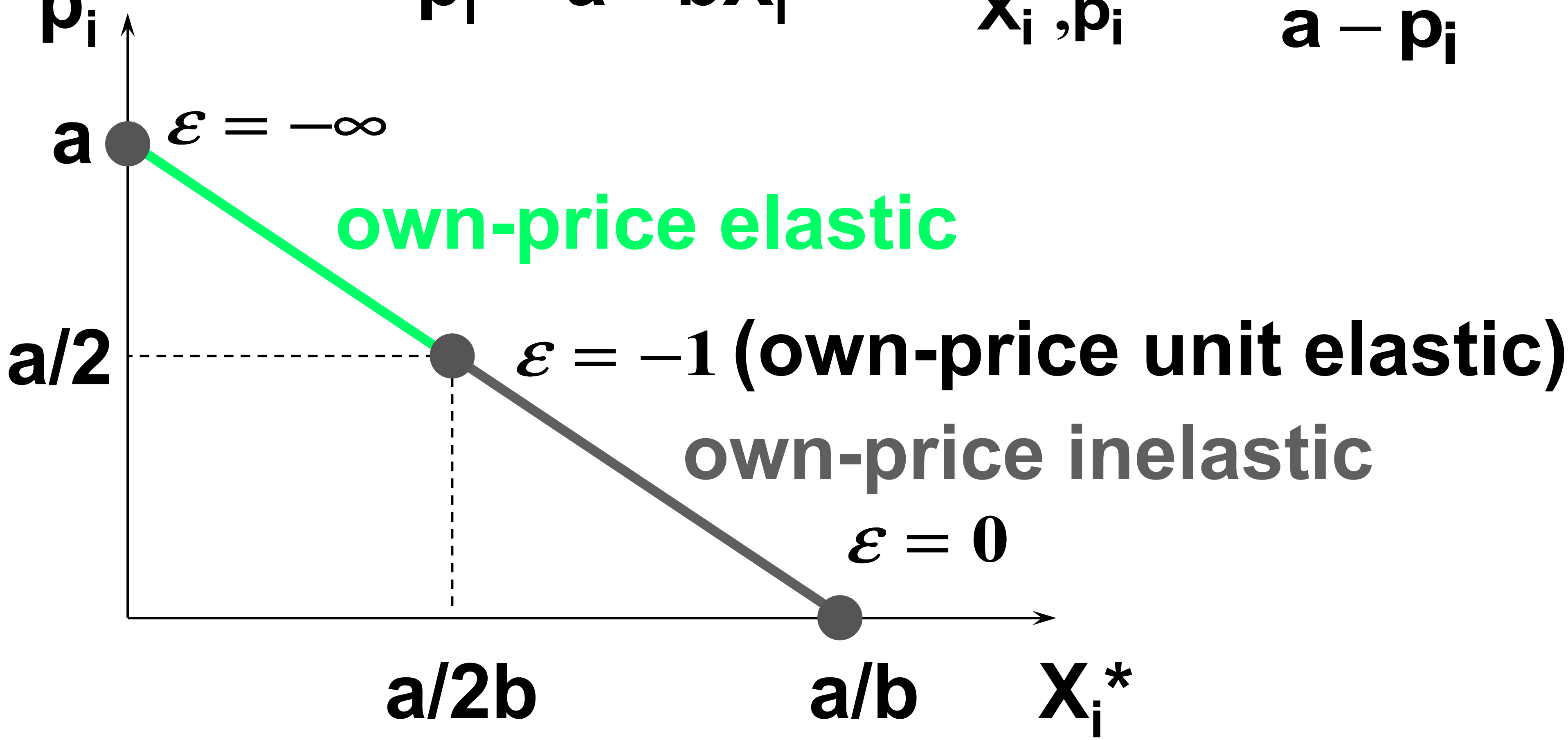


Point Own-Price Elasticity

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

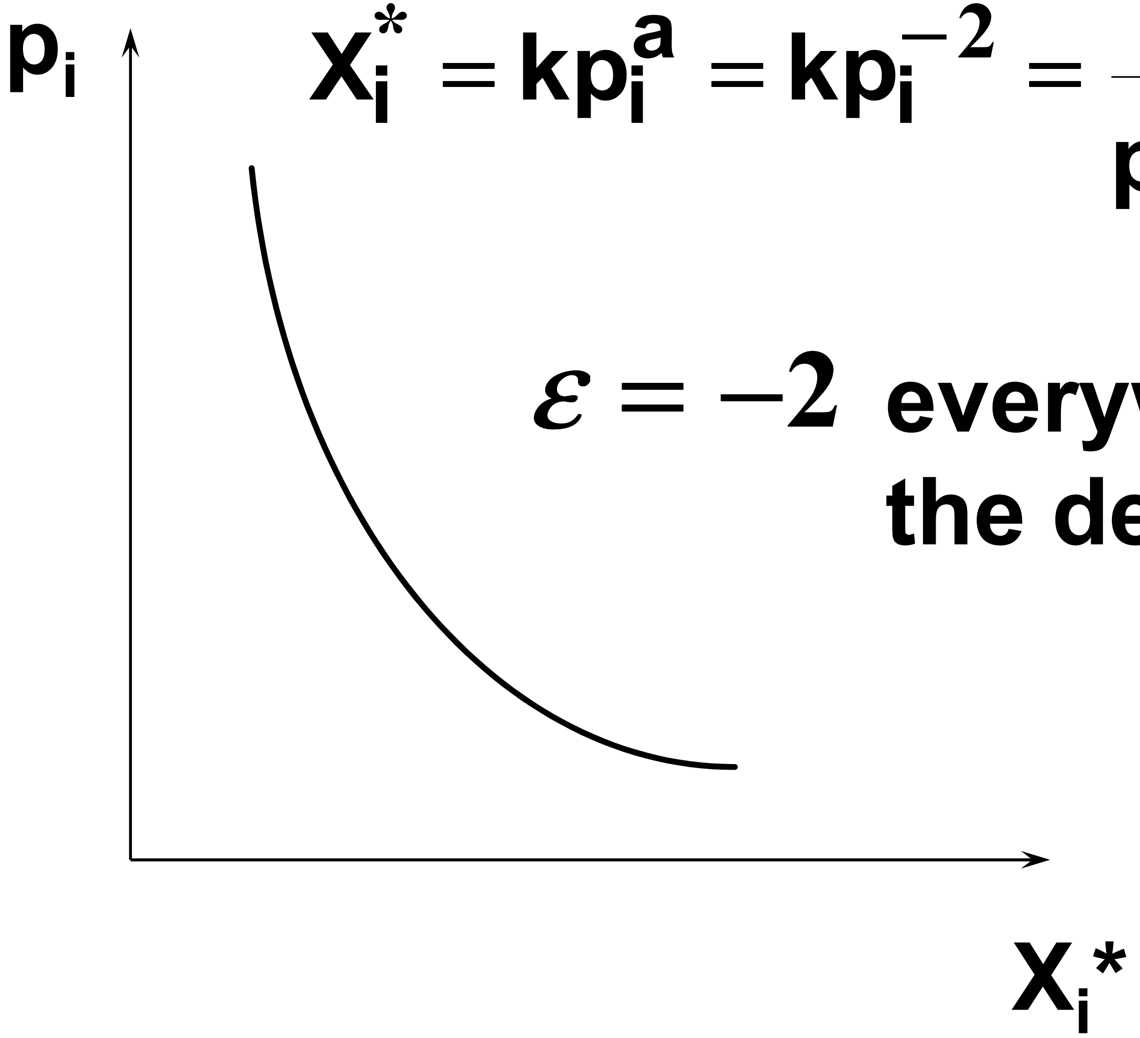
E.g. $X_i^* = kp_i^a$. Then $\dfrac{dX_i^*}{dp_i} = a p_i^{a-1}$

so

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{kp_i^a} \times ka p_i^{a-1} = a \frac{p_i^a}{p_i^a} = a.$$

Point Own-Price Elasticity

$$X_i^* = kp_i^a = kp_i^{-2} = \frac{k}{p_i^2}$$



$\varepsilon = -2$ **everywhere along the demand curve.**

Revenue and Own-Price Elasticity of Demand

- **If raising a commodity's price causes little decrease in quantity demanded, then sellers' revenues rise.**
- **Hence own-price inelastic demand causes sellers' revenues to rise as price rises.**

Revenue and Own-Price Elasticity of Demand

- **If raising a commodity’s price causes a large decrease in quantity demanded, then sellers’ revenues fall.**
- **Hence own-price elastic demand causes sellers’ revenues to fall as price rises.**

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

Revenue and Own-Price Elasticity of Demand

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So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p)$.

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $\dfrac{dR}{dp} = X^*(p) + p\dfrac{dX^*}{dp}$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1 + \varepsilon]$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

so if $\varepsilon = -1$ then $\frac{dR}{dp} = 0$

and a change to price does not alter sellers’ revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

but if $-1 < \varepsilon \le 0$ **then** $\frac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

And if $\varepsilon < -1$ **then** $\frac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Revenue and Own-Price Elasticity of Demand

In summary:

Own-price inelastic demand; $-1 < \varepsilon \leq 0$
price rise causes rise in sellers' revenue.

Own-price unit elastic demand; $\varepsilon = -1$
price rise causes no change in sellers' revenue.

Own-price elastic demand; $\varepsilon < -1$
price rise causes fall in sellers' revenue.

Marginal Revenue and Own-Price Elasticity of Demand

- **A seller's marginal revenue is the rate at which revenue changes with the number of units sold by the seller.**

$$\mathbf{MR(q) = \frac{dR(q)}{dq}.}$$

Marginal Revenue and Own-Price Elasticity of Demand

p(q) denotes the seller's inverse demand function; i.e. the price at which the seller can sell q units. Then

$$R(q) = p(q) \times q$$

so

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$\mathbf{MR(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].}$$

and $$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so $$\mathbf{MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].}$$

Marginal Revenue and Own-Price Elasticity of Demand

$$\mathbf{MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]}$$ **says that the rate at which a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; *i.e.*, upon the of the own-price elasticity of demand.**

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$

If $\varepsilon = -1$ then $MR(q) = 0.$

If $-1 < \varepsilon \leq 0$ then $MR(q) < 0.$

If $\varepsilon < -1$ then $MR(q) > 0.$

Marginal Revenue and Own-Price Elasticity of Demand

If $\varepsilon = -1$ then $MR(q) = 0$. Selling one more unit does not change the seller's revenue.

If $-1 < \varepsilon \leq 0$ then $MR(q) < 0$. Selling one more unit reduces the seller's revenue.

If $\varepsilon < -1$ then $MR(q) > 0$. Selling one more unit raises the seller's revenue.

Marginal Revenue and Own-Price Elasticity of Demand

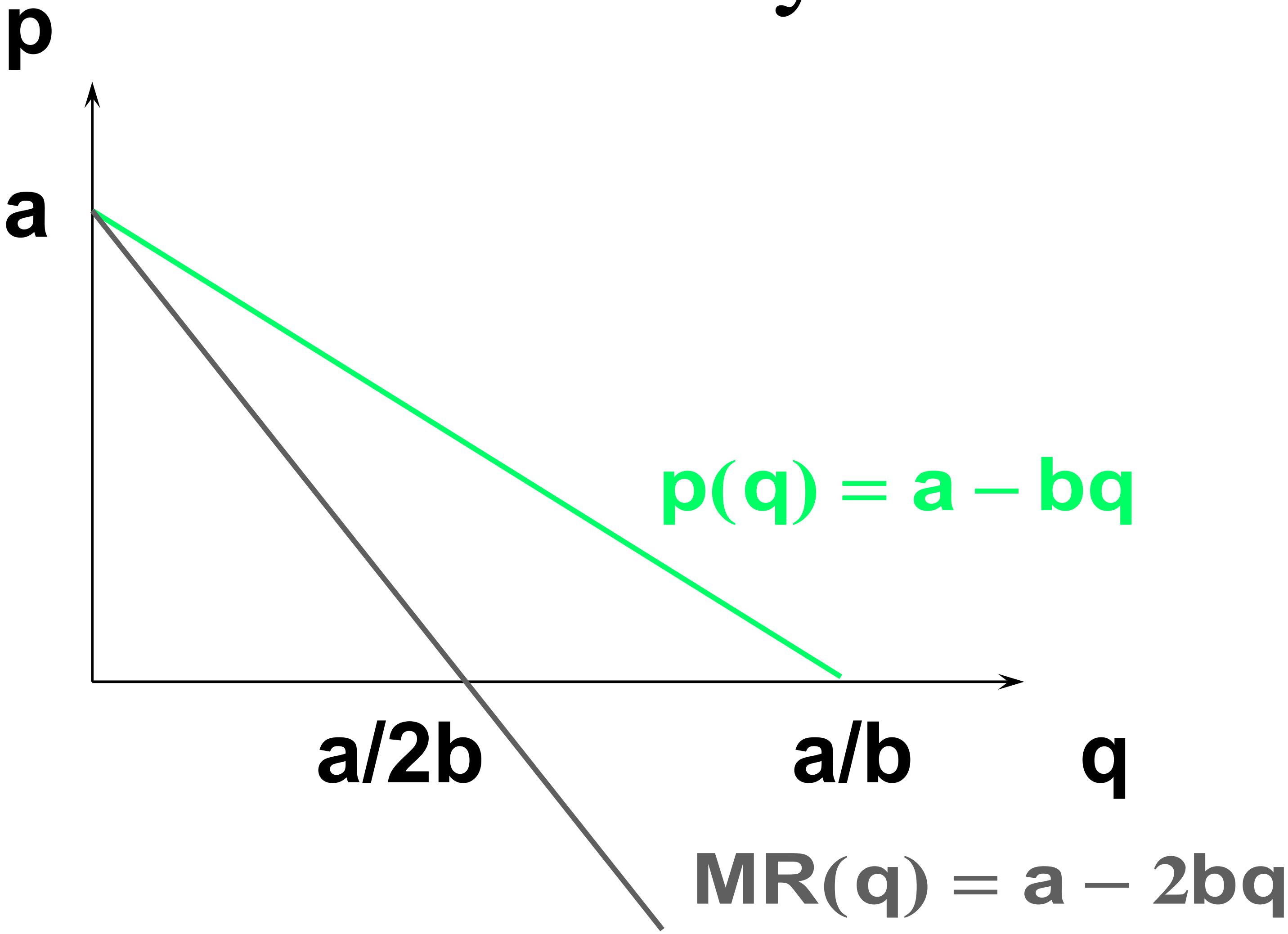
An example with linear inverse demand.

$$p(q) = a - bq.$$

Then $R(q) = p(q)q = (a - bq)q$

and $MR(q) = a - 2bq.$

Marginal Revenue and Own-Price Elasticity of Demand



Marginal Revenue and Own-Price Elasticity of Demand