Linear programming-introduction

Ing.J.Skorkovský,CSc.

USE

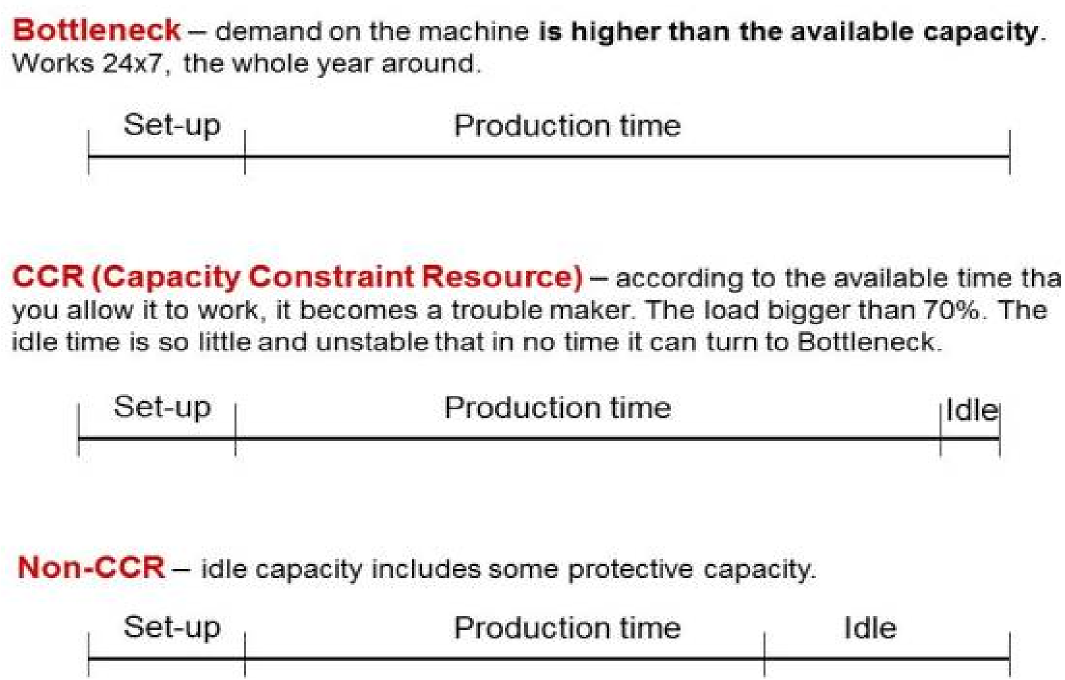
- **Slitting and Levelling of material (coils, bars, sheets)-**Cutting material, trimming,…
- **Blending -** blending, diet, feeding rations for animals, ..
- **Transport problems -** material flow from stock to the destination and route planning - shortest route
- **Assignment of resources with limited capacities - **CCR
- **Sources :** Operation Management, Quality and Competitiveness in a global environment, Russel and Taylor (can be found easily in ESF library)

CCR=Capacity Constraint Resource

See next slide for explanantion of CCR

CCR –additional information

- There are 3 categories of resources from the point of view of capacity:
- Bottleneck
- CCR – Capacity Constraint Resource
- Non-CCR



Formulation of the simple model

Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	x1	1	4	40
Mug	x2	2	3	50

Which combination of products will have the greatest return
at the limits of maximum production capacity type = **40** hours
and moreover, the amount of material that is limited to **120** kg of clay?

Note: A similar task in terms of flow was solved in the P&Q example
(only valid for Czech student), where the limitation in resource
B and with a maximum capacity of 2400 minutes)

Description x1 and x2 stands for variables, Material means e.g. 4 kg for one piece (product)

Basic structures and used terminology

Target function
Z=Cx

- We minimize our target function in the form of:

Z = c1*x1+c2*x2+…..+cn*xn with respect to the matrix of restrictive conditions:
(in our case **c1=40** and **c2=50 which means return/pc**)

A11*x1 + A12*x2+ …+ A1n*xn (<>=) B1
A22*x1 + A22*x2+ …+ A2n*xn (<>=) B2

Am1*x1 + Am2*x2+ …+ Amn*xn (<>=)Bm

- It is a classical system of linear equations is **Ax=B**
- The solving of such a linear equation system, e.g. By use of GAUSS-JORDAN algorithm is not required if we will use **Excel Solver add-on**.
- **xij** : decision variable = level of operation activity specified by this variable
- **Bi** : restrictive conditions , allowed deviations from the norm (in time and materiál as well)
- **cj** : coefficient of the target function (in our case returns, meaning return **40** and **50**)
- **Aij** : restrictive coefficients: work and material for one unit (pcs) of the product

Example I (introduction to the problem – practical demonstration)

Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	x1	**1**	4	**40**
Mug	x2	**2**	3	**50**

Z = c1*x1+c2*x2+.....+cn*xn (classical equation from)

Target function: **Z**= 40*x1+50*x2, which we must maximize

Maximal production capacity = **40** hours and Maximal quantity of material =120 kg
(B1 and B2 in our mathematical expression)

Specifications of task restrictions by use of 2x2 matrix:

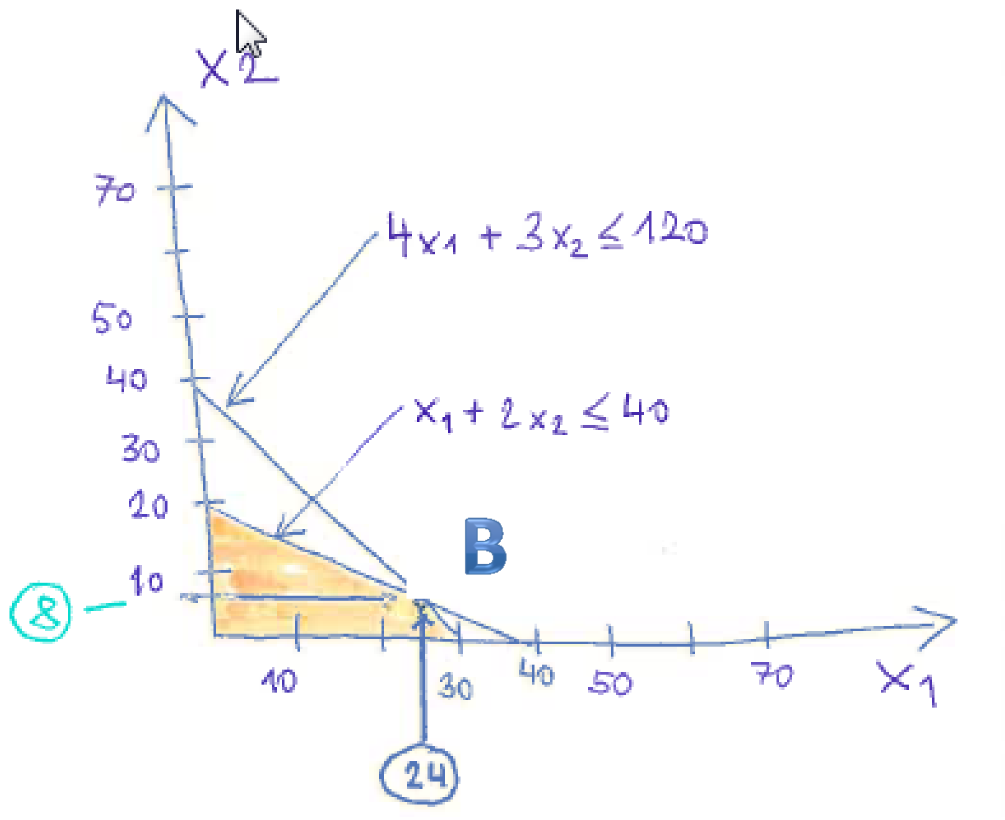
1*x1 + 2*x2 =**40** (work-no more than 40 hours)
4*x1 + 3*x2 =120 (material=kg of clay in our case)->x1=(40-2x2)+3x2=120....

Manual solving : -> x1=24 a x2=8 and after substitution od variables (24 pcs of Dish and 8 pcs of Mug) in target function we will get

Z=40*24+50*8=**1360**

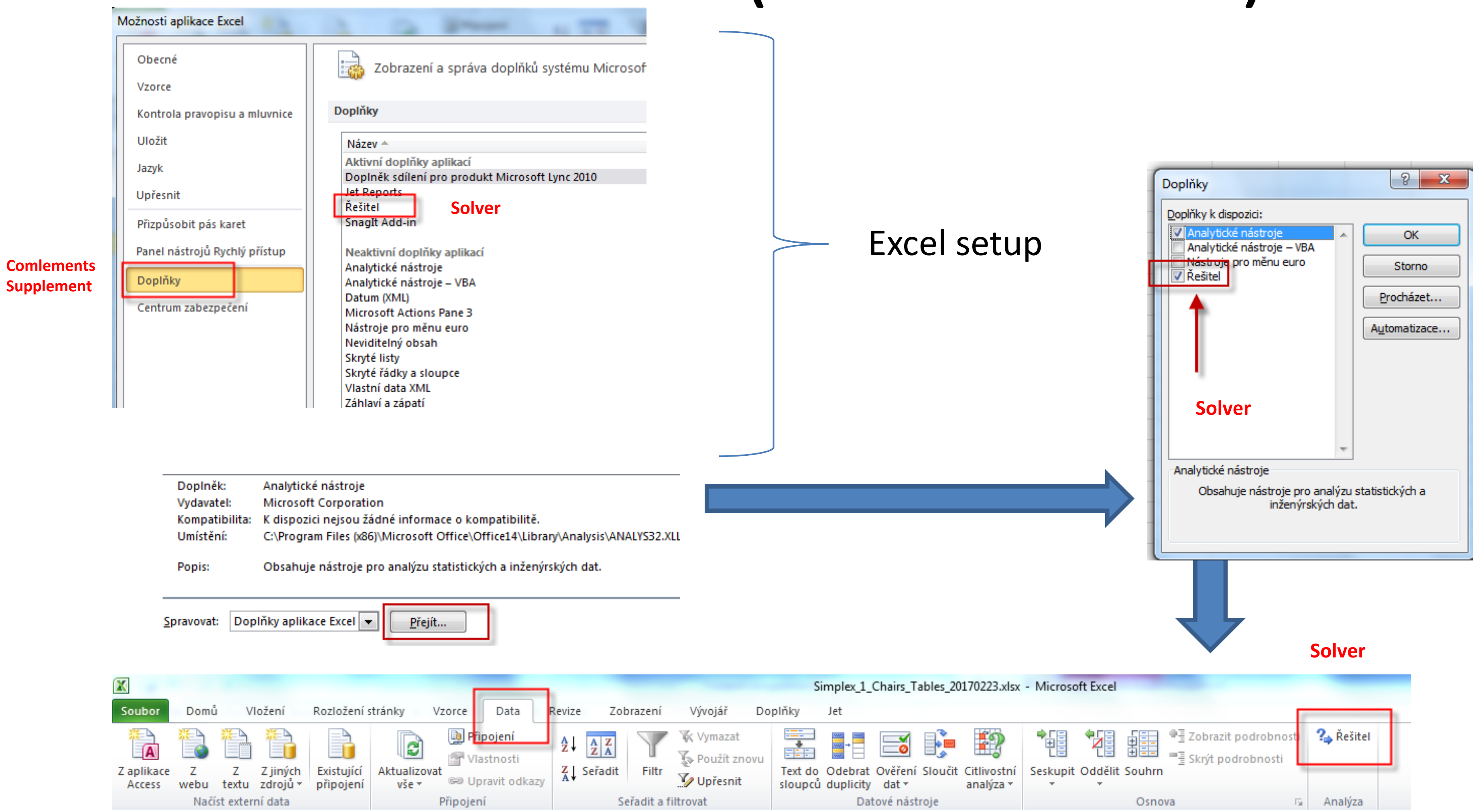
(optimal Return meets the point B – see next slide)

Graphical solution



I apologize for the inappropriate graphic expression….

Use of Solver (Czech EXCEL)



Use o solver (see actual Excel formulas on one of the next slides)

x1 x2

	Dish	Mug	Total	Capacity
Variables (x1, x2)	0	0		
Return	40	50	0	
Material	4	3	0	120
Work	1	2	0	40

=D7*D6+E7*E6

Z

=D10*D6+E10*E6

=D11*D6+E11*E6

Assignment entered in table

x1=Dish , x2=Mug, max 40 hour (B1), max 120 kg (B2)

Assignment

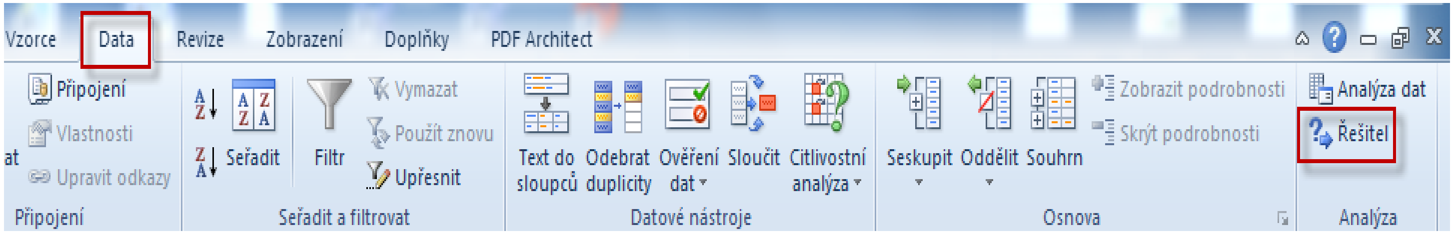
Target function Z = x1*c1 + x2*c2 = 40*x1+50*x2

4 * x1 + 3 *x2 =120 - capacity restrictions= max quantity of material =B1

1 * x1 + 2 *x2 = 40 -capacity restrictions by max work capacity=B2

Product	**Description**	**Work /hour**	**Material/pcs**	**Return/pcs**
Dish	x1	1	4	**40**
Mug	x2	2	3	**50**

Solver start



Use of Solver (Czech- not for MHP_AOPR)

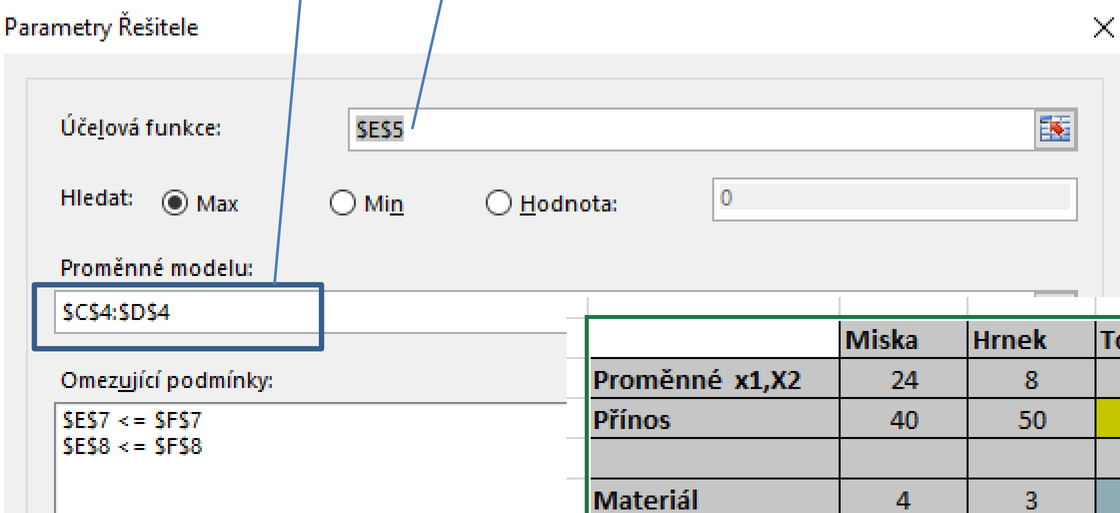
	A	B	C	D	E	F	G
1							
2							
3			Miska	Hrnek	Total	Kapacita	
4		Proměnné x1,X2	0	0			
5		Přínos	40	50	0		
6							
7		Materiál	4	3	0	120	
8		Práce	1	2	0	40	
9							

E5 = =D7*D6+E7*E6

Z = x1*c1 + x2*c2 = 40*x1+30*x2

E7=C7*C4+D7*D4=4*x1+3*x2=120

E8=C8*C4+D8*D4=x1+2*x2=40



	Miska	Hrnek	Total	Kapacita
Proměnné x1,X2	24	8		
Přínos	40	50	1360	
Materiál	4	3	120	120
Práce	1	2	40	40

Use of solver (for MPH_AOPR)

F7 = =D7*D6+E7*E6

	Dish	Mug	Total	Capacity
Varaibles X1, X2	0	0		
Return	40	50	0	
Material	4	3	0	120
Work	1	2	0	40

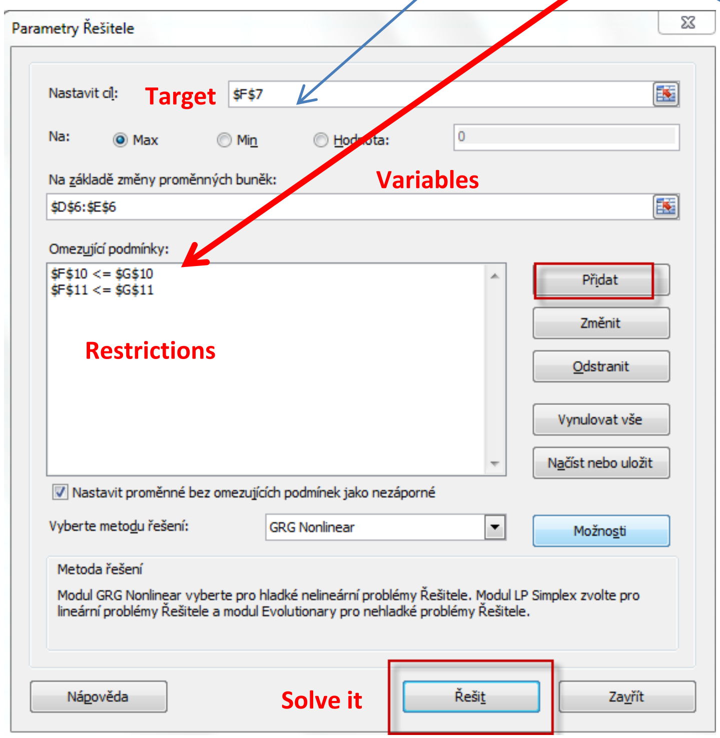
Z = x1*c1 + x2*c2 = 40*x1+30*x2

F10=D10*D6+E10*E6=4*x1+3*x2=120

F11=D11*D6+E11*D6=x1+2*x2=40

=D10*D6+E10*E6

=D11*D6+E11*E6



	Dish	Mug	Total	Capacity
Varaibles X1, X2	24	8		
Return	40	50	1360	
Material	4	3	120	120
Work	1	2	40	40

Využití Řešitele (use of Solver)

Microsoft Excel 15.0 Citlivostní sestava
List: [Simplex_1_Misky_Hrnky_Chairs_Tables_20170228.xlsx]List1
Sestava vytvořena: 9. 3. 2017 16:19:56

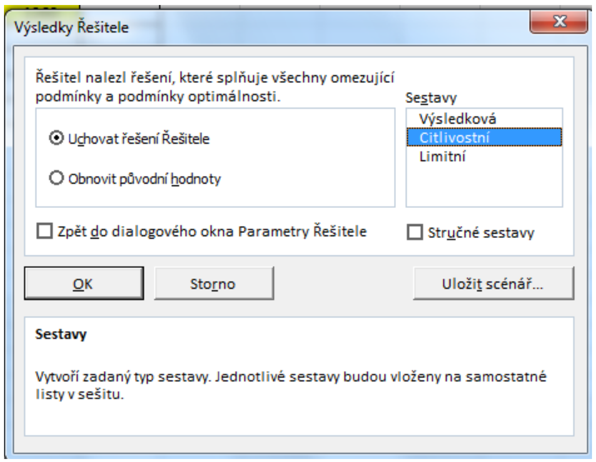
Proměnné

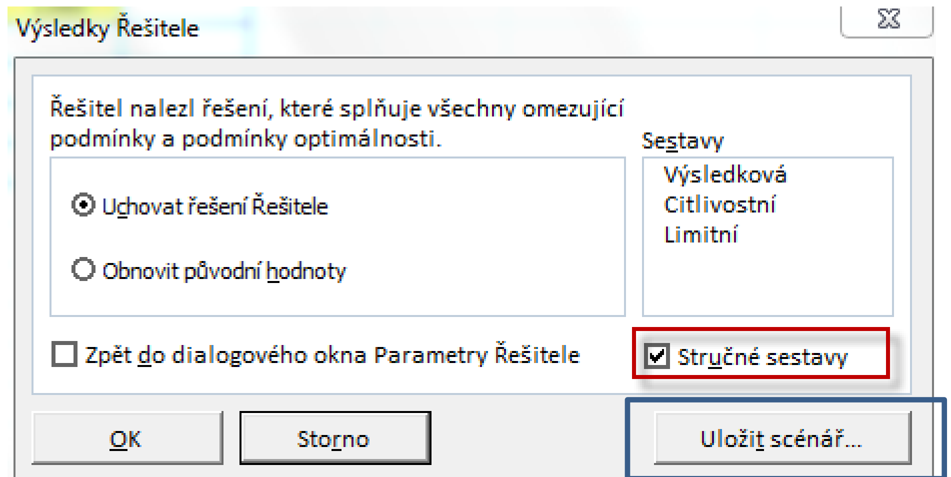
Levá strana omezující podmínky	Název	Konečná Hodnota	Redukovaná náklady	Účelová funkce koeficient	Povolený nárůst	Povolený pokles
C4	Proměnné x1,X2 Miska	24	0	40	26,66666667	15
D4	Proměnné x1,X2 Hrnek	8	0	50	30	20

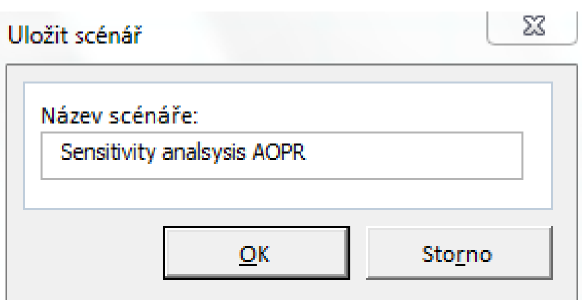
Omezující podmínky

Levá strana omezující podmínky	Název	Konečná Hodnota	Stínová cena	Pravá strana omezující podmínky	Povolený nárůst	Povolený pokles
E7	Materiál Total	120	6	120	40	60
E8	Práce Total	40	16	40	40	10

Use of Solver (English)







Microsoft Excel 14.0 Citlivostní sestava
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Sestava vytvořena: 2.11.2017 8:49:10

Proměnné buňky

Buňka	Název	Konečná Hodnota	Snížené Gradient
D6	Varaibles X1, X2 Dish	24	0
E6	Varaibles X1, X2 Mug	8	0

Omezující podmínky

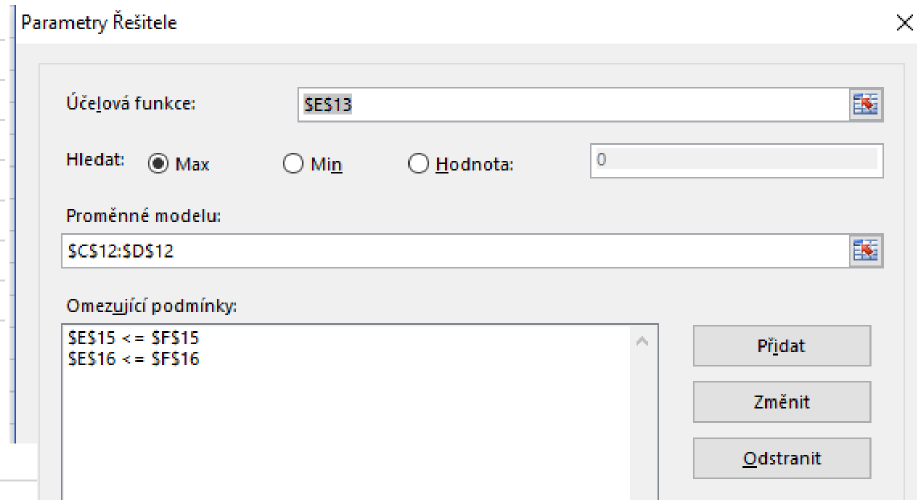
Buňka	Název	Konečná Hodnota	Lagrangeův multiplikátor
F10	Material Total	120	6
F11	Work Total	40	16

New Excel List

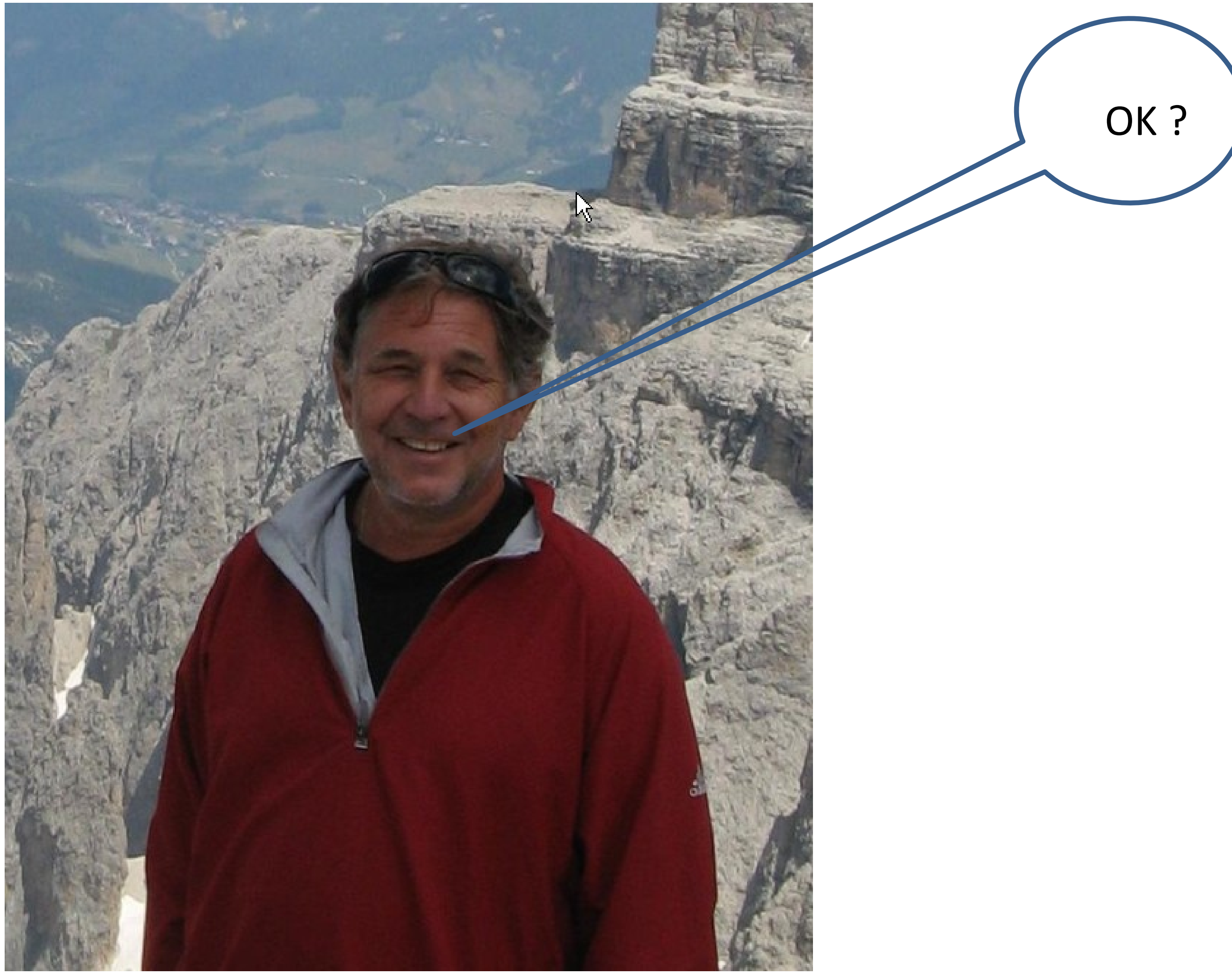
Změna úlohy- jiné výnosy jiná omezení typu práce na dvou strojích a jejich kapacitní omezení

(Change of parameters- not necessary for MPH_AOPR !!!!!)

	Miska	Hrnek	Total	Kapacita
Proměnné x1,x2	0	0		
Přínos	40	50	0	
Stroj 1	7	5	0	200
Stroj 1	5	5	0	400



	Miska	Hrnek	Total	Kapacita
Proměnné x1,x2	0	40		
Přínos	40	50	2000	
Stroj 1	7	5	200	200
Stroj 1	5	5	200	400


OK ?