

Little's law basics

Ing. J.Skorkovský,CSc.
KPH-ESF-MU BRNO

Different times used in Little's law

(used in different resources)

- Lead time (LT)
- Flow time (FT)
- Cycle time (CT)

It is **essential** to define precisely all mentioned times to better understand the principles of Little's law- see time variables explanation in the following slides

$$WIP = TH \times CT$$

Definitions – time variables

- **CT**=average time from when the job is released into the station (machine or line) to when it exits
- **LT**=management **constant** indicating the time allotted (assigned) for production of a part on a given routing – used often in planning
- **CT = FT** (in different publications they use **FT** instead of **CT**), where FT stands for=Flow Time
- **CT=Throughput Time** (in different publications they use Throughput Time instead of CT)

Other two variables of Little's law

- **WIP**= Work in Process (Work in Progress)
- **TH**=Throughput=Throughput Rate = average output of the production process (machine, workstations) per unit time

Routing (Business Central)

1000 · Bicycle

General

No.: ...

Description:

Type: ▾

Status: ▾

Search Description:

Version Nos.:

Active Version:

Last Date Modified:

Lines

Operation ▾ Find Filter Clear Filter

Operati... No.	Type	No.	Description	Setup Time	Run Time	Wait Time	Move Time
10	Work Center	100	Wheel assembly	110	12	0	0
20	Machine Ce...	120	Chain assembly	15	15	0	0
30	Machine Ce...	130	Final assembly	10	20	0	0
40	Machine Ce...	110	Control	10	8	0	0

10 pcs of Bicycle)

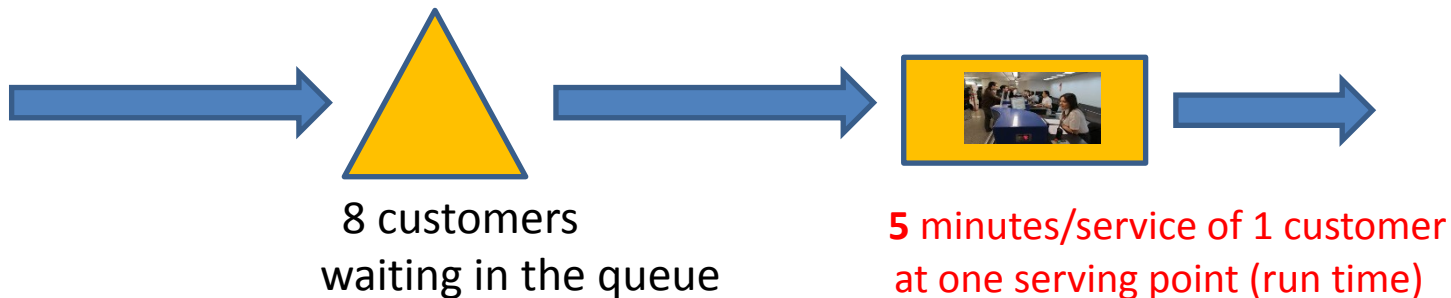
$$110+15+10+10 +10*(12+15+20+8)$$

Facilities



One example to be solved

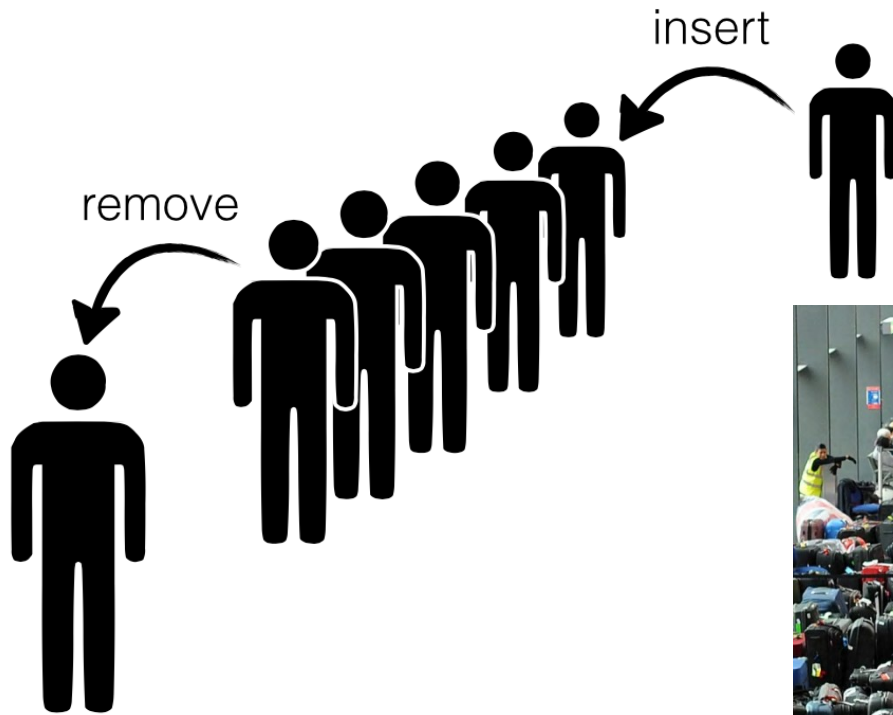
- **30** customers/hour – (max capacity of the facility). The **Facility** may be a hairdresser, fast food, bank counter, airport checking, massages and so on.
- 8 customers waiting in the queue (buffer) – **see next slide**
- **5 minutes** take service of one customer



- **You need to remove all times that do not add value to the process**
- 1 **serving point** = **12** customers per hour -> (**60** minutes/**5** = **12**), for **30** customers/hour the needed **2.5** = **30/12** of serving points

$$WIP = TH \times CT$$





Buffering

A [buffer](#) is used to temporarily keep data (customers in our model) while it is being moved from one place to another. A buffer often adjusts timing by implementing a queue or **FIFO** algorithm in memory, simultaneously writing data into the queue at one rate and reading it at another rate.

Questions to be answered

- How long does the customer wait in the queue?
- How many people on an average can be served at once?
- How many customers are in the facility just in time (both pending and those served by facility staff)?
- What is the average time of the "flow" of the customer by the facility (wait and service)

Simplifying conditions :

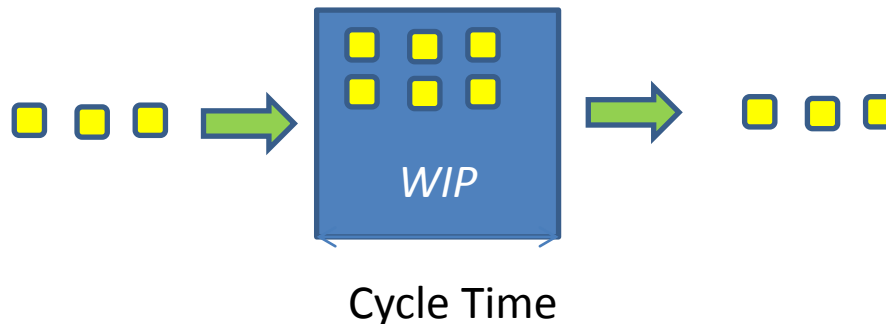
"Input flow" (average) = "Output flow" (average)- **steady flow** (see video at the end of this PWP)

We do not consider fluctuation due to averaging (see flip a coin situation)

Key metrics and variables (completion of definitions)

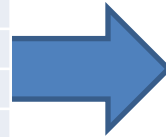
- **CT = Cycle Time** (how long takes the whole process) = **5 minutes take the service of one customer**
- **Work In Progress = WIP** (how many units are in process at any moment = **Work In Progress**)
- **Throughput = TH** (how many customers/time unit) – e.g. *In our case 30/hour = 30/60*

- These metrics are tied together by Little's law **WIP = TH x CT**
 - Our example : **TH = 30 customers/hour**, service of one takes **5 minutes**, **WIP = (30/60) * (5/1) = (1/2) * 5 = 2,5**
- ↑
TH in minutes



Solution (home study, C=Customer)

Proces	WIP	TH (Cust/hour)	CT
Buffer	8	30	
Service		30	5
Total		30	



Proces	WIP	TH (Cust/hour)	CT (min/Cust)	Time
Buffer	8	30	5	
Service		30	5	5
Total		30	5	

$$WIP = TH \times CT$$

Proces	WIP	TH (C/h)	CT (min/C)	Time
Buffer	8	30	5	
Service	2,5	30	5	5
Total	10,5	30	5	

$$CT = WIP / TH \text{ (third column is only see the units)}$$

Proces	WIP	TH (C/h)	CT (/min/C)	Time
Buffer	8	30	5	16
Service	2,5	30	5	5
Total	10,5	30	5	21

Input data (from previous slides)

30 customers/hour -> (max Capacity of the facility) = Throughput = TH

8 customers are waiting in the queue = WIP = buffer

5 minutes per customer service = CT

$WIP = TH \times CT = ((30/60) * 5) = (3 * 5) / 6 = 2,5$, so how many customers can be served at the same time and total quantity is after that $10,5 = 8,0$ (queue) + $2,5$ and then:

$CT = WIP / TH = 8 / (3/6) = (8 * 6) / 3 = 48/3 = 16$ (as long as the customer waits in the queue=buffer) and finally for control $CT = 5 = (2,5 / (3/6)) = 2,5 * 6/3 = 15/3$ is the service time (already entered). So total time is $16+5=21$

Questions

- How long does the customer wait in the queue? -> **16 minutes**
- How many average people can be served at once? -> **>2,5 customers**
- How many customers are in the facility just in time (both pending and those just served by facility staff)? **10,5 customers**
- What is the average time of the "flow" of the customer by the facility (wait and service) -> **21 minutes**

Little's law-2nd part

Skorkovský ,KPH,ESF.MU

Based on resource : Factory Physics (Hopp and Spearman)

Little's law - definition (formula)

- Fundamental relationships among :
 - WIP (Work In Process)
 - Cycle Time (CT)
 - Throughput (T or sometimes TH)

- Formula

$$WIP = TH \times CT$$

- Can be applied to :
 - Single machine station
 - Complex production line
 - Entire plant

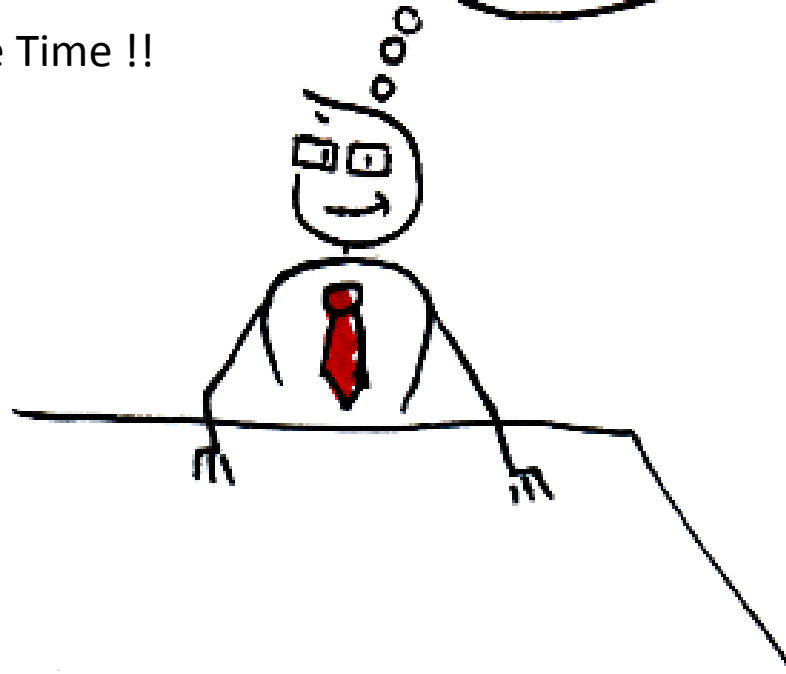
Relationships among these variables will serve to se clearly precise (quantitative) description of behaviour of the single production line . It helps user to use a given scale to benchmark actual production systems

I finally figured it out !!!!

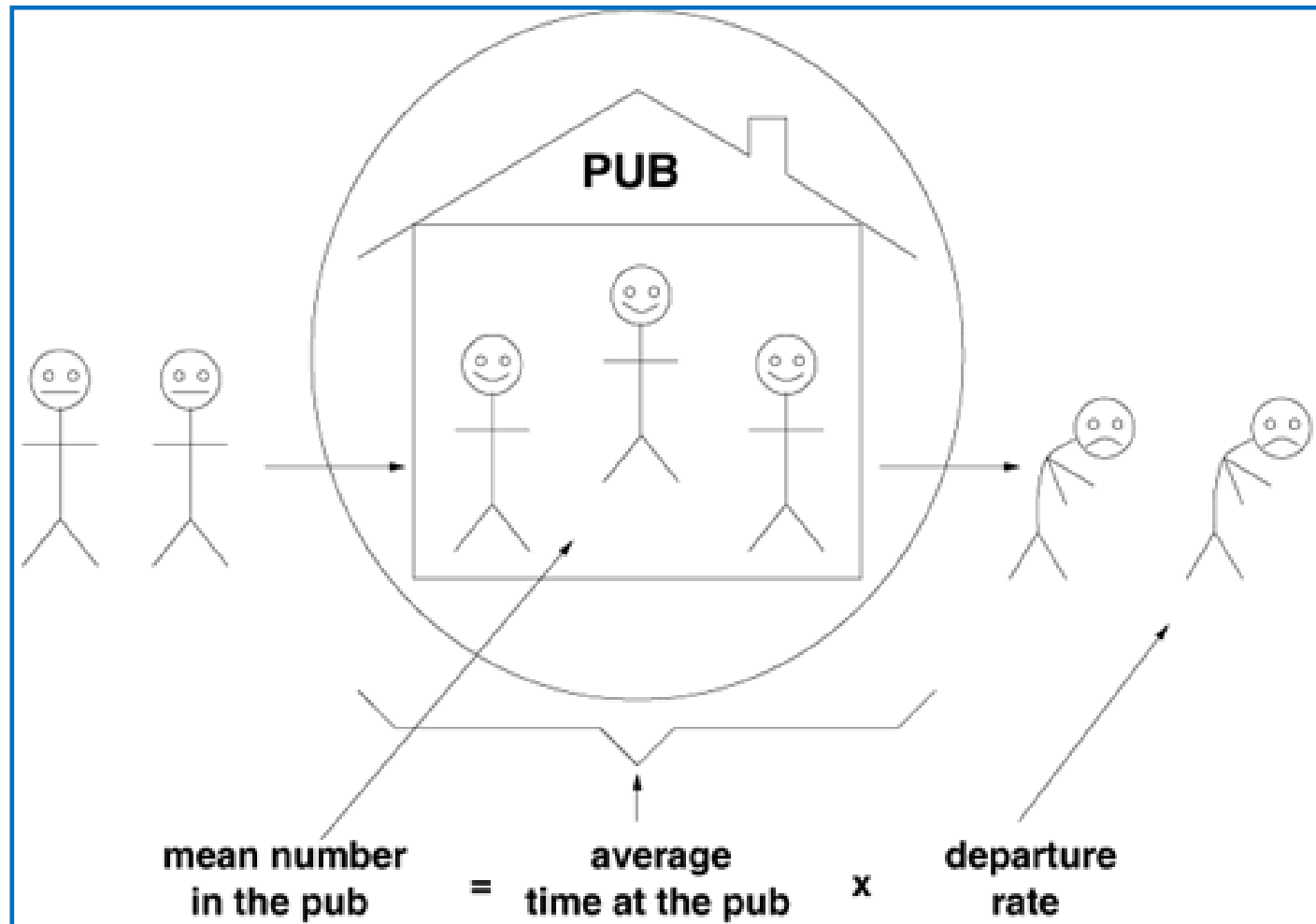
Little's Law

$$\text{avg. Lead Time} = \frac{\text{avg. Work in Progress}}{\text{avg. Throughput}}$$

Avg. Lead Time = Cycle Time !!



Daily application of the law....



Definition of basic parameters (supplements)

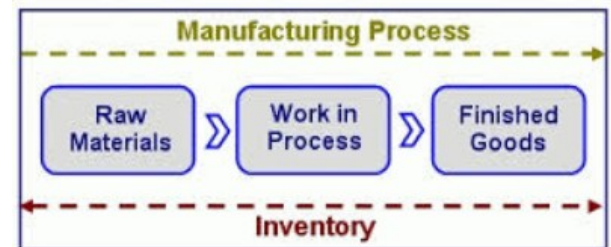
- **Throughput (Throughput rate, TH)** : production per unit time that is sold (see TOC definition)
- If **TH** is measured in **cost dollars** , it is typically called :

Cost of good sold (COGS)

- **The upper limit** of **TH** in the production process is the **capacity**
- If you release more raw material above the capacity of the line (machine), **the system becomes unstable → WIP goes up !! See later ...**

Definition of basic parameters (supplements)

- **WIP (Work In Process)** : inventory between start and end points of the product routing
- **WIP** can be used as one parameter to calculate (measure) an **efficiency**
- **Efficiency** can be defined as **Turnover Ratio** = TH/FGI for warehouses or $\text{TH}/(\text{FGI}+\text{WIP})$ for production plants where **FGI**=Finished goods inventory
- **WIP** : inventory still in line
- **FGI** : inventory waiting for dispatch (shipping)



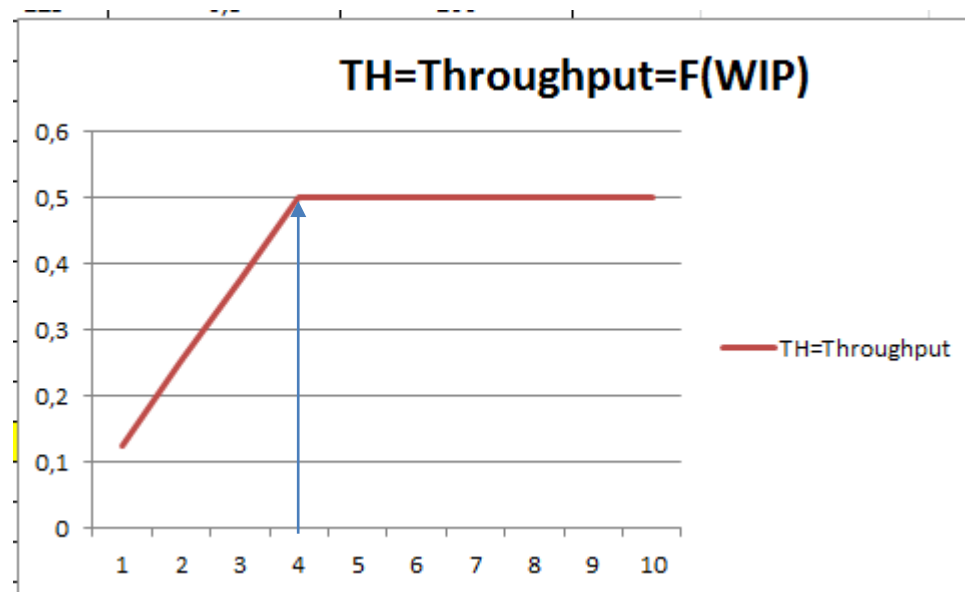
Definition of basic parameters (supplements-already mentioned)

- **CT (Cycle Time) or so called Throughput Rate**: average time from the release of the job of the beginning of the routing until it reaches an inventory point at the end of the routing or time that part spends as a **WIP** in the line.
- **LT (Lead Time)** : managerial constant used for planning of production
- **Service Level** (especially for M-T-O lines, where the plant has to satisfy orders with specific due dates) :

Service level $P\{\text{Cycle time} \leq \text{Lead Time}\}$

M-T-O =Make to Order

Best case performance



Resources

• **WIP=TH * CT**

- *Source : Factory Physics, Wallace J Hopp and Mark L. Spearman ; ISBN 13: 978-1-57766-739-1 or ISBN 10 :1-57766-739-5*

<http://www.factoryphysics.com/principle/littleslaw.htm>

Example 1 (home study)

- **Estimating Waiting Times:** If are in a grocery queue behind 10 persons and estimate that the clerk is taking around 5 minutes/per customer, we can calculate that it will take us 50 minutes (10 persons x 5 minutes/person) to start service.
- This is essentially **Little's law**. We take the number of persons in the **queue** (10) as the "**inventory**".
- The inverse of the average time per customer (1/5 customers/minute) provides us the rate of service or the Throughput.
- Finally, we obtain the waiting time as equal to number of persons in the queue divided by the processing rate $10/(1/5) = 50$ minutes).

Example 2 (home study)

- **Planned Inventory Time:** Suppose a product is scheduled so that we expect it to wait for 2 days in finished goods inventory before shipping to the customer. This two days is called **planned inventory time** and is sometimes used as protection against system variability to ensure high delivery service. Using Little's law the total size of inventory in finished goods can be computed as :
- **FGI = throughput × planned inventory time**

Youtube examples (6 minutes)

- <http://www.youtube.com/watch?v=VU8TUSnQ-vw>
- <http://www.youtube.com/watch?v=rtGihR-bm-U>