

#### **Chapter 7**

#### Revealed Preference

### **Revealed Preference Analysis**

Suppose we observe the demands (consumption choices) that a consumer makes for different budgets. This reveals information about the consumer's preferences. We can use this information to ...

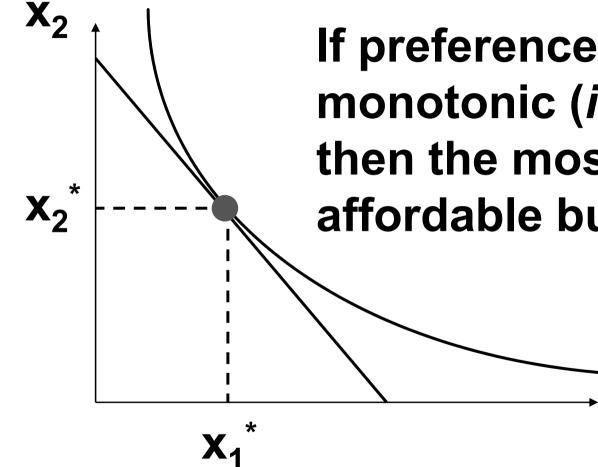
## **Revealed Preference Analysis**

- -Test the behavioral hypothesis that a consumer chooses the most preferred bundle from those available.
- -Discover the consumer's preference relation.

# Assumptions on PreferencesPreferences

- do not change while the choice data are gathered.
- -are strictly convex.
- -are monotonic.
- Together, convexity and monotonicity imply that the most preferred affordable bundle is unique.

### Assumptions on Preferences



If preferences are convex and monotonic (*i.e.* well-behaved) then the most preferred affordable bundle is unique.

Suppose that the bundle x\* is chosen when the bundle y is affordable. Then x\* is revealed directly as preferred to y (otherwise y would have been chosen).

**X**<sub>2</sub>

The chosen bundle x<sup>\*</sup> is revealed directly as preferred to the bundles y and z.

Xı

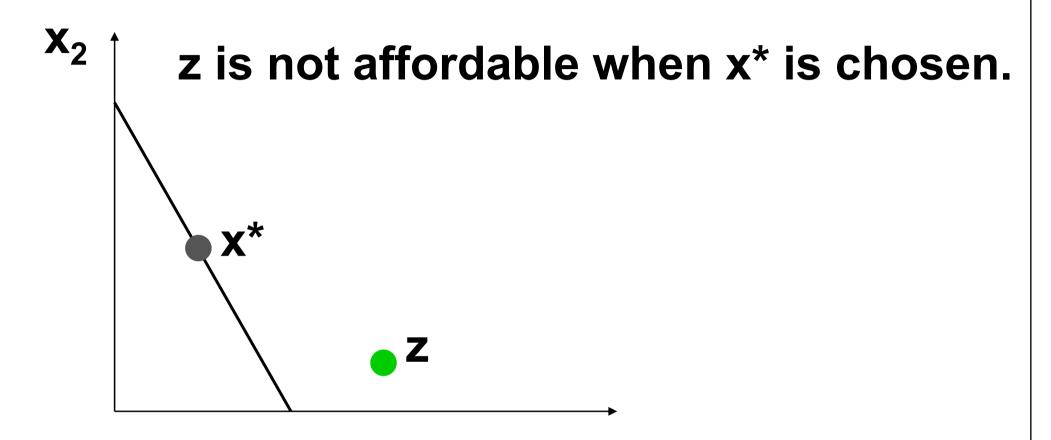
## That x is revealed directly as preferred to y will be written as

x ≻ y. D

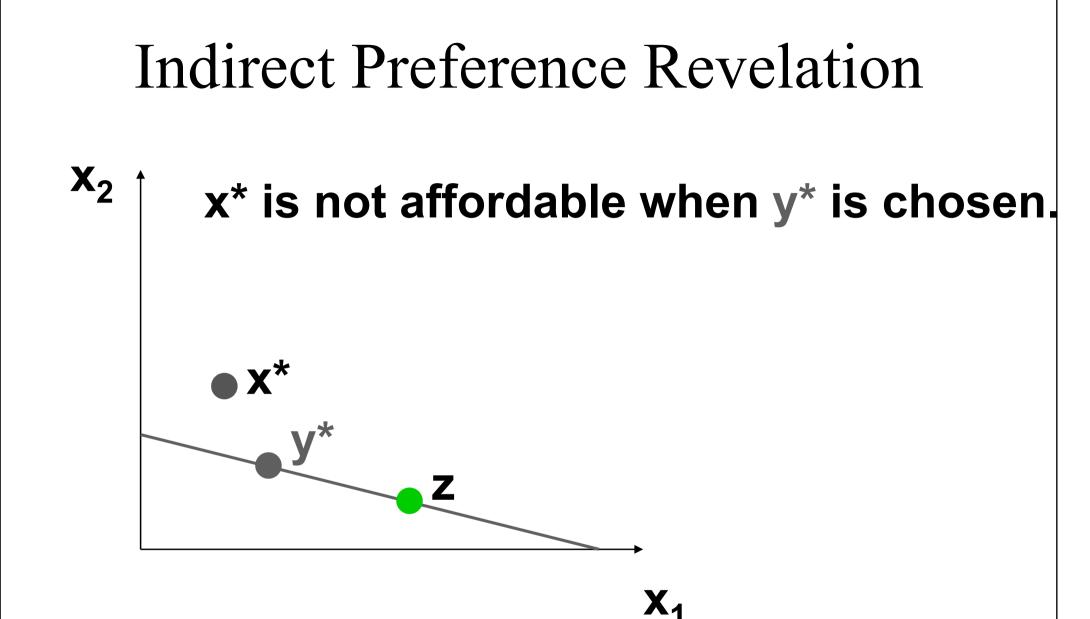
Suppose x is revealed directly preferred to y, and y is revealed directly preferred to z. Then, by transitivity, x is revealed indirectly as preferred to z. Write this as

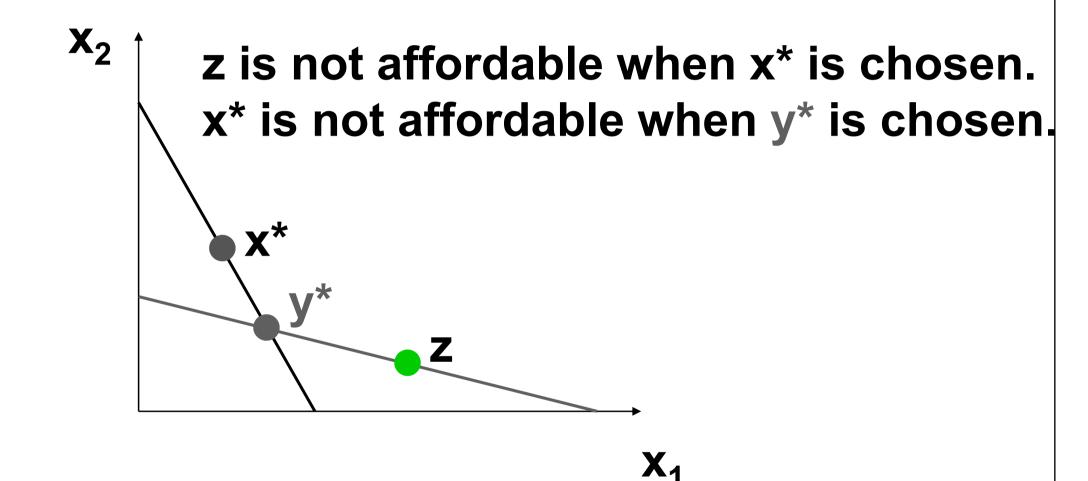
so 
$$\mathbf{x} \succeq \mathbf{y}$$
 and  $\mathbf{y} \succeq \mathbf{z} \implies \mathbf{x} \succeq \mathbf{z}$ .

x ≻ z



**X**<sub>1</sub>

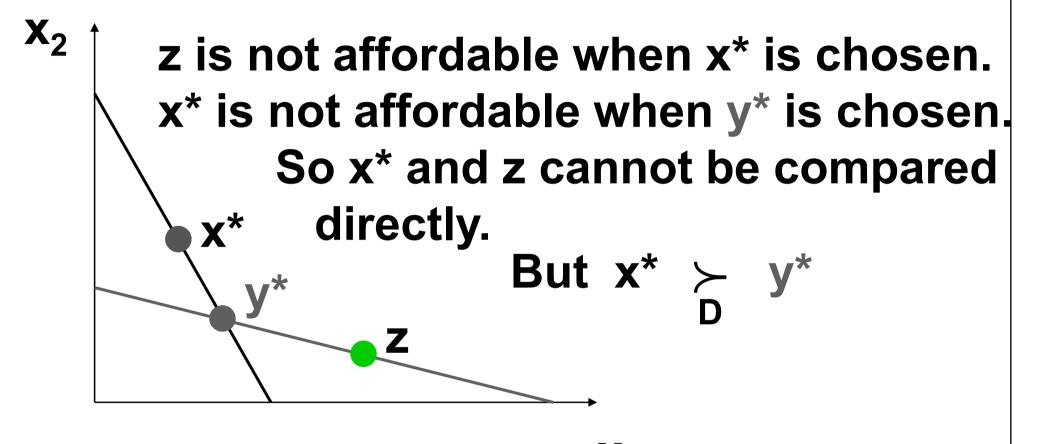




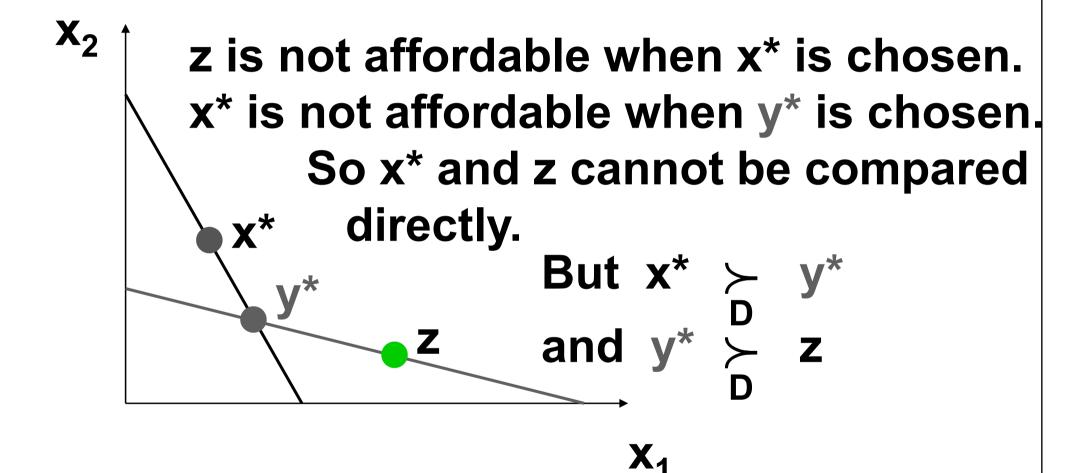
**X**<sub>2</sub>

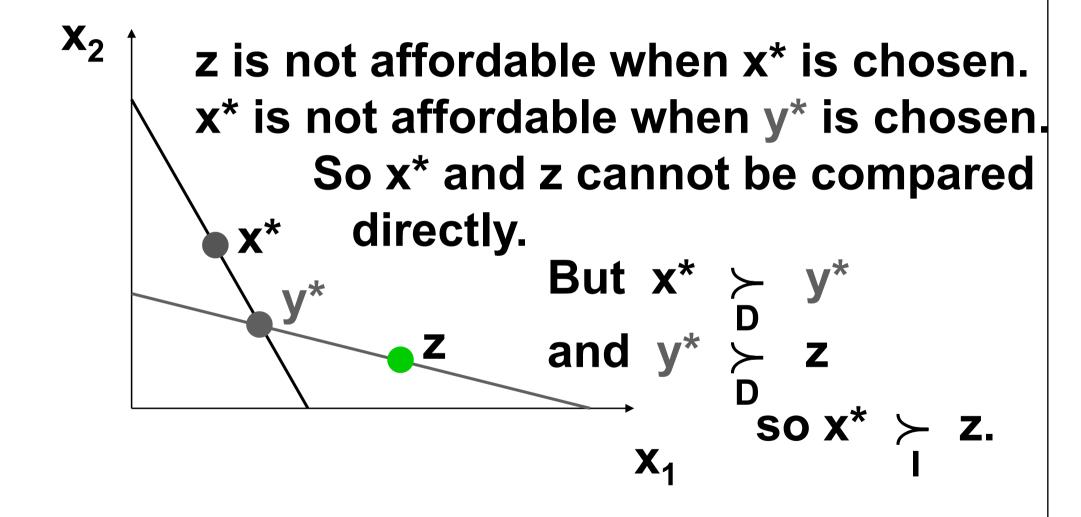
z is not affordable when x\* is chosen. x\* is not affordable when y\* is chosen. So x\* and z cannot be compared x\* directly.

**X**<sub>1</sub>



**X**<sub>1</sub>





Two Axioms of Revealed Preference

To apply revealed preference analysis, choices must satisfy two criteria -- the Weak and the Strong Axioms of Revealed Preference. The Weak Axiom of Revealed Preference (WARP)

If the bundle x is revealed directly as preferred to the bundle y then it is never the case that y is revealed directly as preferred to x; *i.e.* 

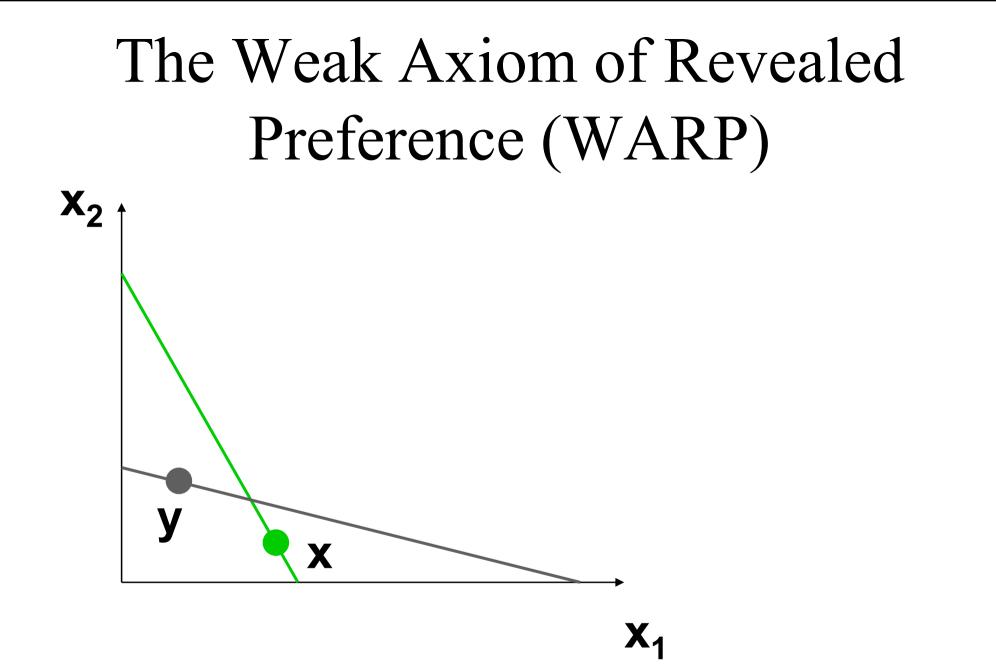
$$\mathbf{x} \succeq_{\mathbf{D}} \mathbf{y} \implies \operatorname{not}(\mathbf{y} \succeq_{\mathbf{D}} \mathbf{x}).$$

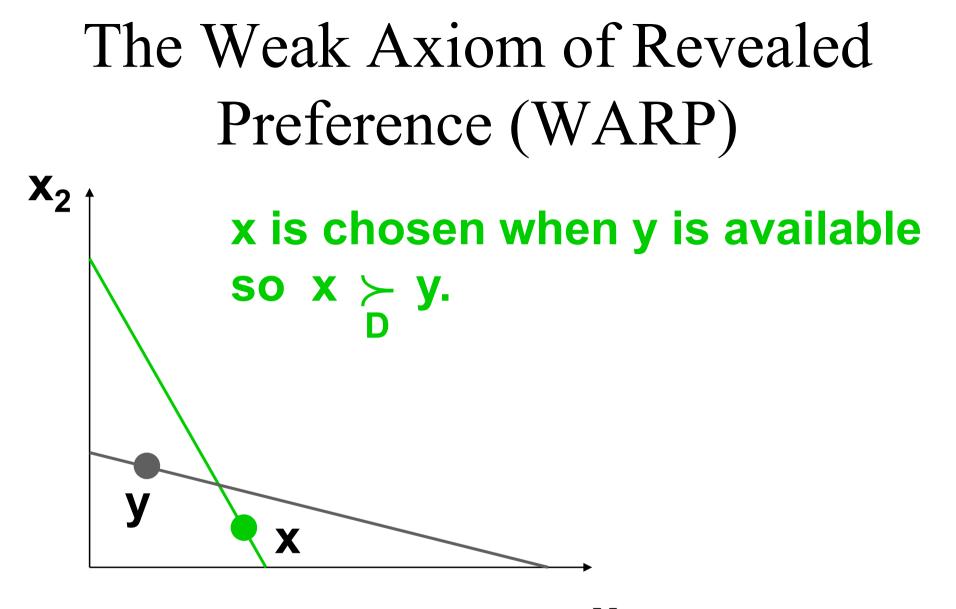
## The Weak Axiom of Revealed Preference (WARP)

- Choice data which violate the WARP are inconsistent with economic rationality.
- The WARP is a necessary condition for applying economic rationality to explain observed choices.

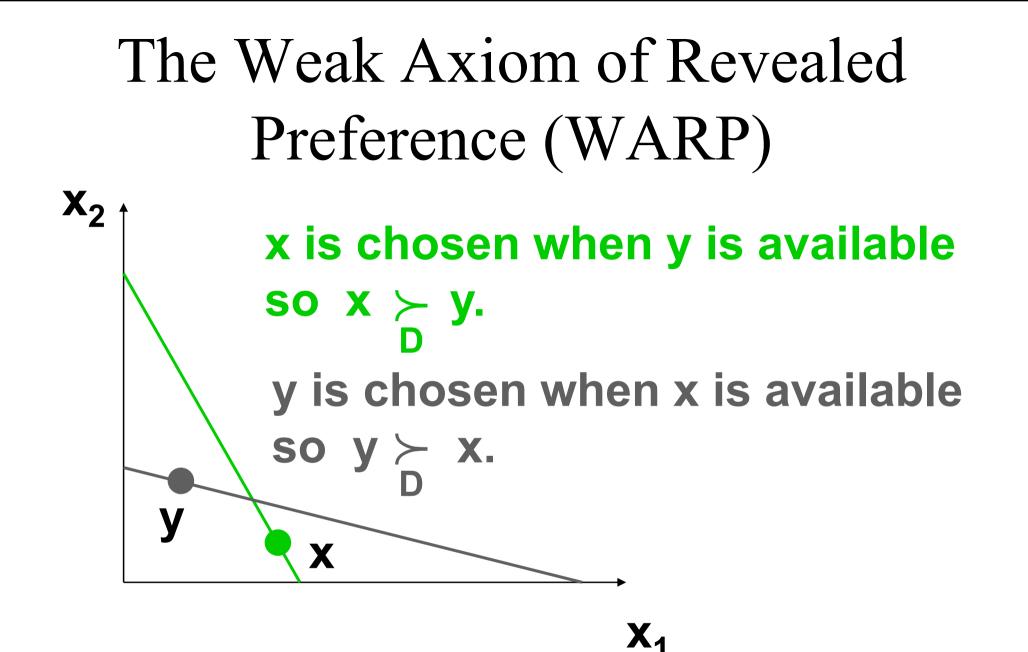
## The Weak Axiom of Revealed Preference (WARP)

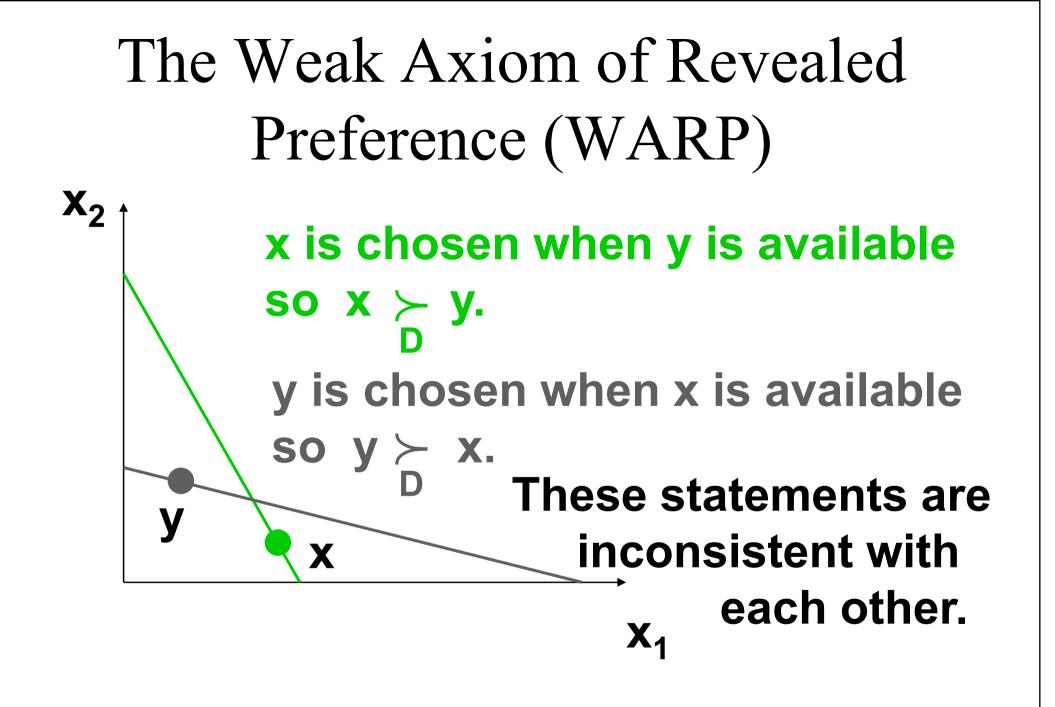
#### What choice data violate the WARP?





**X**<sub>1</sub>





- A consumer makes the following choices:
  - -At prices  $(p_1, p_2)=(\$2, \$2)$  the choice was  $(x_1, x_2) = (10, 1)$ .
  - -At  $(p_1, p_2)=(\$2, \$1)$  the choice was  $(x_1, x_2) = (5, 5)$ .
  - -At  $(p_1, p_2)=(\$1, \$2)$  the choice was  $(x_1, x_2) = (5, 4)$ .
- Is the WARP violated by these data?

<b>Choices</b> <b>Prices</b>	(10, 1)	(5, 5)	(5, 4)		
(\$2, \$2)	<b>\$22</b>	<b>\$20</b>	<b>\$18</b>		
(\$2, \$1)	<b>\$21</b>	<b>\$15</b>	<b>\$14</b>		
(\$1, \$2)	<b>\$12</b>	<b>\$15</b>	<b>\$13</b>		

<b>Choices</b> <b>Prices</b>	(10, 1)	(5, 5)	(5, 4)		
(\$2, \$2)	<b>\$22</b>	<b>\$20</b>	<b>\$18</b>		
(\$2, \$1)	<b>\$21</b>	<b>\$15</b>	<b>\$14</b>		
(\$1, \$2)	<b>\$12</b>	<b>\$15</b>	<b>\$13</b>		

Red numbers are costs of chosen bundles.

<b>Choices</b> <b>Prices</b>	(10, 1)	(5, 5)	(5, 4)		
(\$2, \$2)	<b>\$22</b>	<b>§20</b>	<b>§18</b>		
(\$2, \$1)	<mark>\$21</mark>	<b>\$15</b>	<b>\$14</b>		
(\$1, \$2)	<b>\$12</b>	<b>\$15</b>	<b>\$13</b>		

Circles surround affordable bundles that were not chosen.



Circles surround affordable bundles that were not chosen.

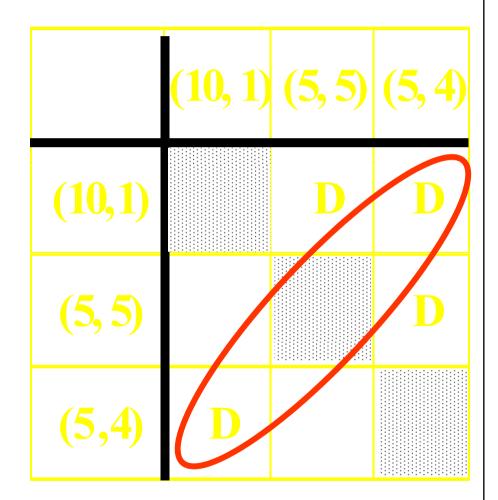


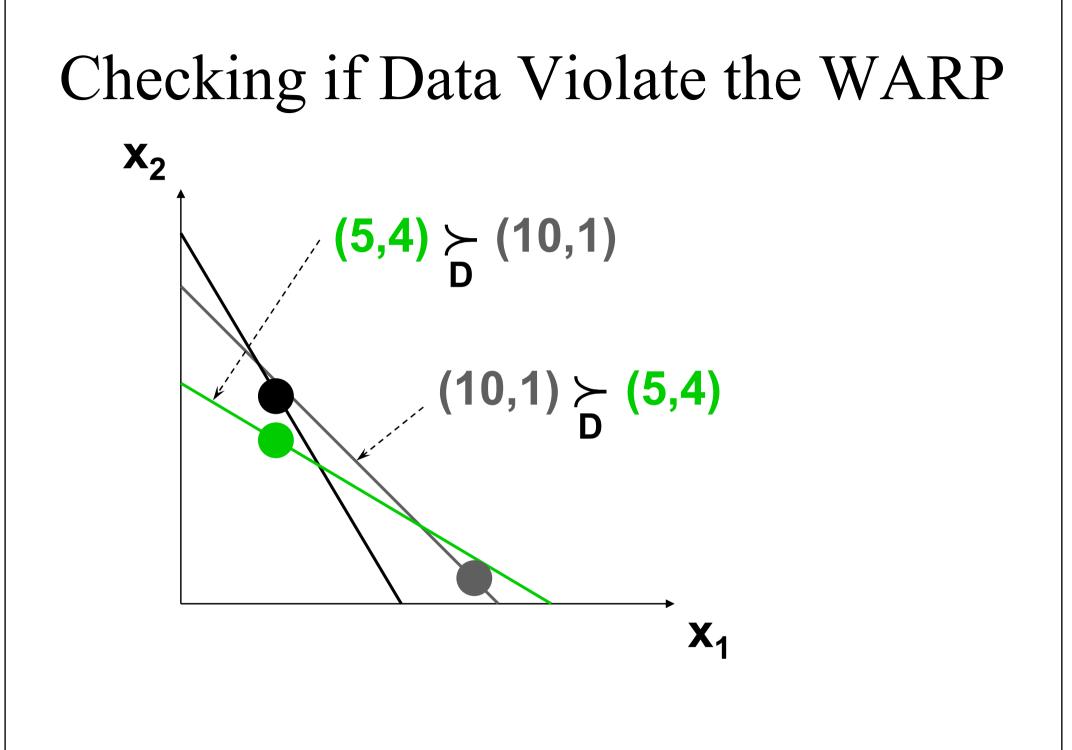
Circles surround affordable bundles that were not chosen.

<b>Choices</b> <b>Prices</b>	(10,1)	(5,5)	(5,4)			<b>(10, 1</b> )	(5, 5)	(5, 4)
(\$2,\$2)	<b>\$22</b>	<b>\$20</b>	<b>§18</b>	(10	),1)		D	D
(\$2,\$1)	<b>\$21</b>	<b>\$15</b>	<b>§14</b>	(5	, 5)			D
(\$1,\$2)	<b>§12</b>	<b>\$15</b>	<b>\$13</b>	(5	,4)	D		



(10,1) is directly revealed preferred to (5,4), but (5,4) is directly revealed preferred to (10,1), so the WARP is violated by the data.





## The Strong Axiom of Revealed Preference (SARP) ◆ If the bundle x is revealed (directly or indirectly) as preferred to the bundle y and x ≠ y, then it is never the case

that the y is revealed (directly or indirectly) as preferred to x; *i.e.* 

$$\begin{array}{c} \mathbf{x} \succ \mathbf{y} \text{ or } \mathbf{x} \succ \mathbf{y} \\ \mathbf{D} & \mathbf{v} \succ \mathbf{x} \text{ or } \mathbf{y} \succ \mathbf{x} \\ \mathbf{D} & \mathbf{v} \succ \mathbf{x} \text{ or } \mathbf{y} \succ \mathbf{x} \end{array}$$

## The Strong Axiom of Revealed Preference

## What choice data would satisfy the WARP but violate the SARP?

#### Consider the following data:

A: 
$$(p_1, p_2, p_3) = (1, 3, 10) \& (x_1, x_2, x_3) = (3, 1, 4)$$

B: 
$$(p_1, p_2, p_3) = (4, 3, 6)$$
 &  $(x_1, x_2, x_3) = (2, 5, 3)$ 

C: 
$$(p_1, p_2, p_3) = (1, 1, 5)$$
 &  $(x_1, x_2, x_3) = (4, 4, 3)$ 

A: (\$1,\$3,\$10) (3,1,4).

B: (\$4,\$3,\$6) (2,5,3).

C: (\$1,\$1,\$5) (4,4,3).

Choice Prices	A	B	С
A	<b>\$46</b>	<b>\$47</b>	<b>\$46</b>
B	<b>\$39</b>	<b>\$41</b>	<b>\$46</b>
С	<b>\$24</b>	<b>\$22</b>	<b>\$23</b>

Choices Prices	Α	В	С
Α	<b>\$46</b>	\$47	<b>\$46</b>
В	<mark>\$39</mark>	\$41	<b>\$46</b>
С	<b>\$24</b>	<mark>\$22</mark>	\$23



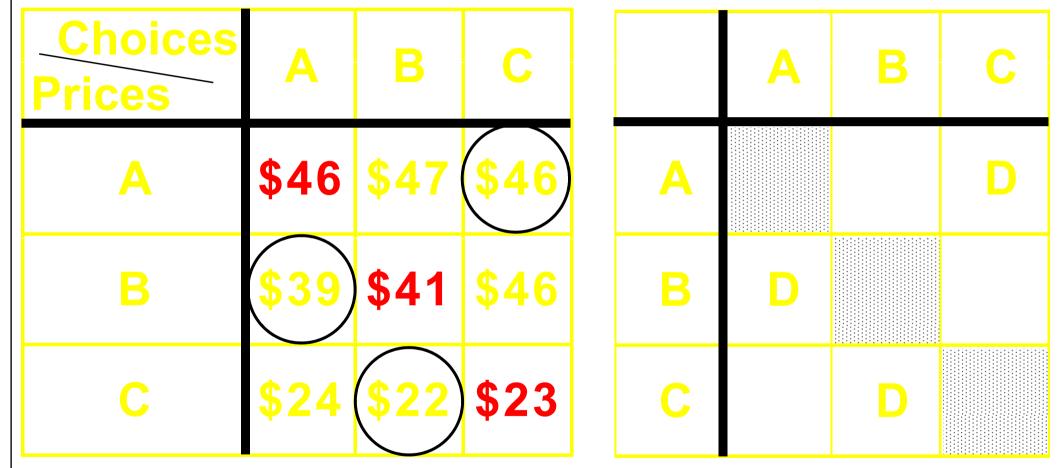
In situation A, bundle A is directly revealed preferred to bundle C;  $A \succeq C.$ 

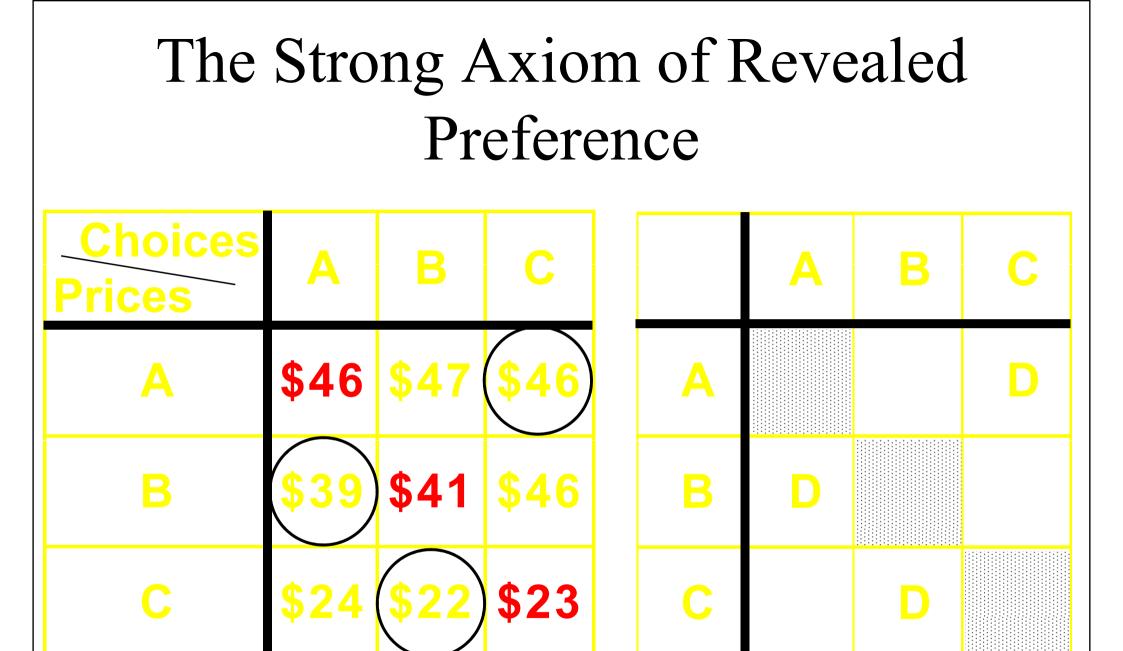


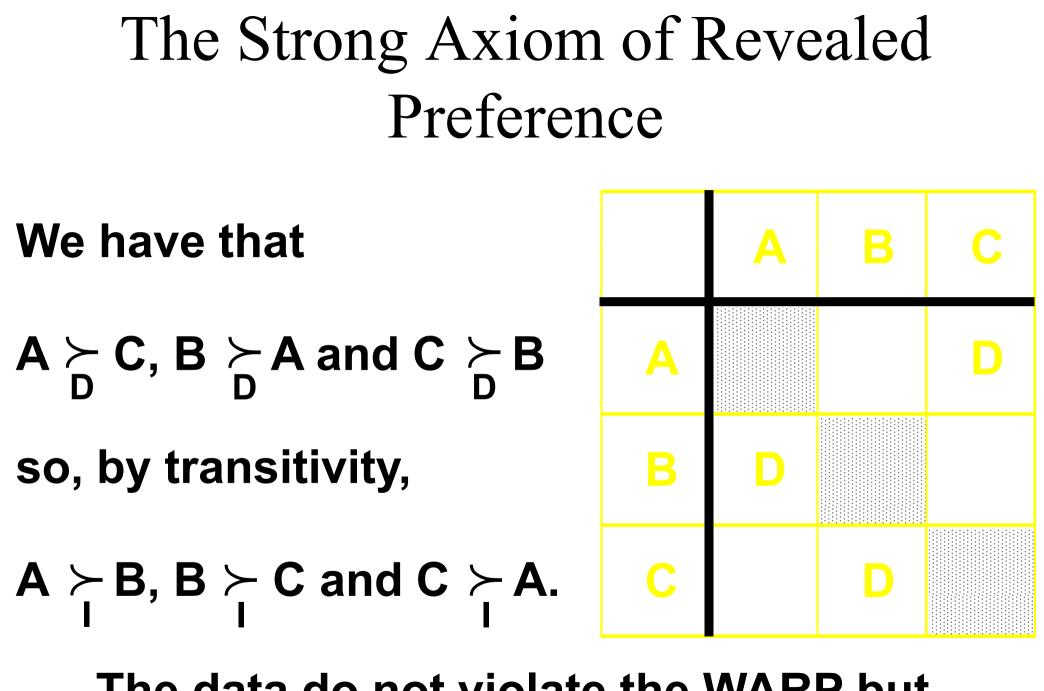
In situation B, bundle B is directly revealed preferred to bundle A;  $B \succeq A.$ 

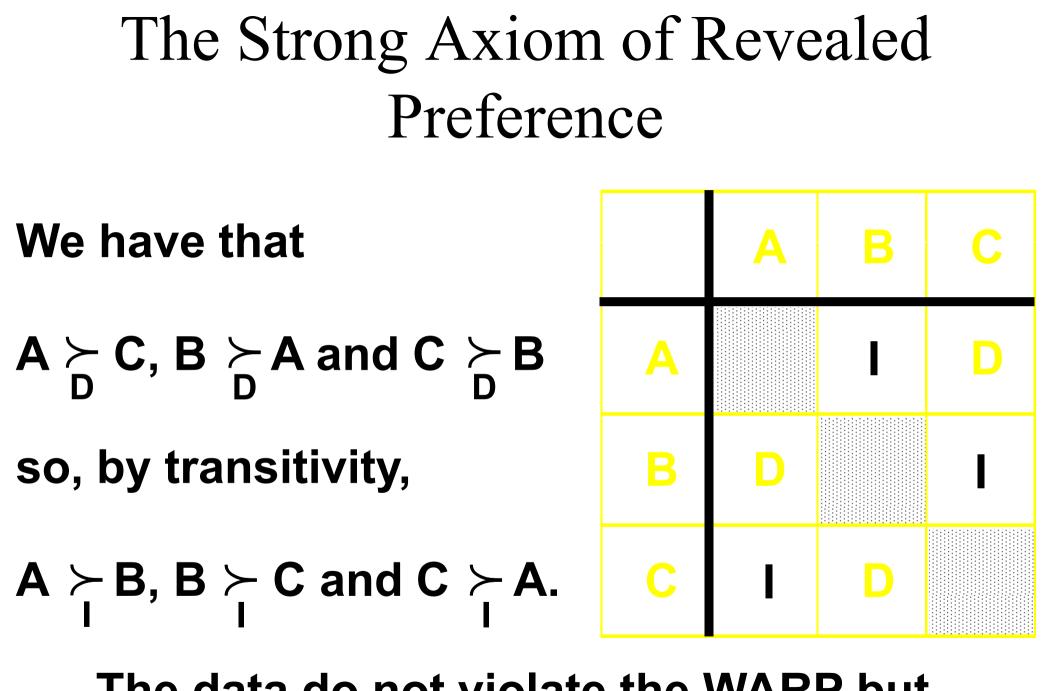


In situation C, bundle C is directly revealed preferred to bundle B;  $C \succeq B.$ 

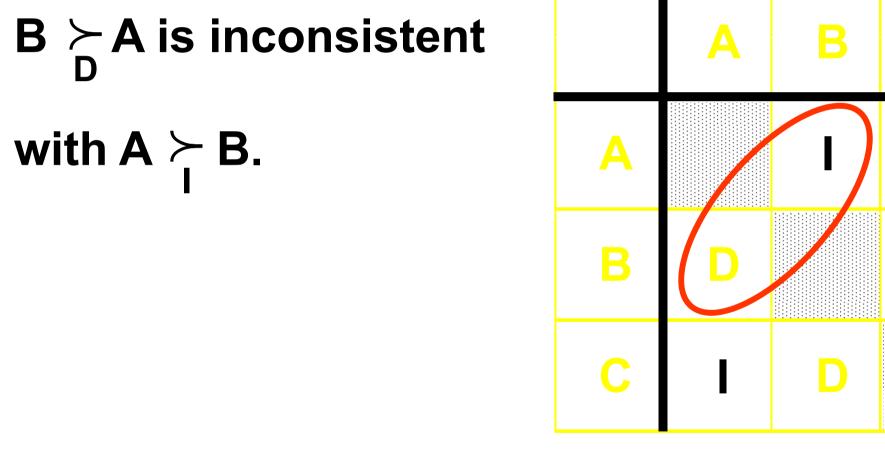


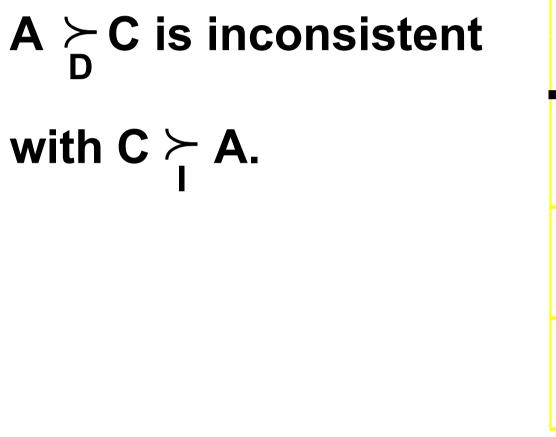


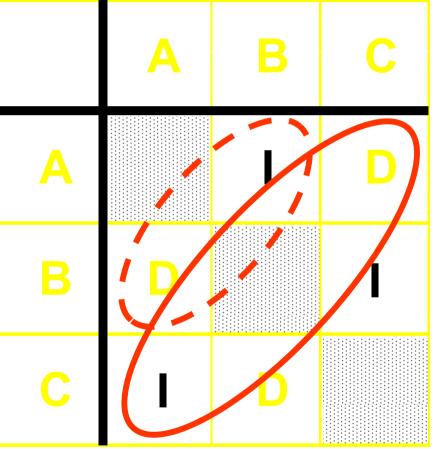


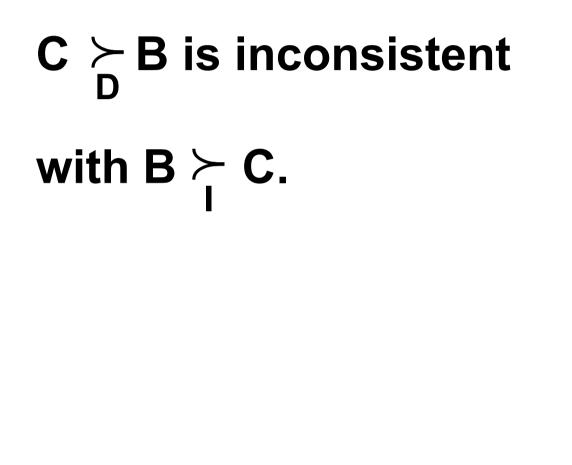


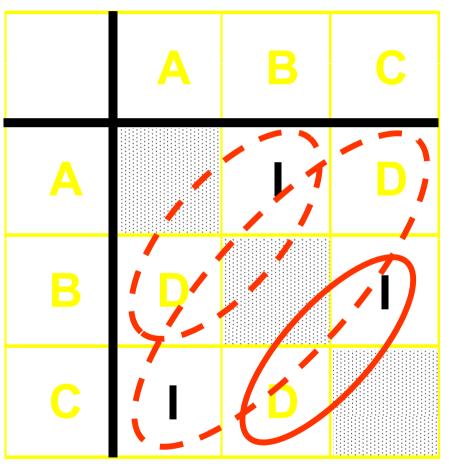
С



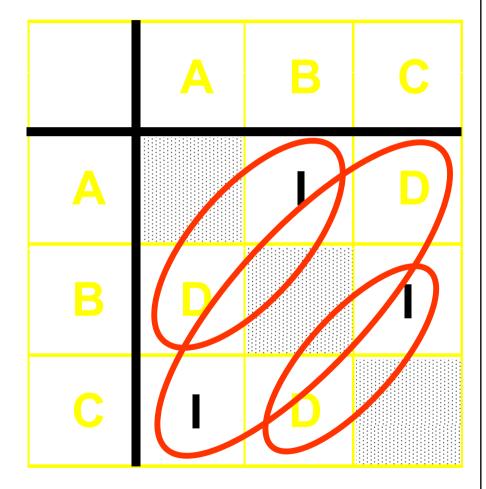








#### The data do not violate the WARP but there are 3 violations of the SARP.



- That the observed choice data satisfy the SARP is a condition necessary and sufficient for there to be a wellbehaved preference relation that "rationalizes" the data.
- So our 3 data cannot be rationalized by a well-behaved preference relation.

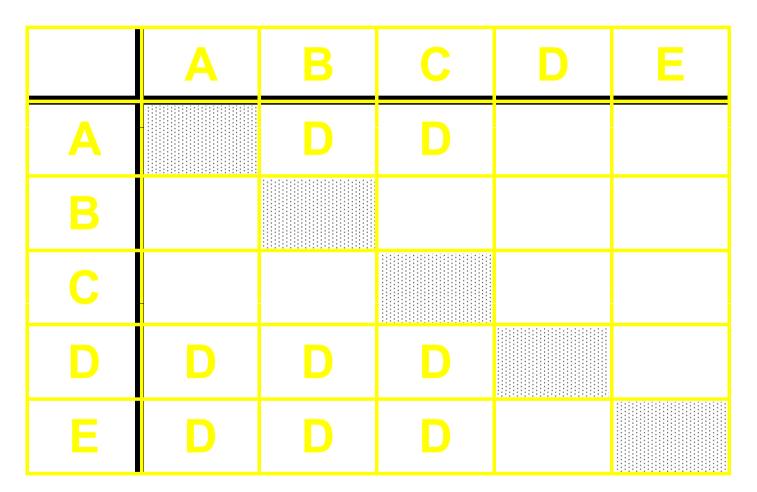
- Suppose we have the choice data satisfy the SARP.
- Then we can discover approximately where are the consumer's indifference curves.
- How?

#### Suppose we observe:

A:  $(p_1, p_2) = (\$1, \$1) \& (x_1, x_2) = (15, 15)$ B:  $(p_1, p_2) = (\$2, \$1) \& (x_1, x_2) = (10, 20)$ C:  $(p_1, p_2) = (\$1, \$2) \& (x_1, x_2) = (20, 10)$ D:  $(p_1, p_2) = (\$2, \$5) \& (x_1, x_2) = (30, 12)$ E:  $(p_1, p_2) = (\$5, \$2) \& (x_1, x_2) = (12, 30)$ .

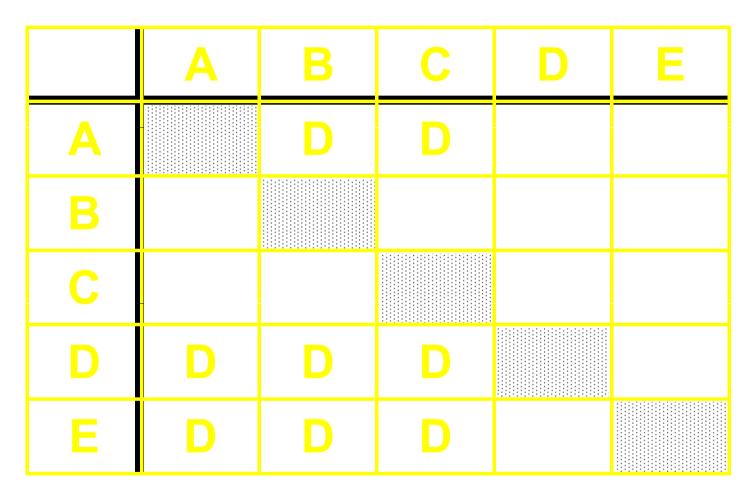
Where lies the indifference curve containing the bundle A = (15,15)?

The table showing direct preference revelations is:



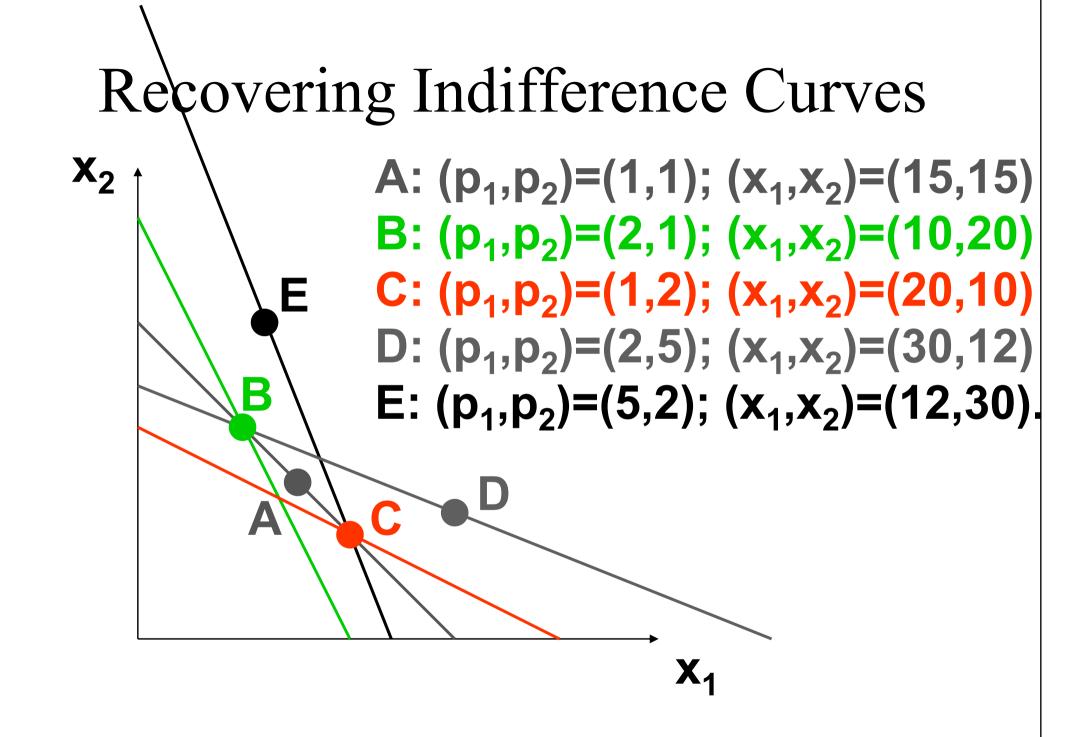
Direct revelations only; the WARP is not violated by the data.

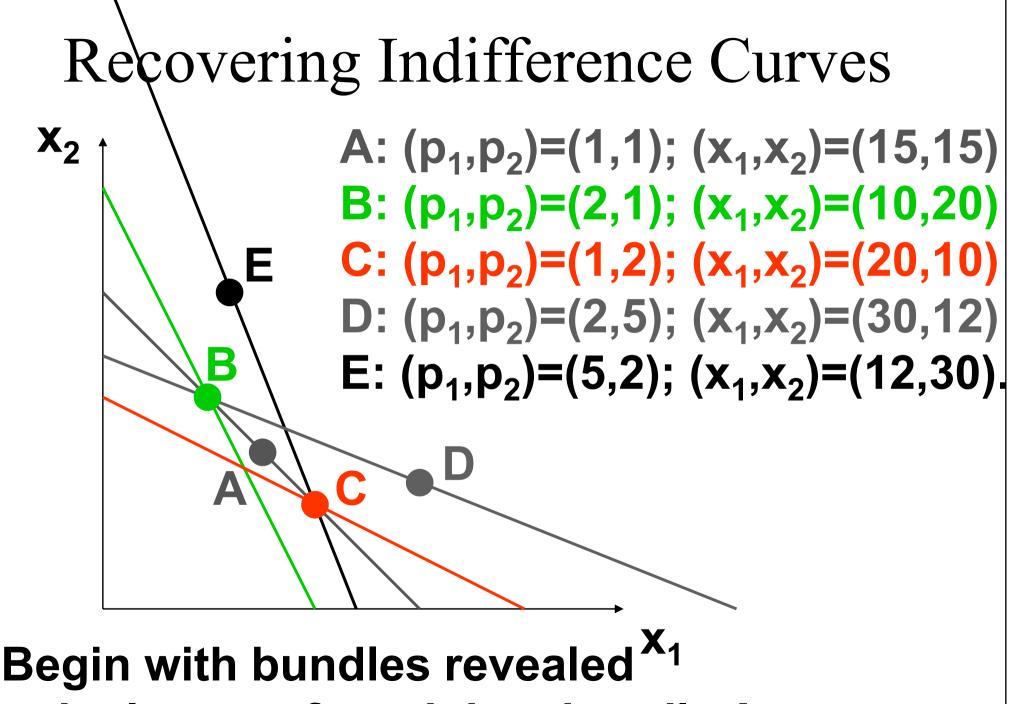
Indirect preference revelations add no extra information, so the table showing both direct and indirect preference revelations is the same as the table showing only the direct preference revelations:



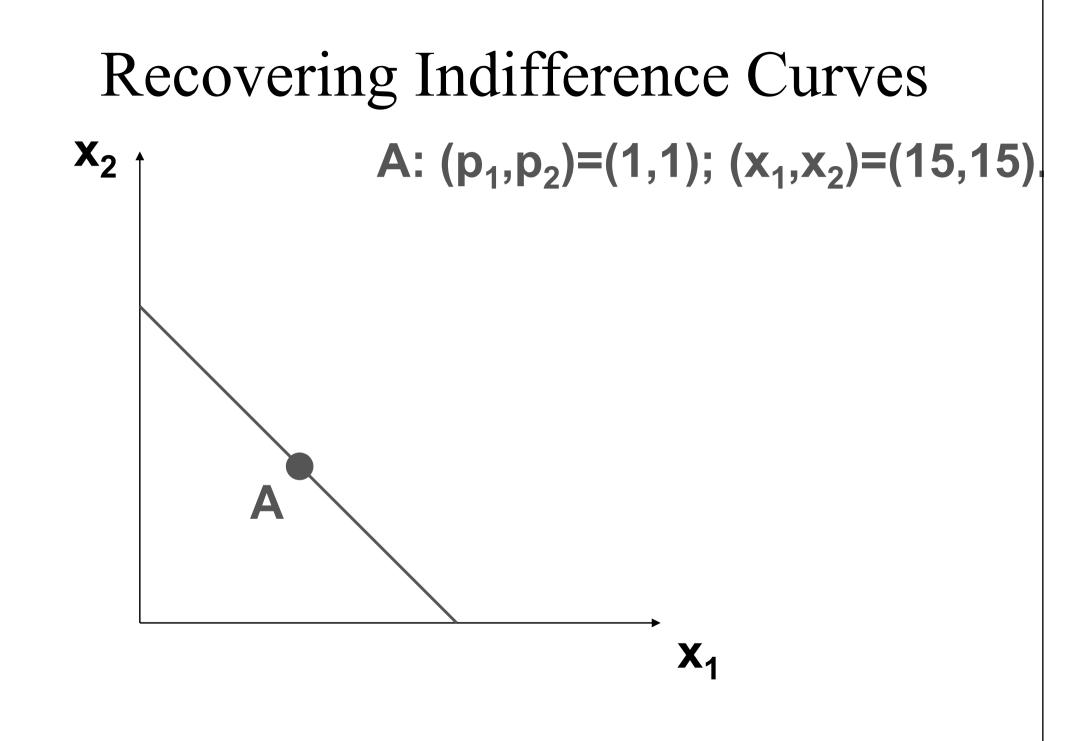
Both direct and indirect revelations; neither WARP nor SARP are violated by the data.

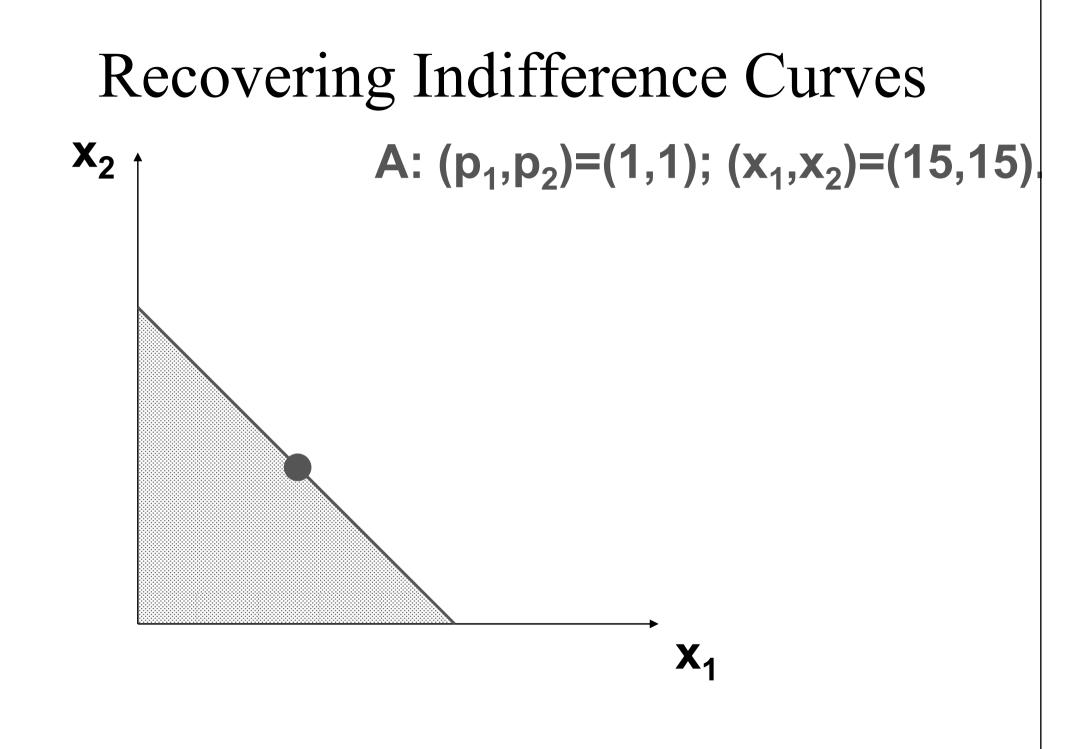
Since the choices satisfy the SARP, there is a well-behaved preference relation that "rationalizes" the choices.

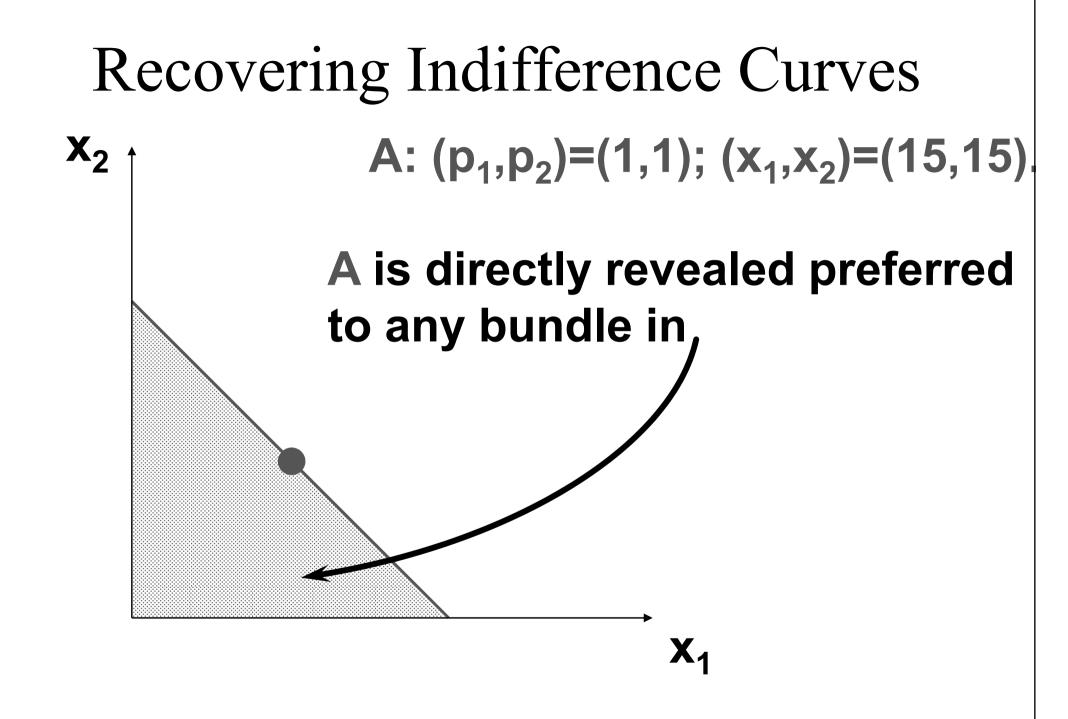


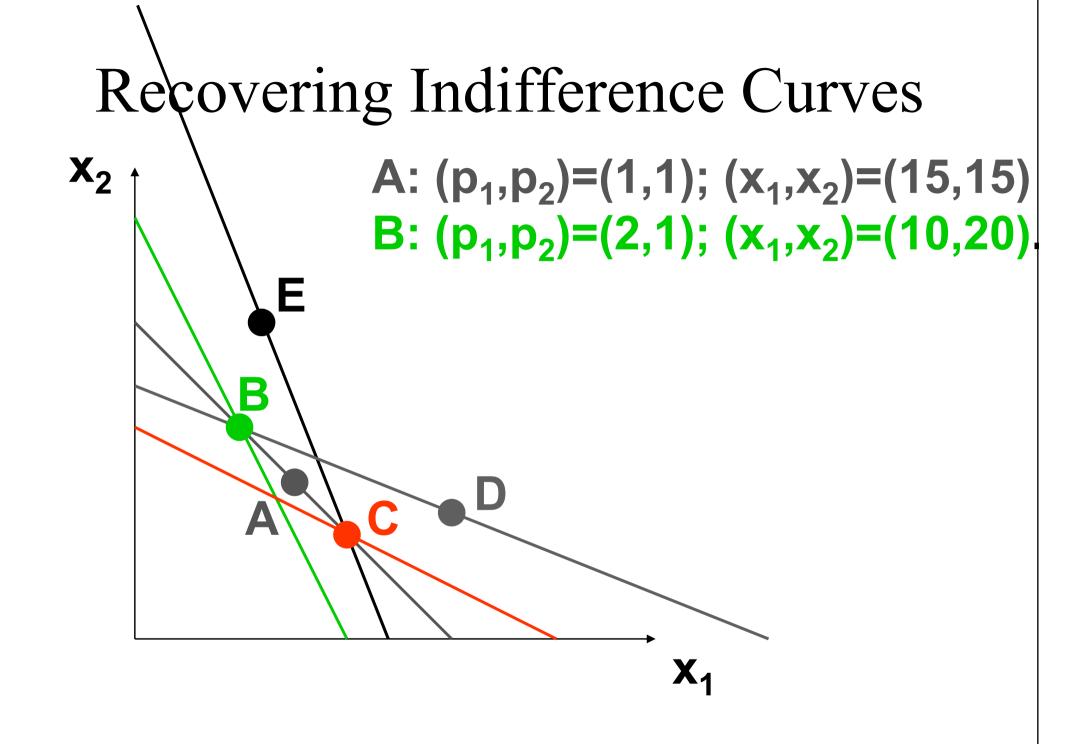


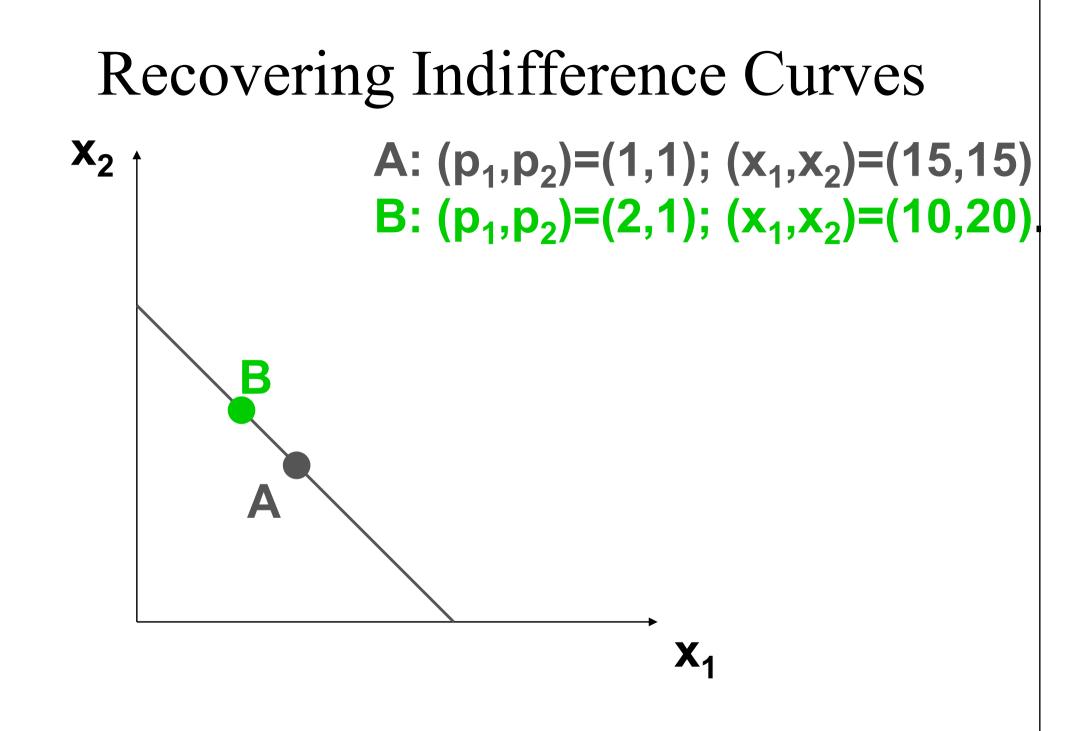
to be less preferred than bundle A.

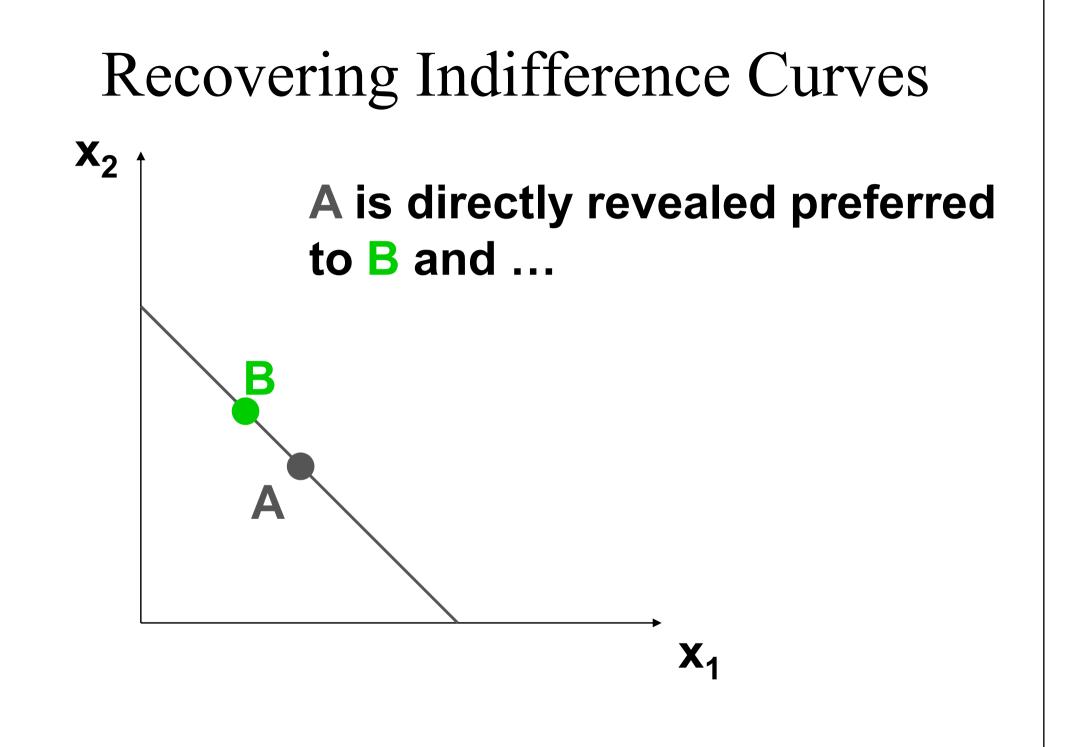


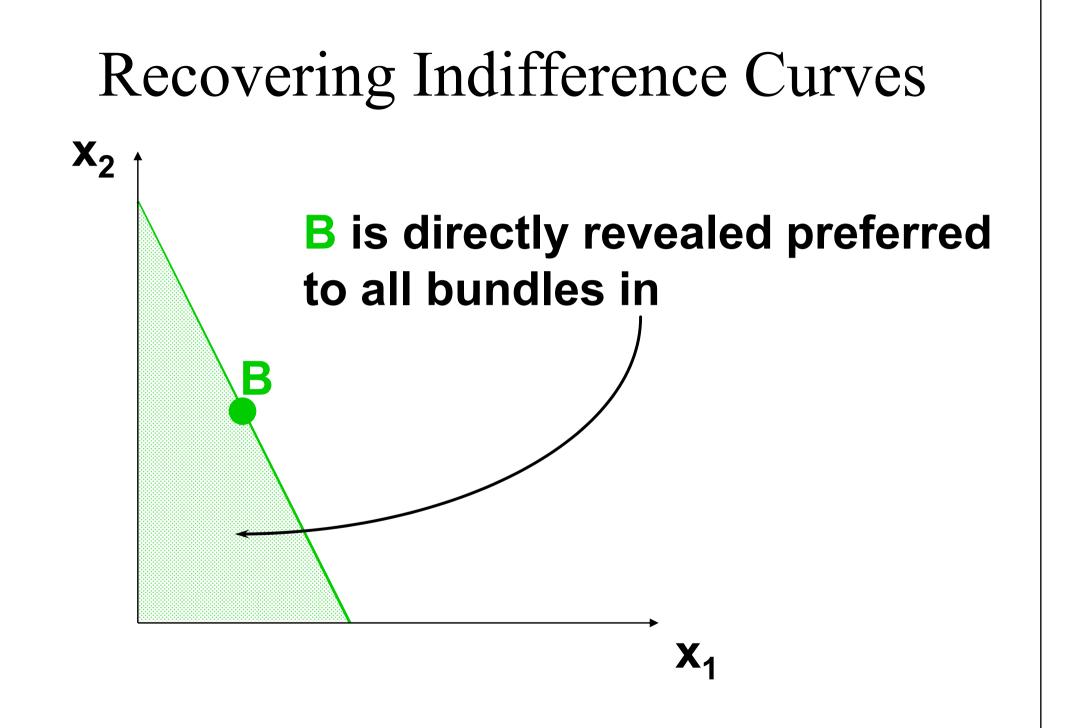


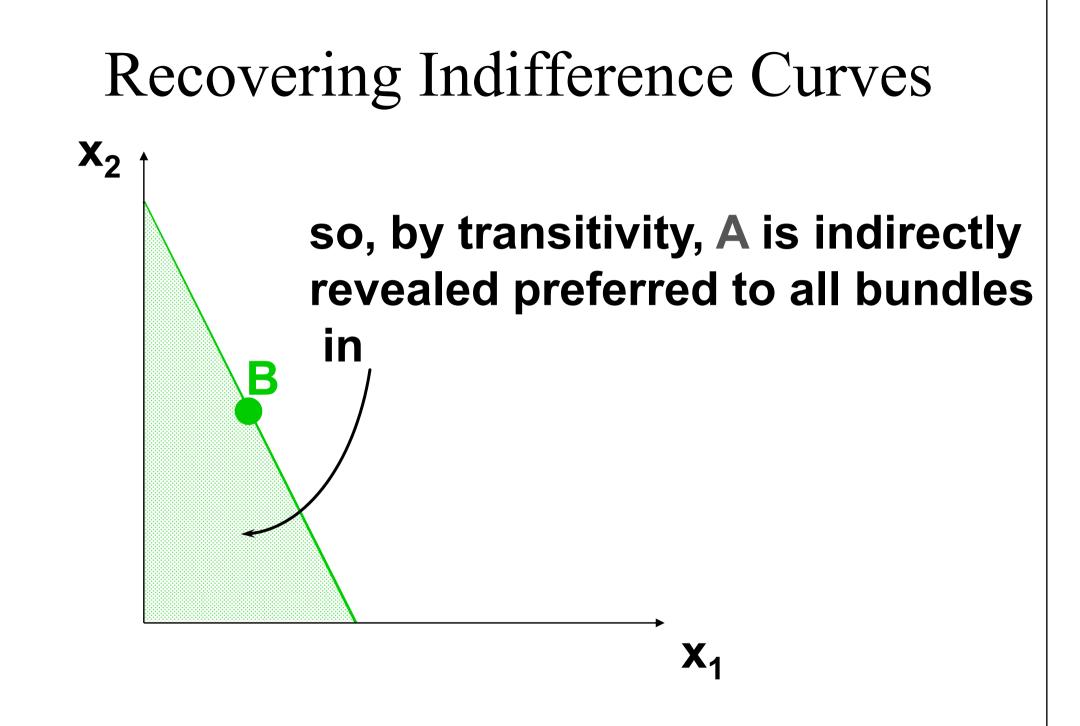


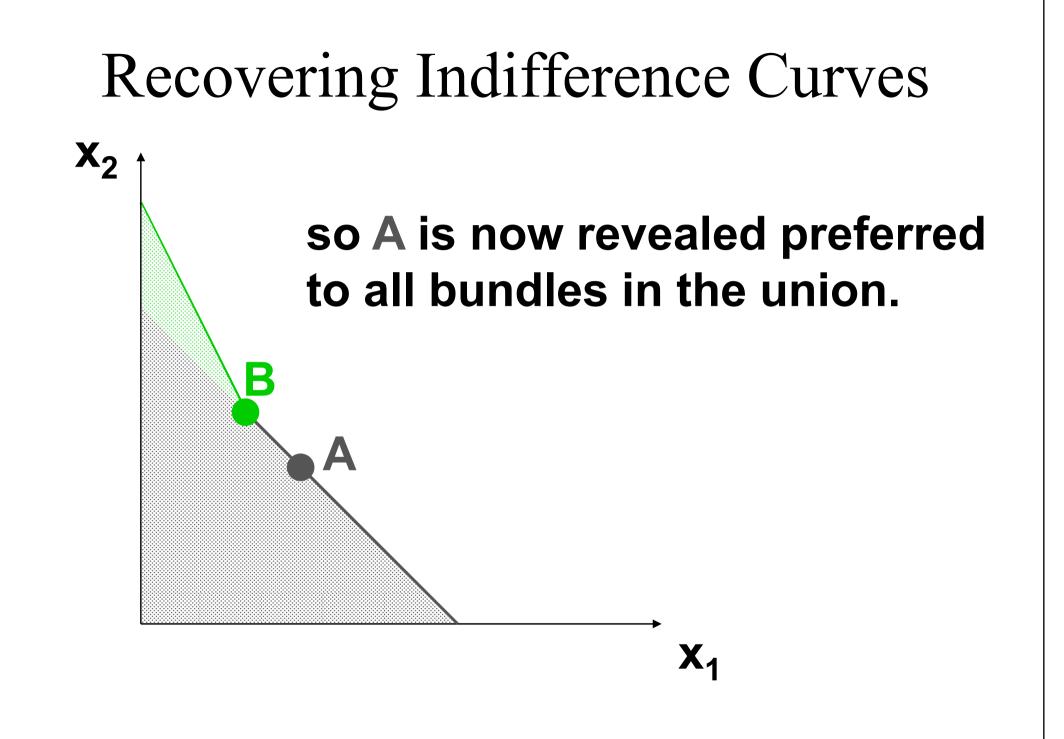


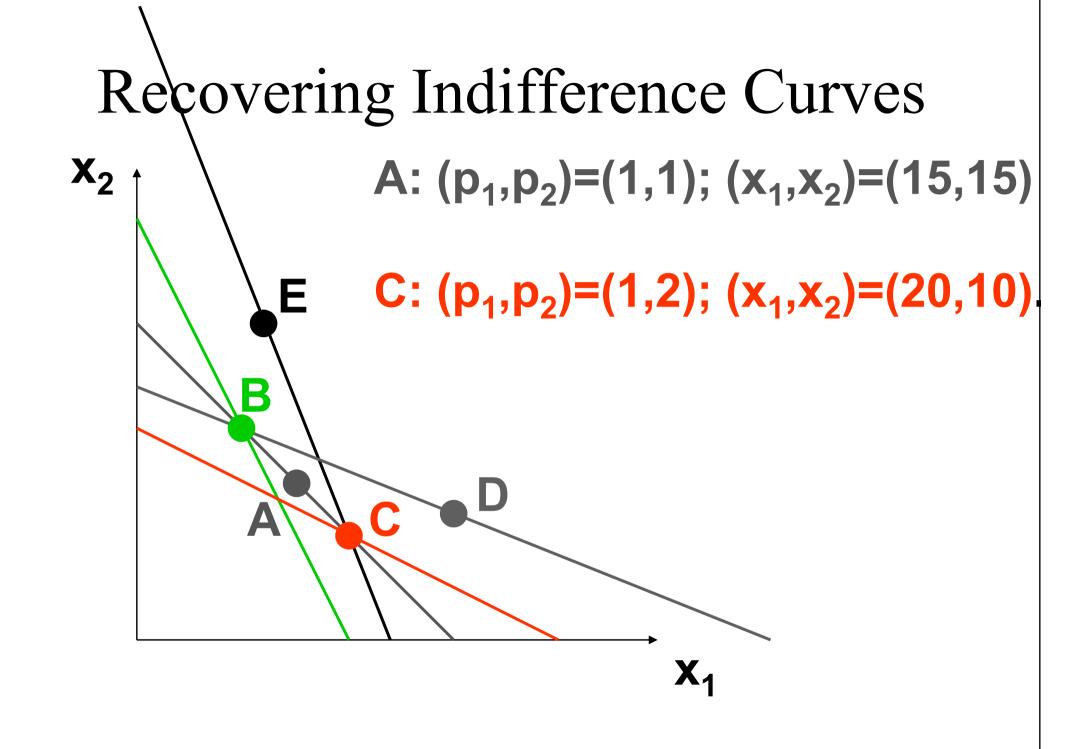


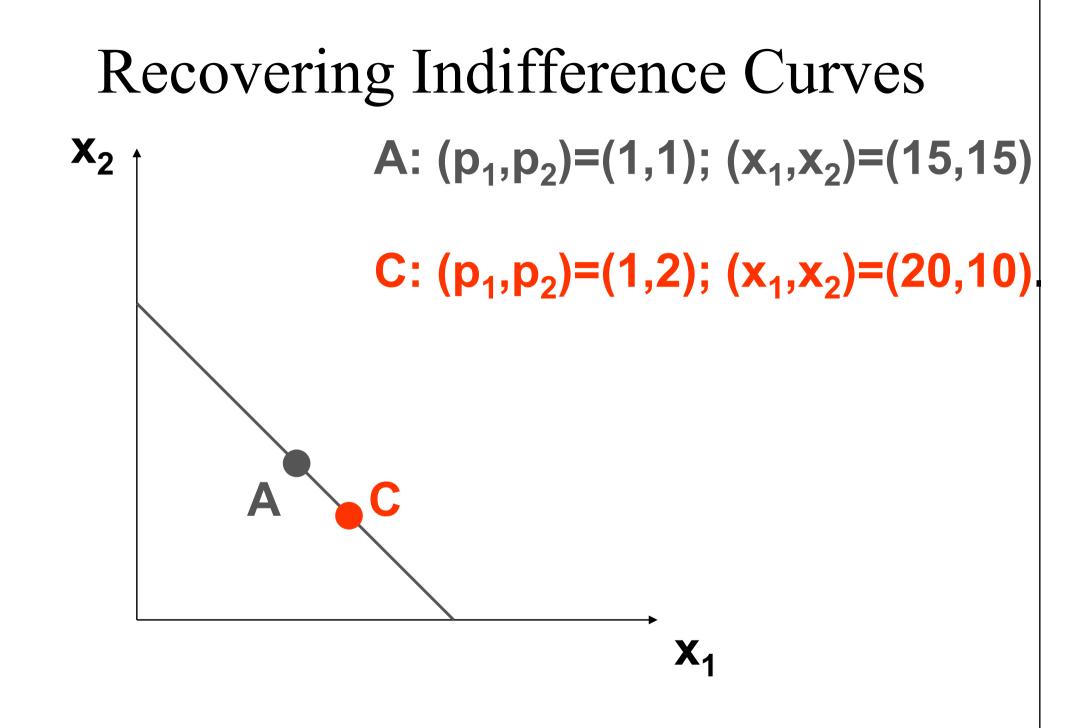


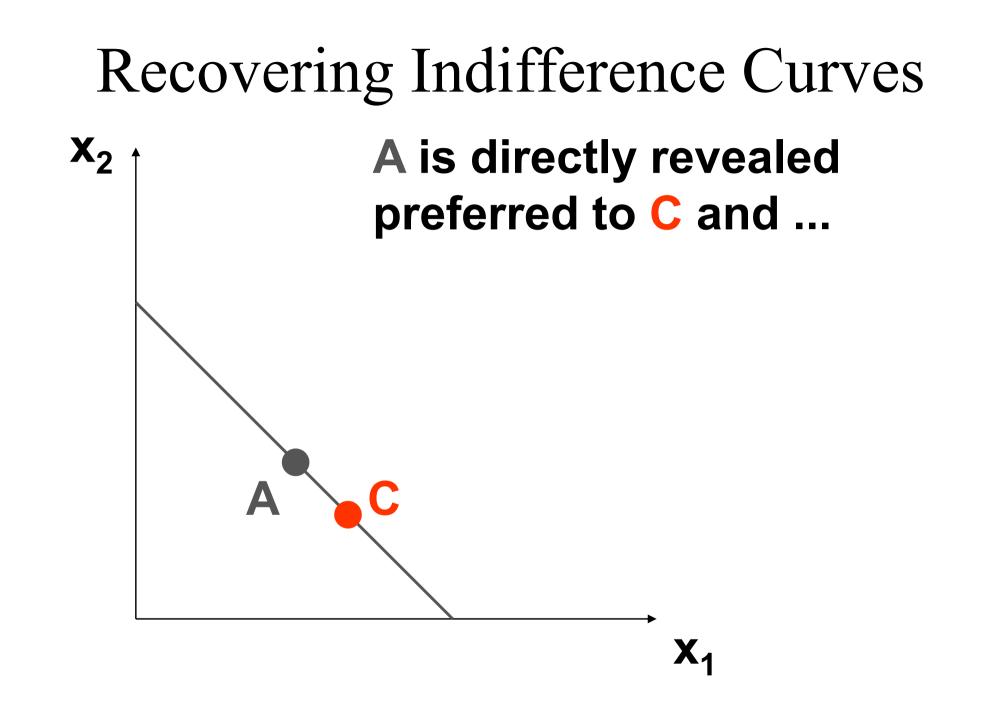


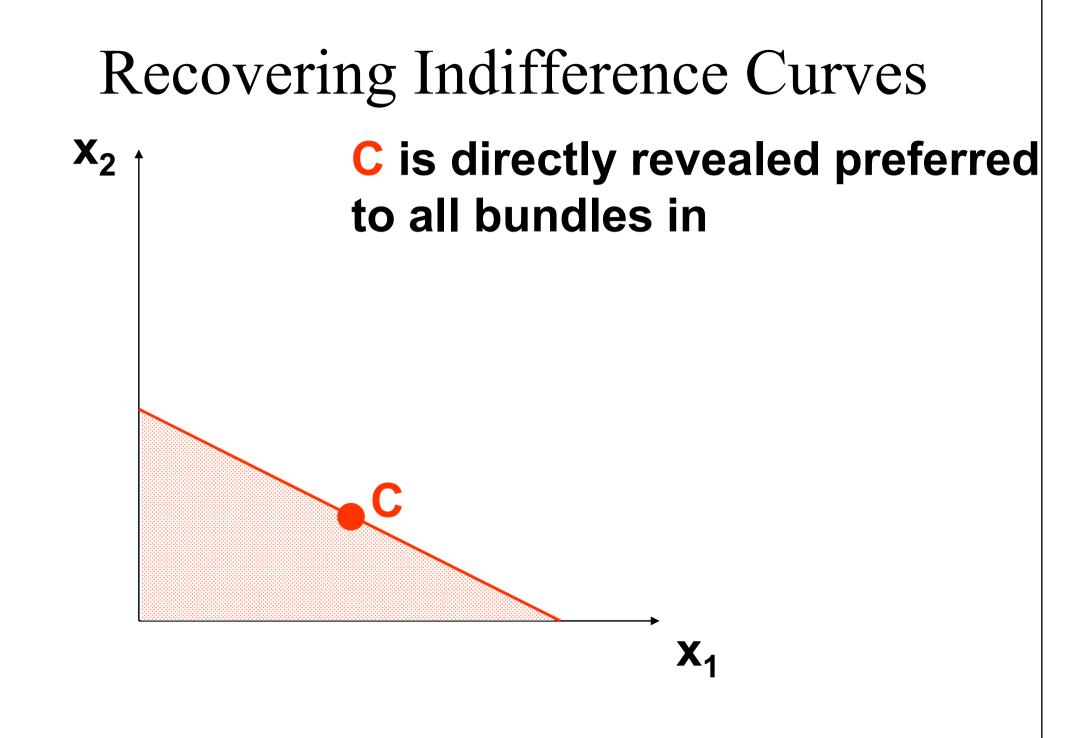


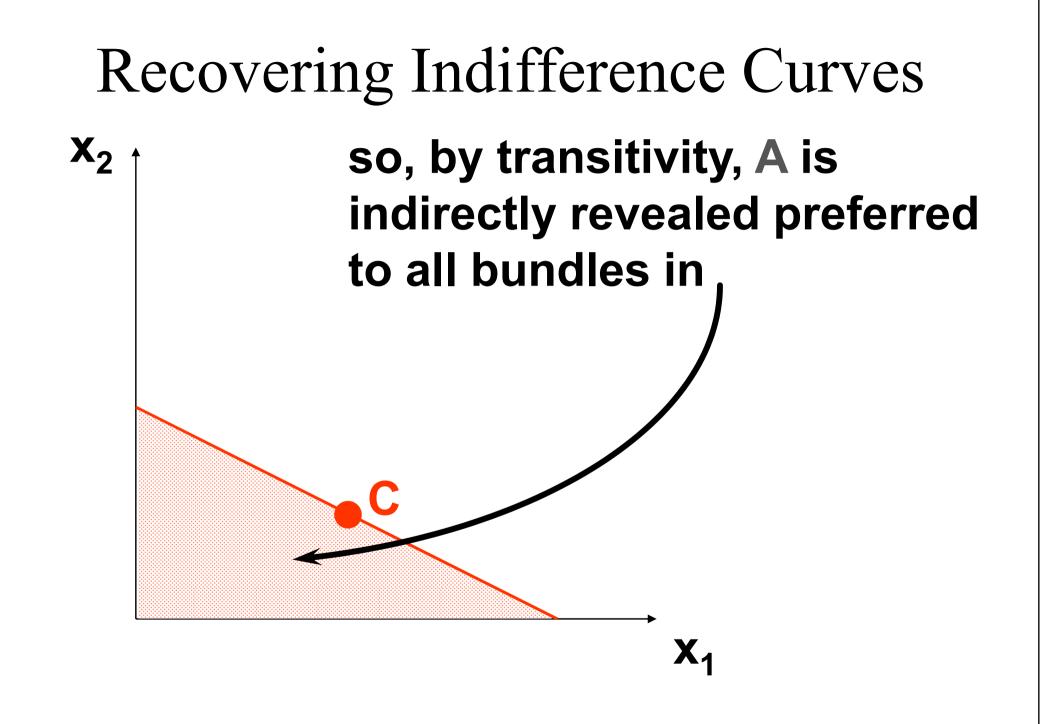


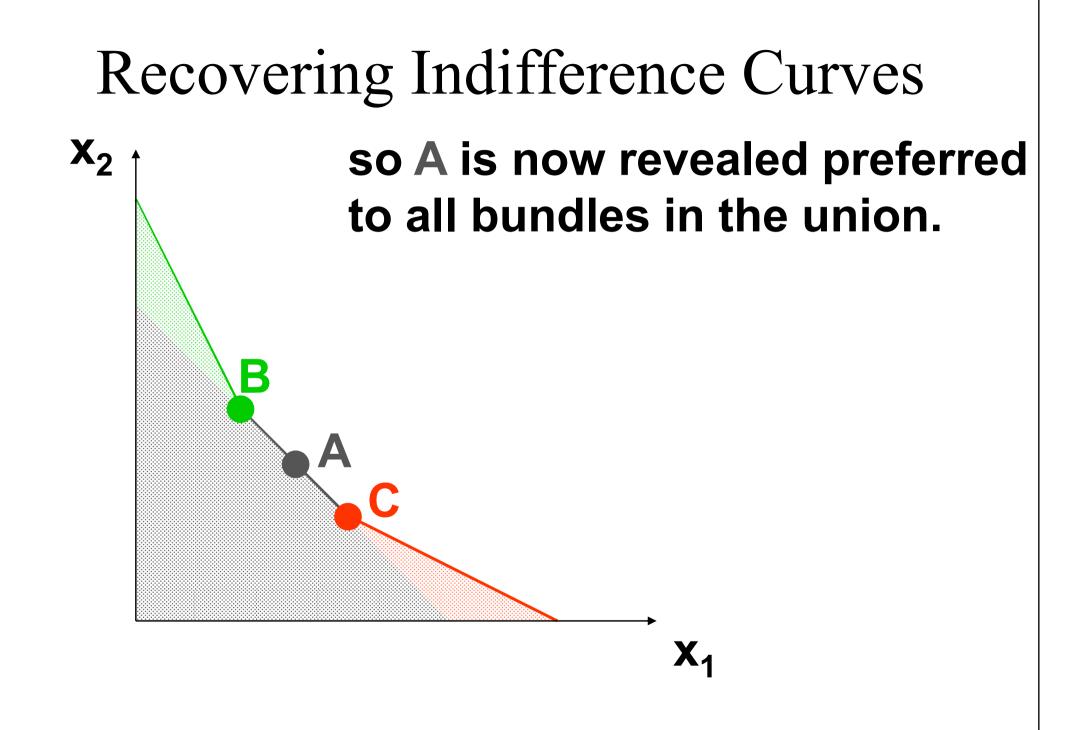












# Recovering Indifference Curves

 $X_2$ 

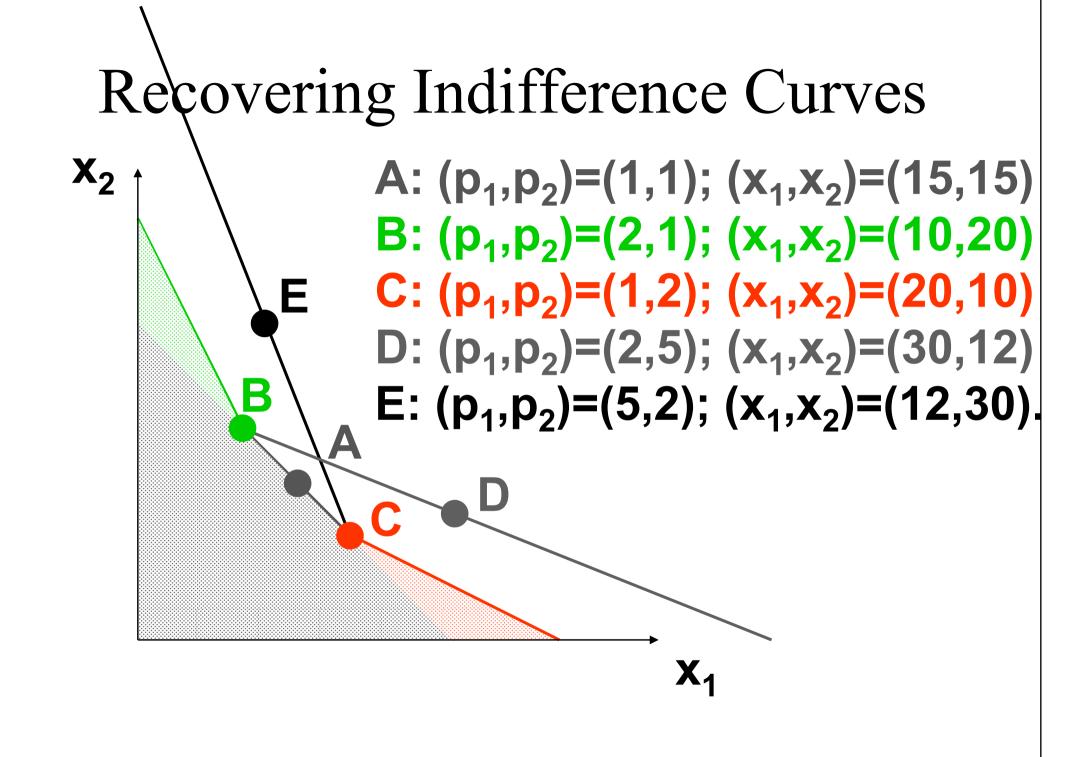
B

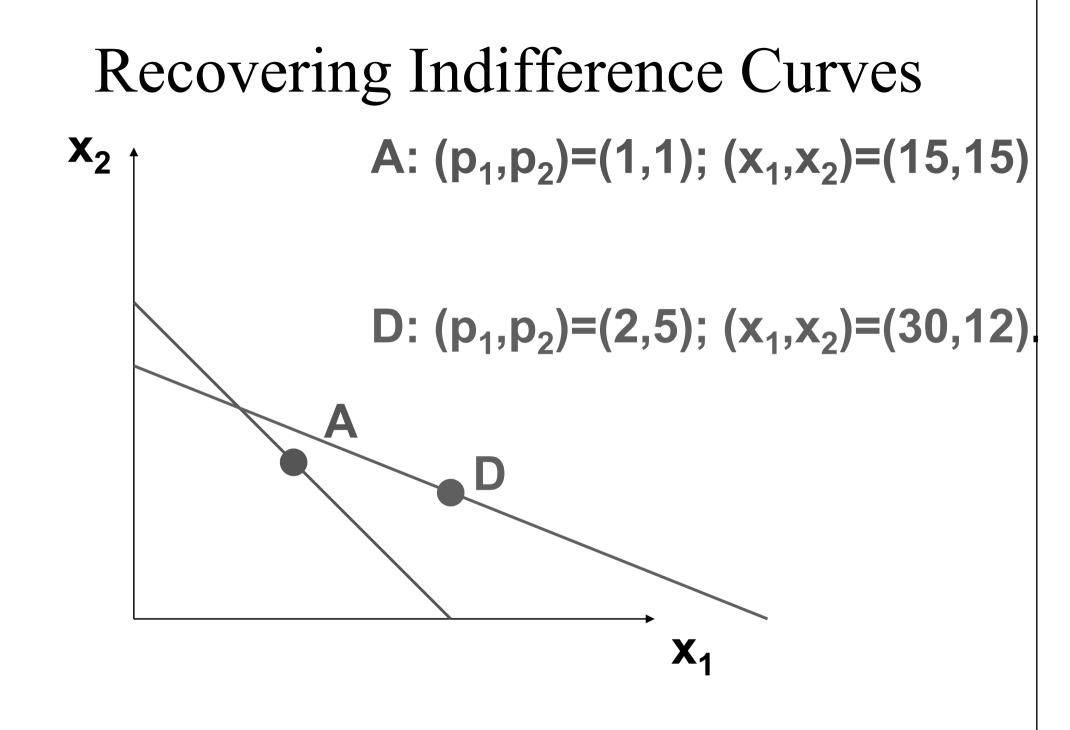
so **A** is now revealed preferred to all bundles in the union.

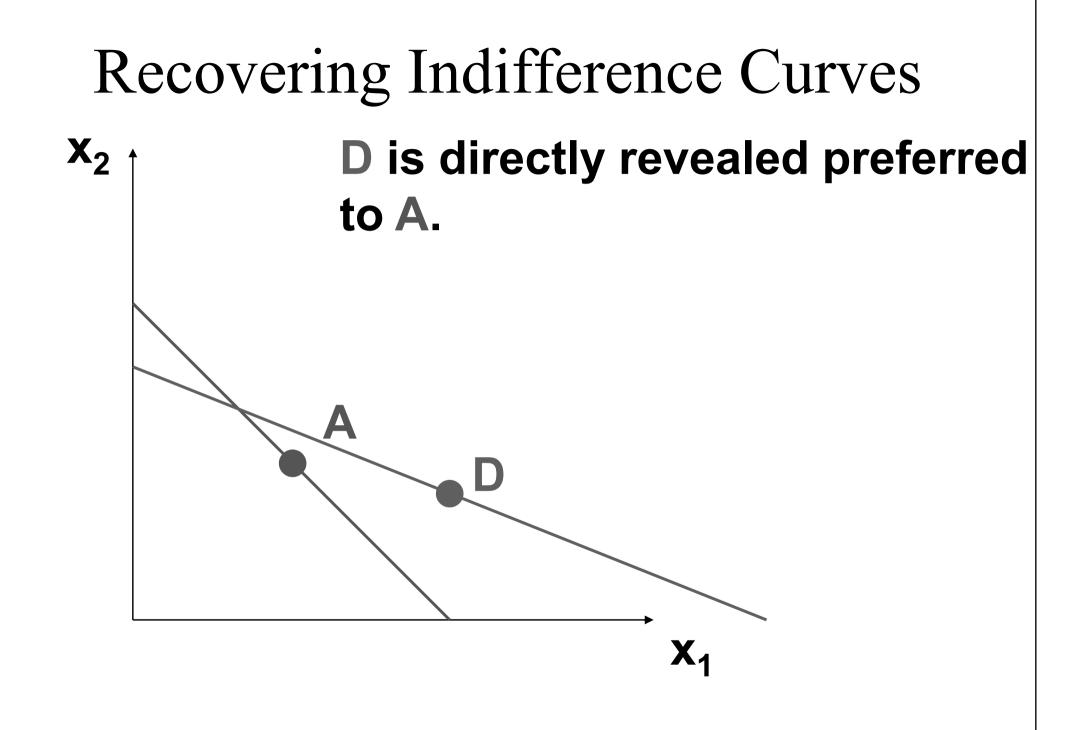
Therefore the indifference curve containing A must lie everywhere else above this shaded set.

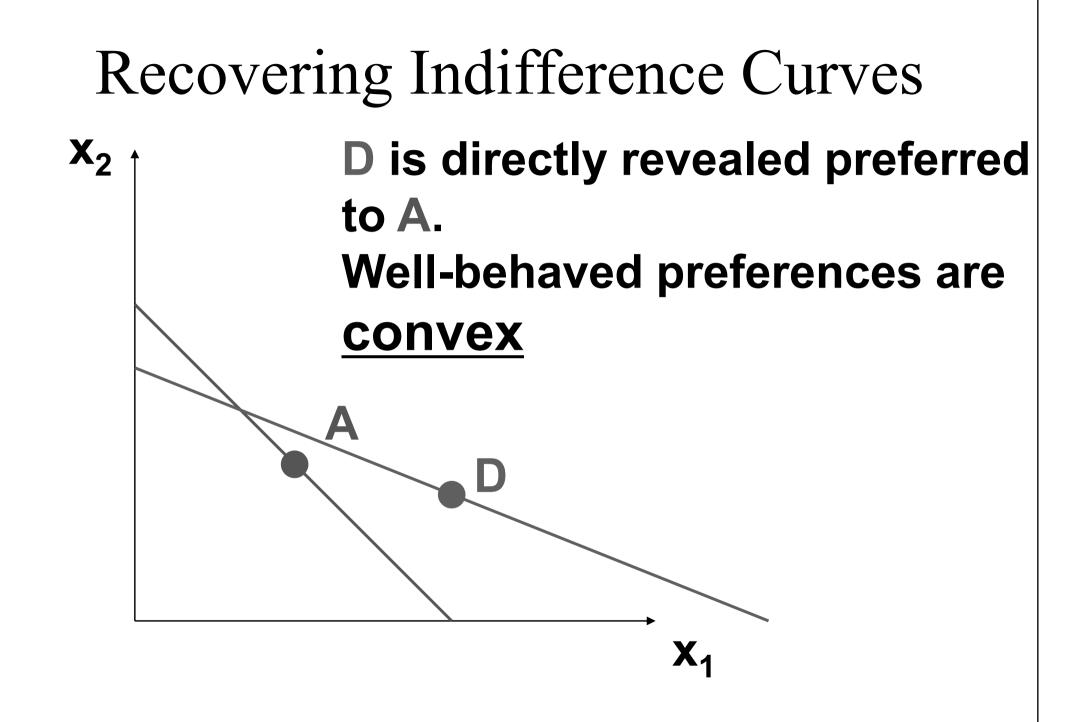
# Recovering Indifference Curves

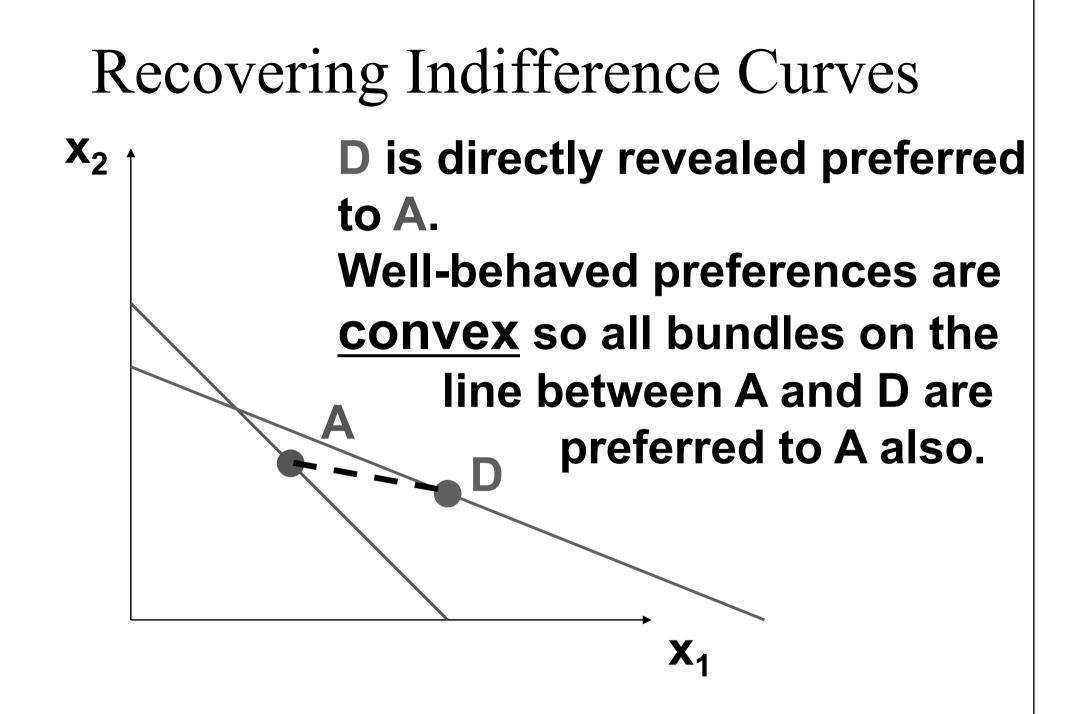
Now, what about the bundles revealed as more preferred than A?

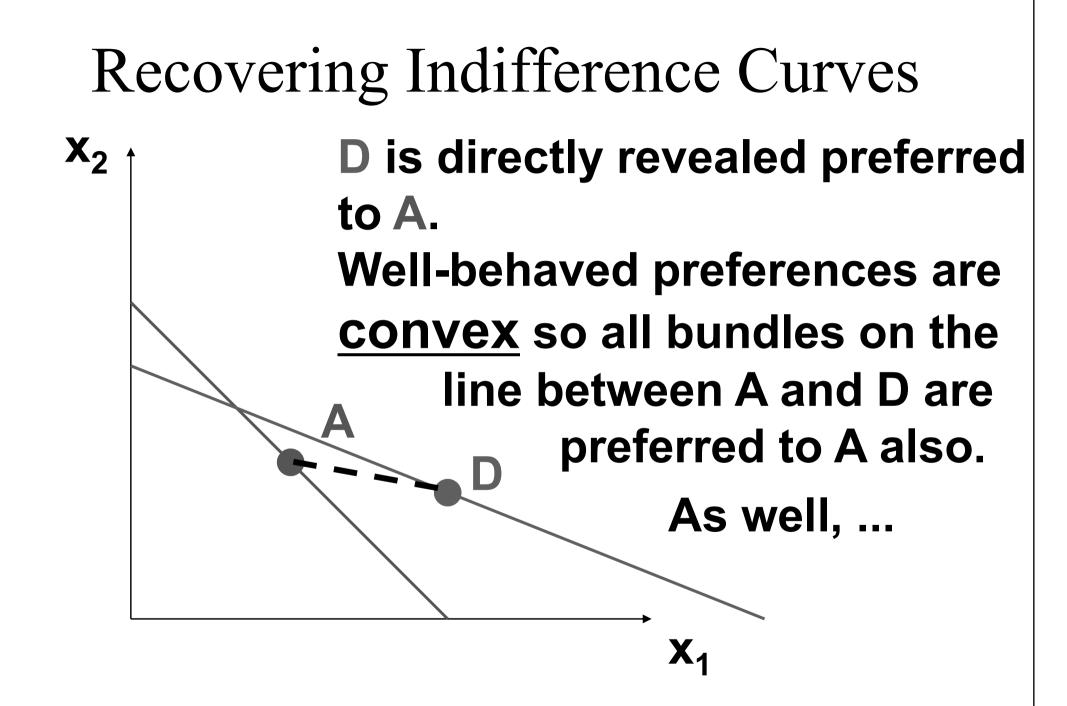


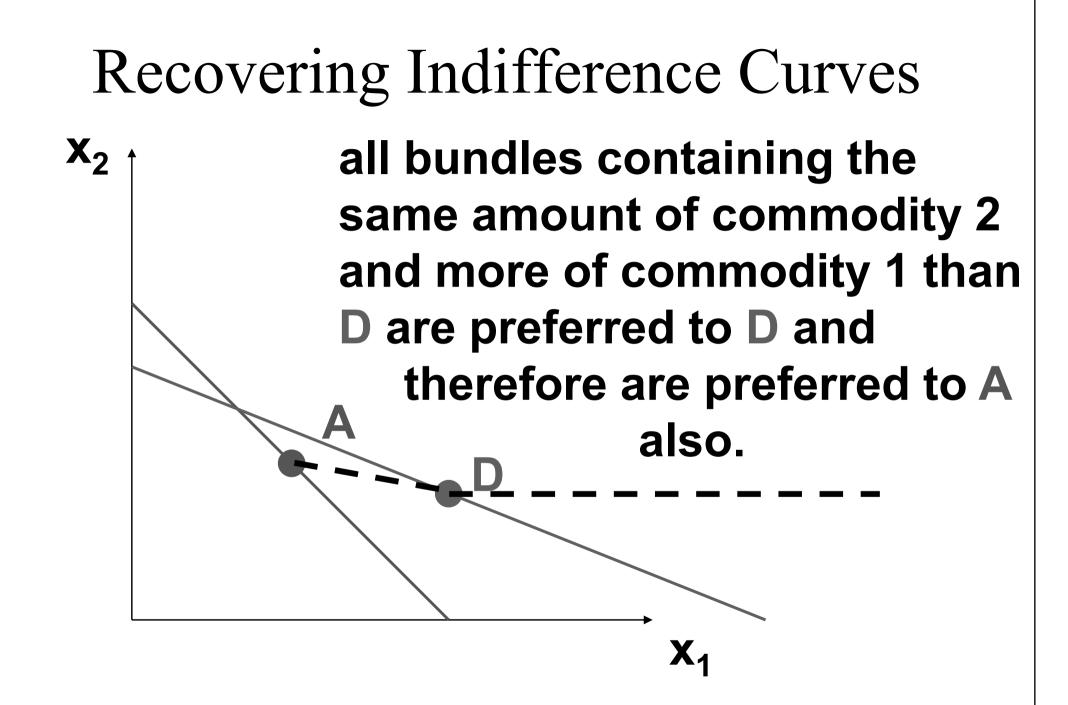


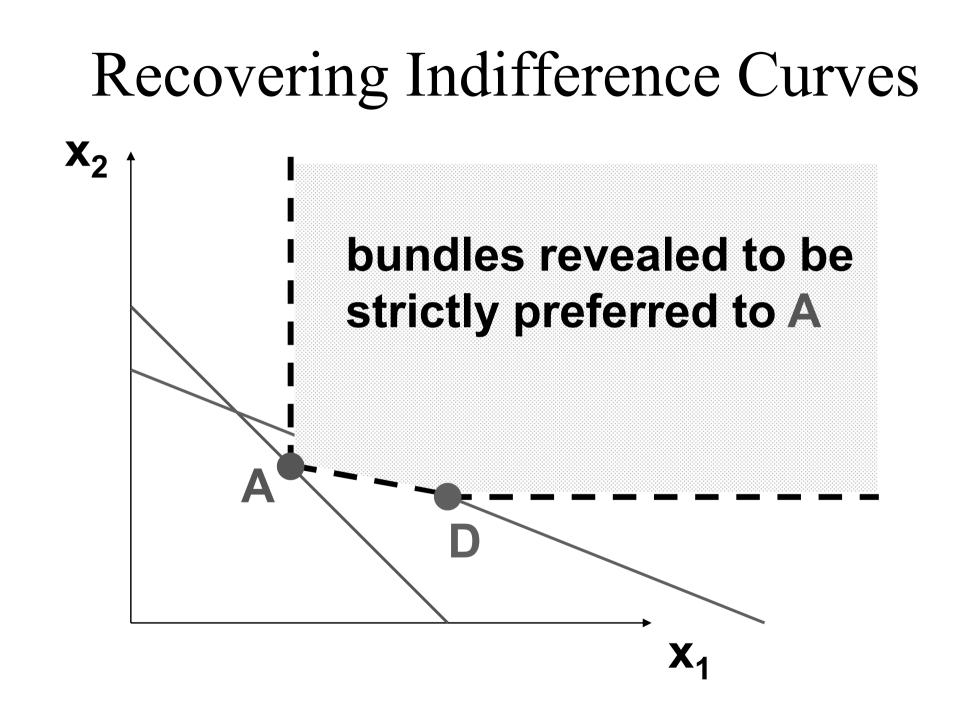


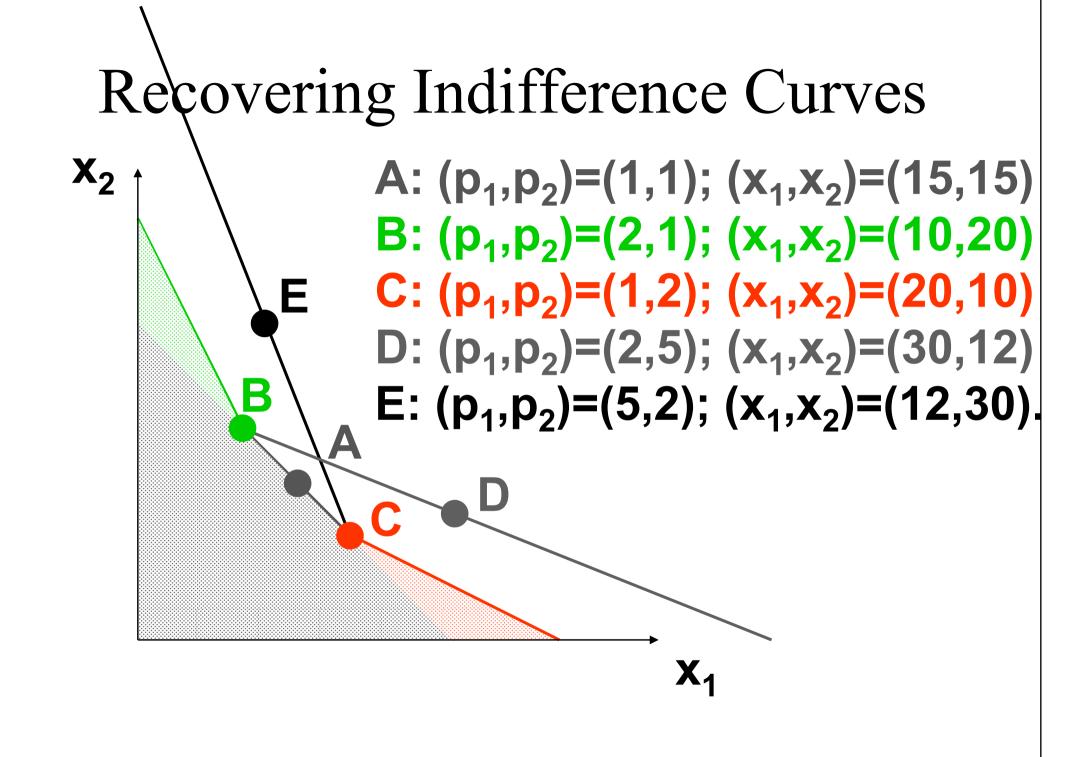


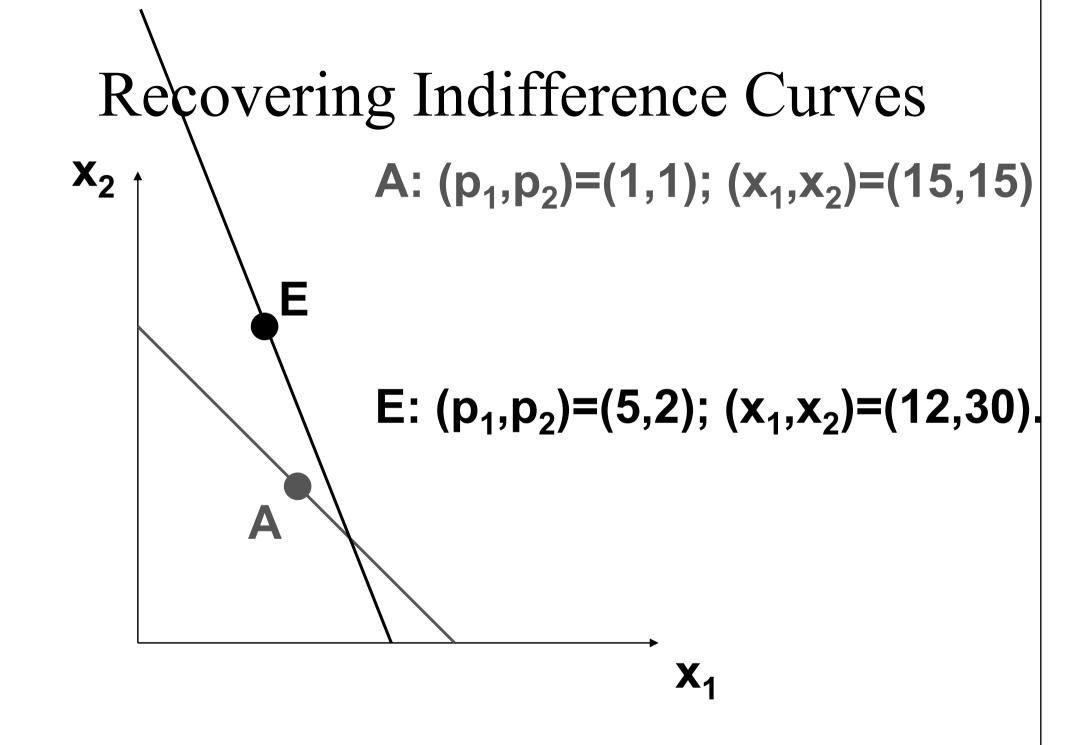


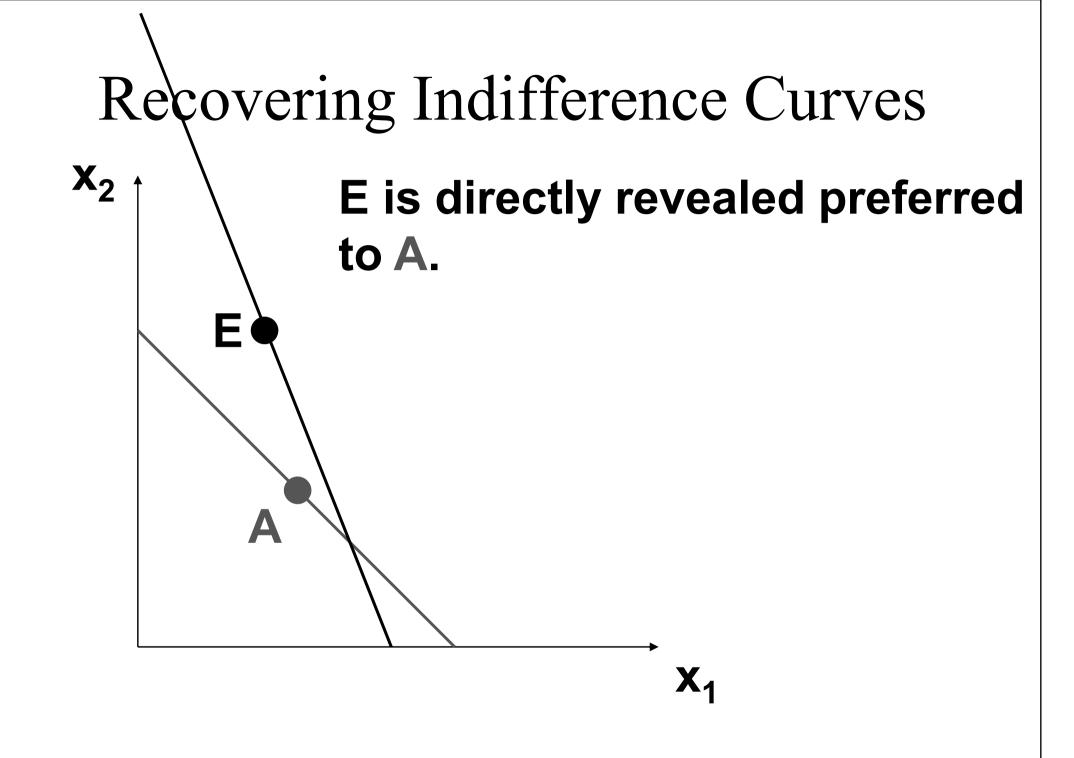


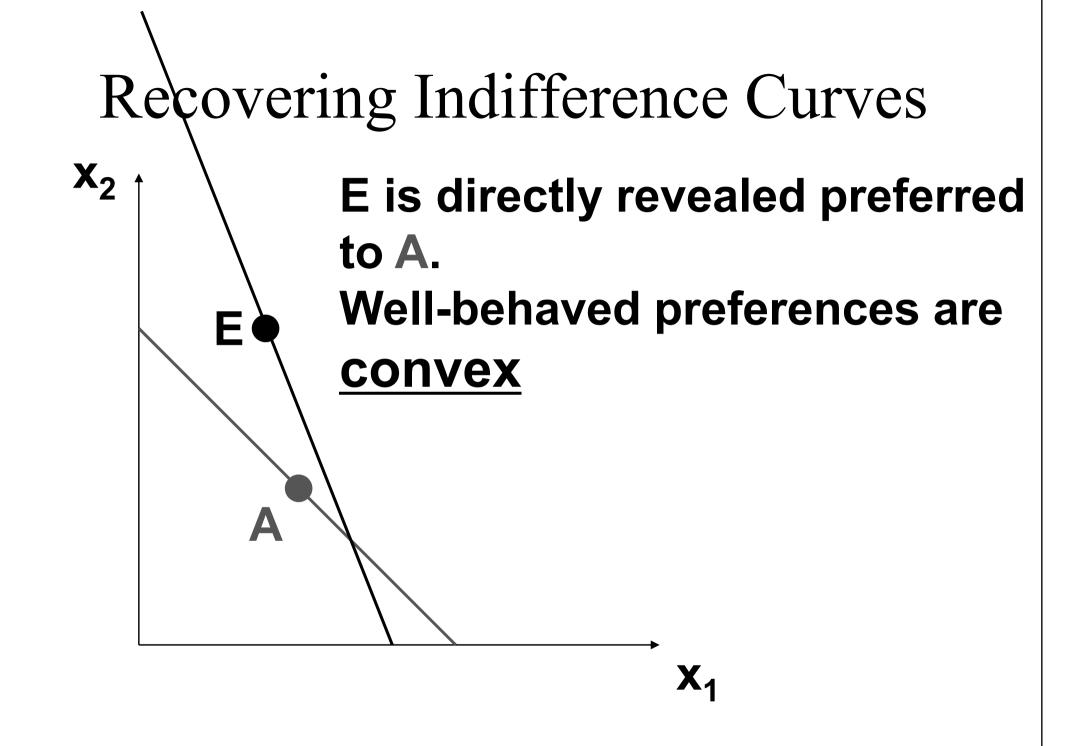


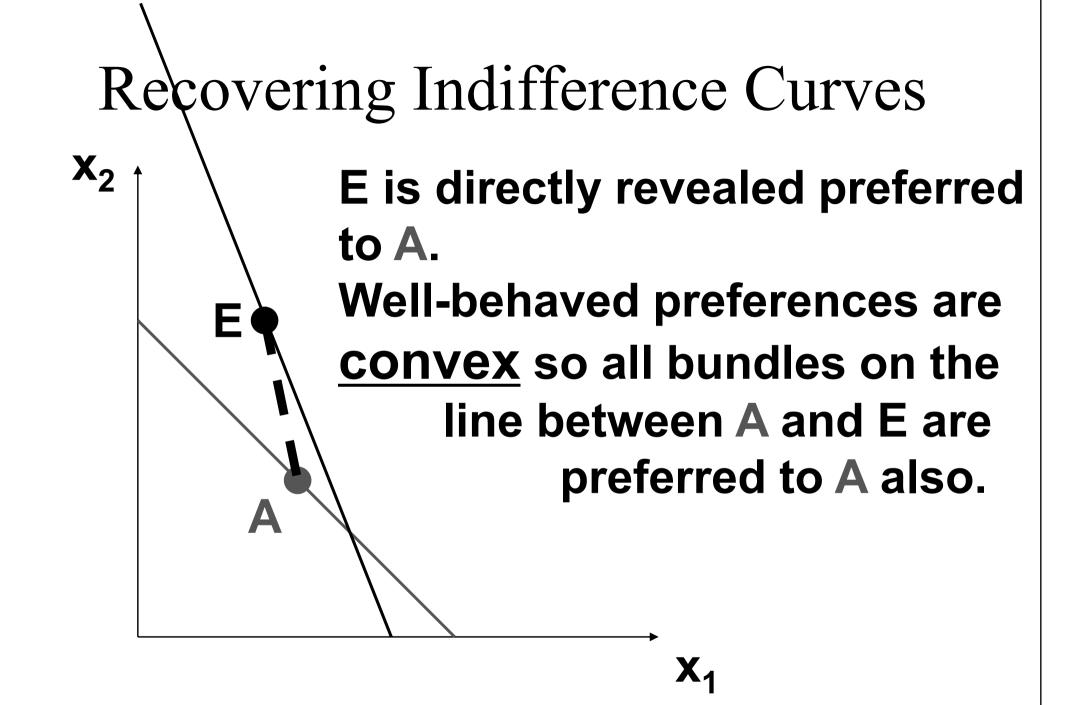


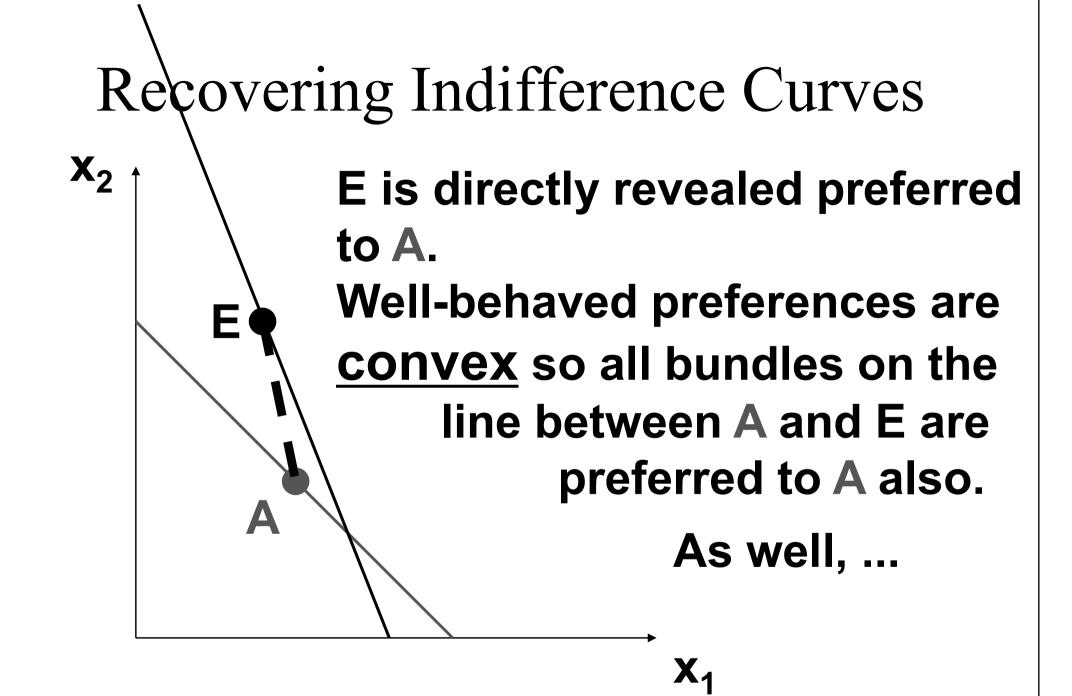


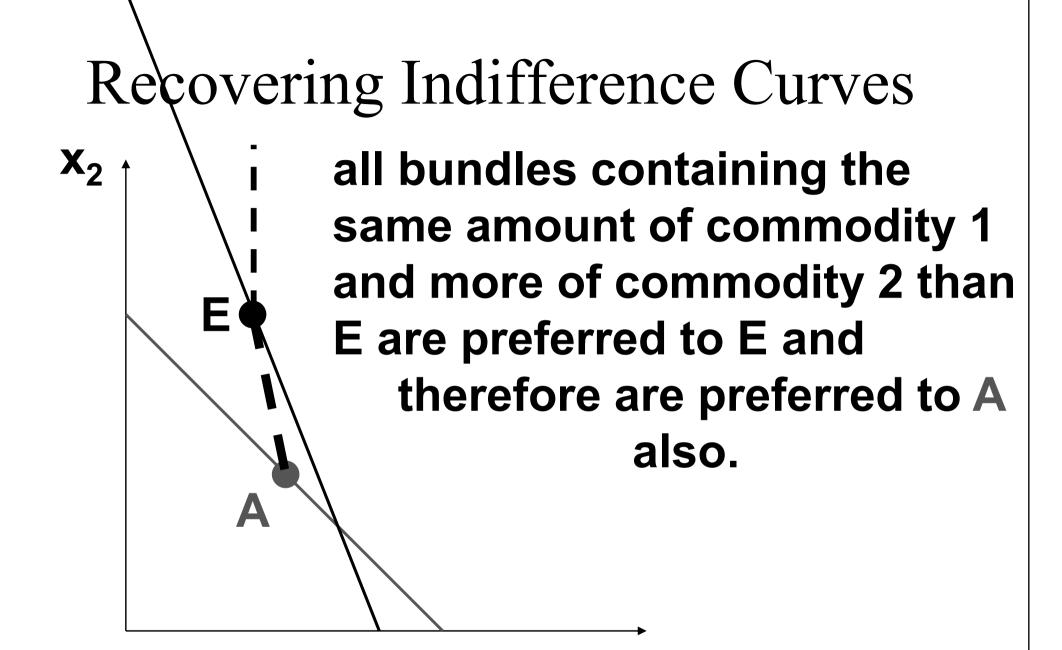


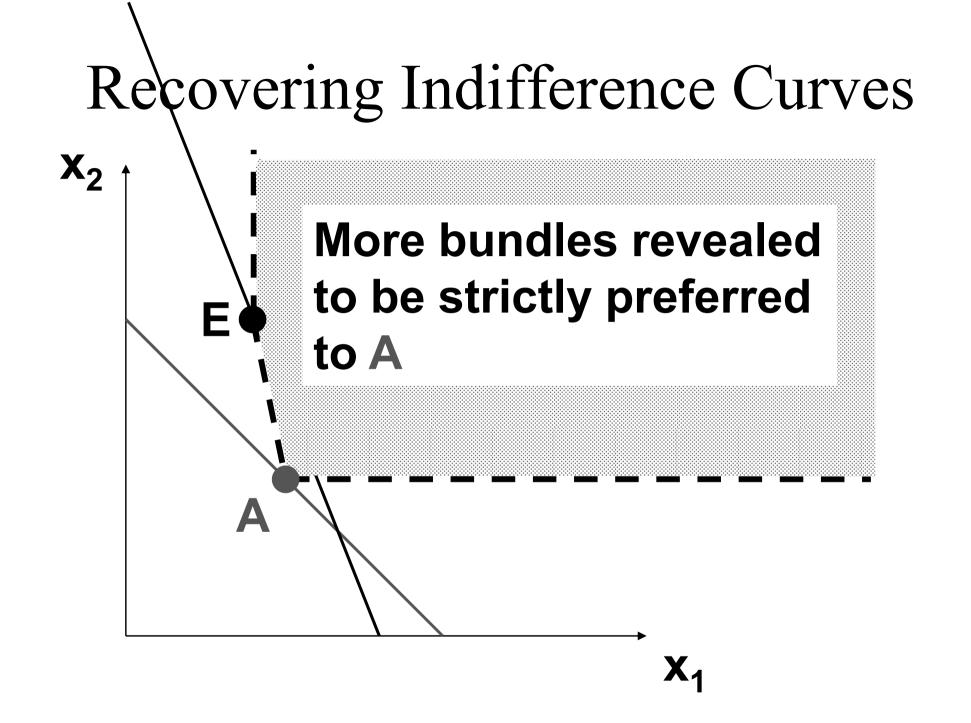


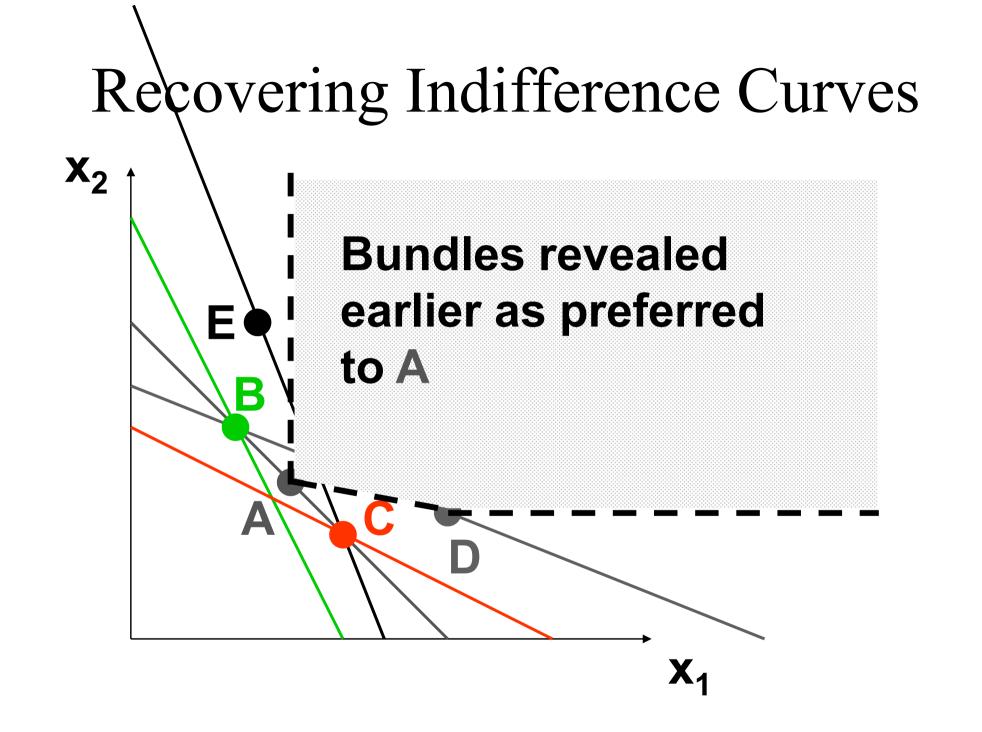


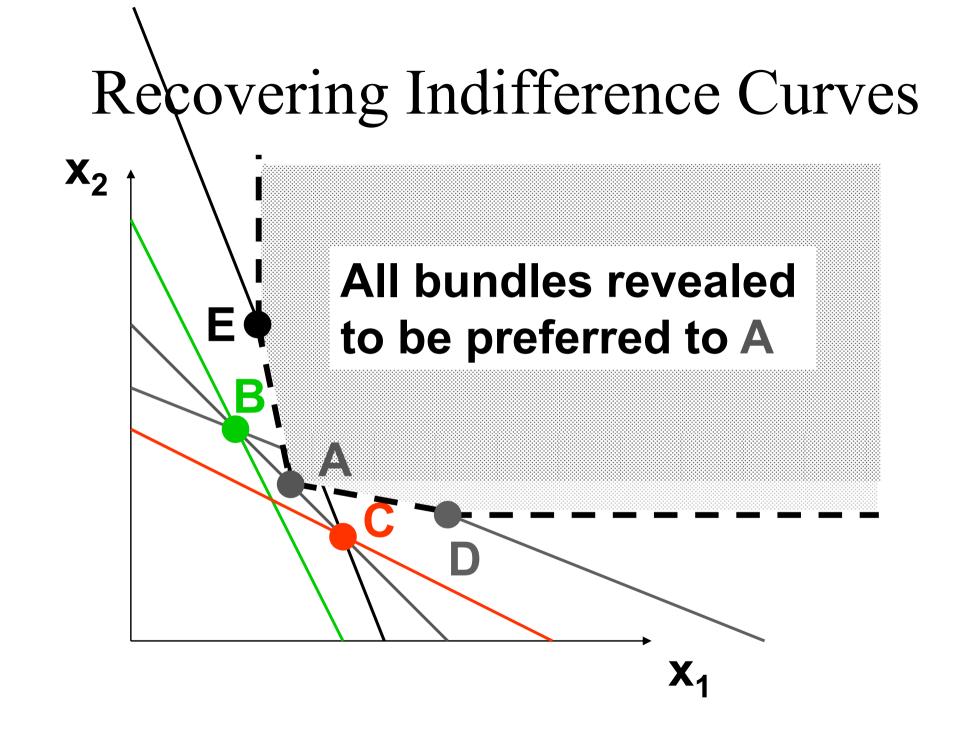






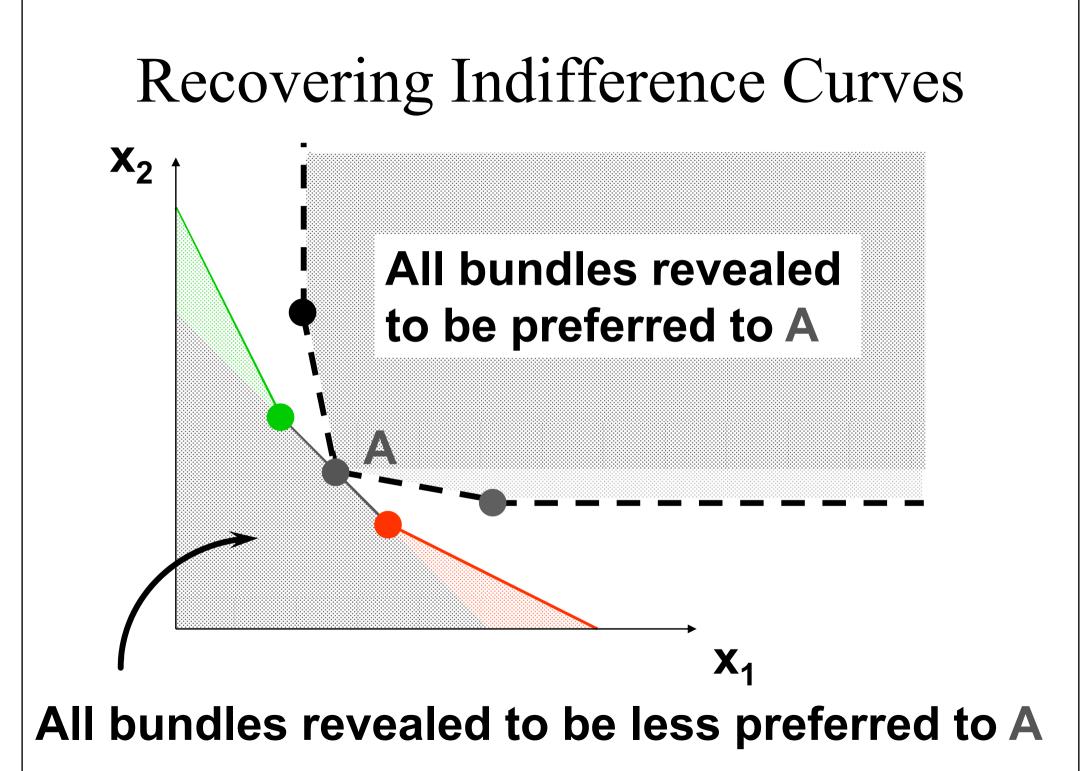


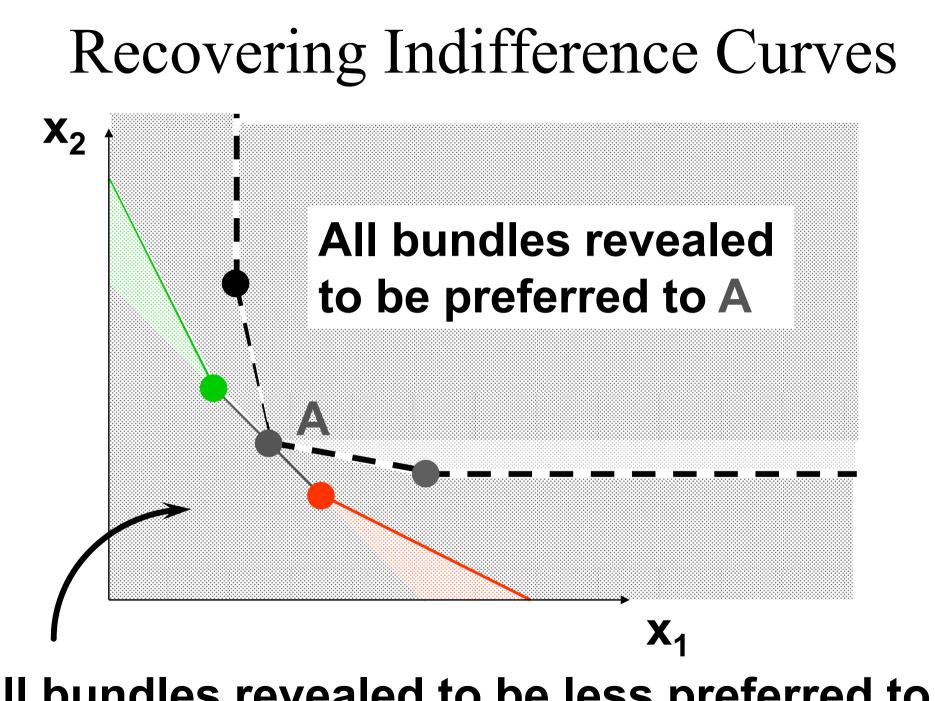




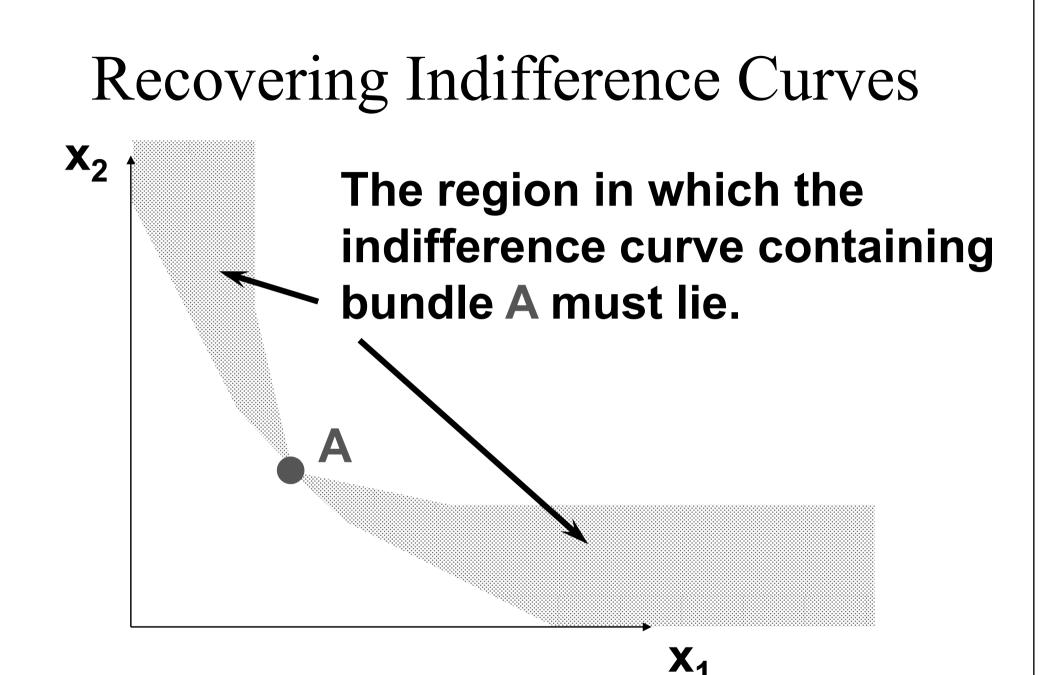
# Recovering Indifference Curves

Now we have upper and lower bounds on where the indifference curve containing bundle A may lie.





### All bundles revealed to be less preferred to A



# Index Numbers

- Over time, many prices change. Are consumers better or worse off "overall" as a consequence?
- Index numbers give approximate answers to such questions.

# Index Numbers

## Two basic types of indices

- -price indices, and
- -quantity indices

Each index compares expenditures in a base period and in a current period by taking the ratio of expenditures.

# ♦ A quantity index is a price-weighted average of quantities demanded; *i.e.* $I_q = \frac{p_1 x_1^t + p_2 x_2^t}{p_1 x_1^b + p_2 x_2^b}$

(p<sub>1</sub>,p<sub>2</sub>) can be base period prices (p<sub>1</sub><sup>b</sup>,p<sub>2</sub><sup>b</sup>) or current period prices (p<sub>1</sub><sup>t</sup>,p<sub>2</sub><sup>t</sup>).

If (p<sub>1</sub>,p<sub>2</sub>) = (p<sub>1</sub><sup>b</sup>,p<sub>2</sub><sup>b</sup>) then we have the Laspeyres quantity index;

$$L_{q} = \frac{p_{1}^{b}x_{1}^{t} + p_{2}^{b}x_{2}^{t}}{p_{1}^{b}x_{1}^{b} + p_{2}^{b}x_{2}^{b}}$$

If (p<sub>1</sub>,p<sub>2</sub>) = (p<sub>1</sub><sup>t</sup>,p<sub>2</sub><sup>t</sup>) then we have the Paasche quantity index;

$$P_{q} = \frac{p_{1}^{t}x_{1}^{t} + p_{2}^{t}x_{2}^{t}}{p_{1}^{t}x_{1}^{b} + p_{2}^{t}x_{2}^{b}}$$

How can quantity indices be used to make statements about changes in welfare?

# Quantity Index Numbers If $L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b} < 1$ then $p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$

so consumers overall were better off in the base period than they are now in the current period.

# Quantity Index Numbers If $P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1$ then $p_1^t x_1^t + p_2^t x_2^t > p_1^t x_1^b + p_2^t x_2^b$

so consumers overall are better off in the current period than in the base period.

## Price Index Numbers

♦ A price index is a quantity-weighted average of prices; *i.e.*  $I_p = \frac{p_1^t x_1 + p_2^t x_2}{p_1^b x_1 + p_2^b x_2}$ 

 (x<sub>1</sub>,x<sub>2</sub>) can be the base period bundle (x<sub>1</sub><sup>b</sup>,x<sub>2</sub><sup>b</sup>) or else the current period bundle (x<sub>1</sub><sup>t</sup>,x<sub>2</sub><sup>t</sup>).

♦ If (x<sub>1</sub>,x<sub>2</sub>) = (x<sub>1</sub><sup>b</sup>,x<sub>2</sub><sup>b</sup>) then we have the Laspeyres price index;

$$L_{p} = \frac{p_{1}^{t}x_{1}^{b} + p_{2}^{t}x_{2}^{b}}{p_{1}^{b}x_{1}^{b} + p_{2}^{b}x_{2}^{b}}$$

If (x<sub>1</sub>,x<sub>2</sub>) = (x<sub>1</sub><sup>t</sup>,x<sub>2</sub><sup>t</sup>) then we have the Paasche price index;

$$P_{p} = \frac{p_{1}^{t}x_{1}^{t} + p_{2}^{t}x_{2}^{t}}{p_{1}^{b}x_{1}^{t} + p_{2}^{b}x_{2}^{t}}$$

- How can price indices be used to make statements about changes in welfare?
- ♦ Define the expenditure ratio  $M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$

$$\begin{array}{l} \textbf{Price Index Numbers} \\ \bullet \mbox{ If } \\ L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b} < & \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M \\ \\ \mbox{ then } \\ & p_1^t x_1^b + p_2^t x_2^b < & p_1^t x_1^t + p_2^t x_2^t \end{array}$$

so consumers overall are better off in the current period.

♦ But, if

$$P_{p} = \frac{p_{1}^{t}x_{1}^{t} + p_{2}^{t}x_{2}^{t}}{p_{1}^{b}x_{1}^{t} + p_{2}^{b}x_{2}^{t}} > \frac{p_{1}^{t}x_{1}^{t} + p_{2}^{t}x_{2}^{t}}{p_{1}^{b}x_{1}^{t} + p_{2}^{b}x_{2}^{t}} = M$$

then

# $p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$

so consumers overall were better off in the base period.

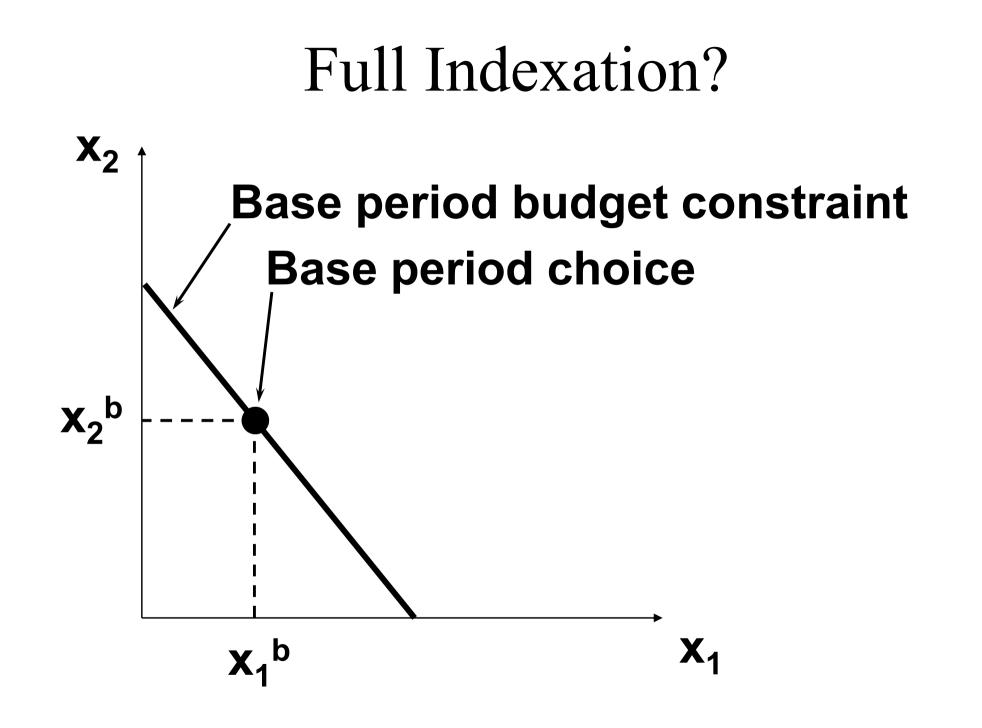
- Changes in price indices are sometimes used to adjust wage rates or transfer payments. This is called "indexation".
- Full indexation" occurs when the wages or payments are increased at the same rate as the price index being used to measure the aggregate inflation rate.

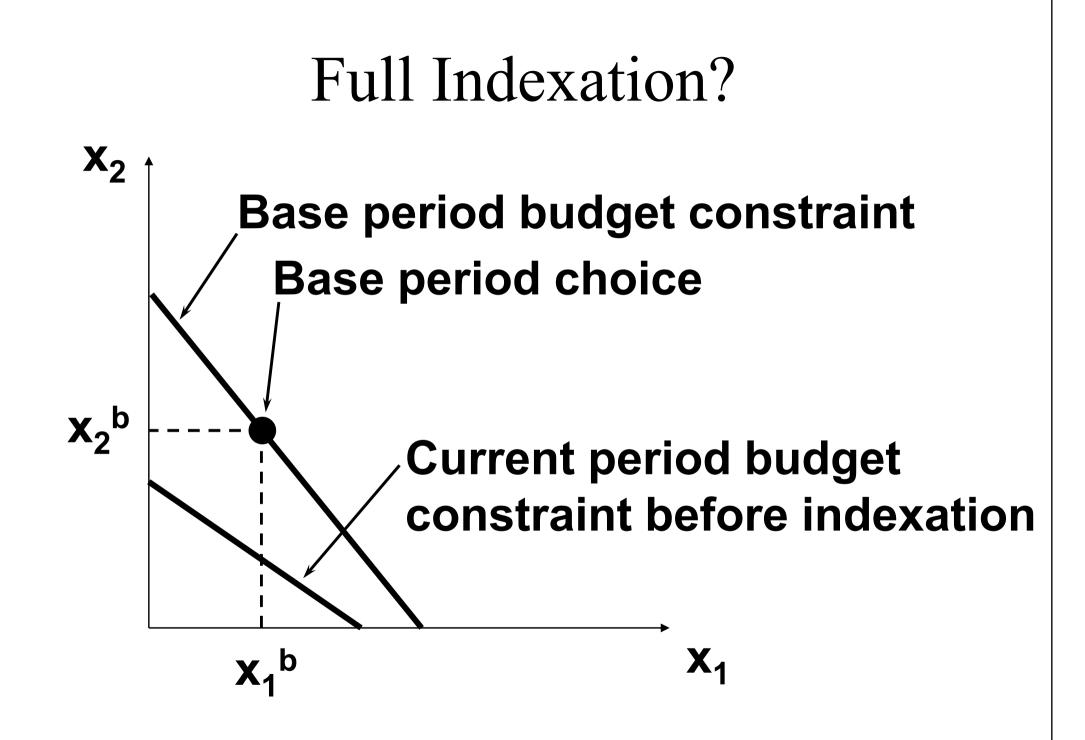
- Since prices do not all increase at the same rate, relative prices change along with the "general price level".
- A common proposal is to index fully Social Security payments, with the intention of preserving for the elderly the "purchasing power" of these payments.

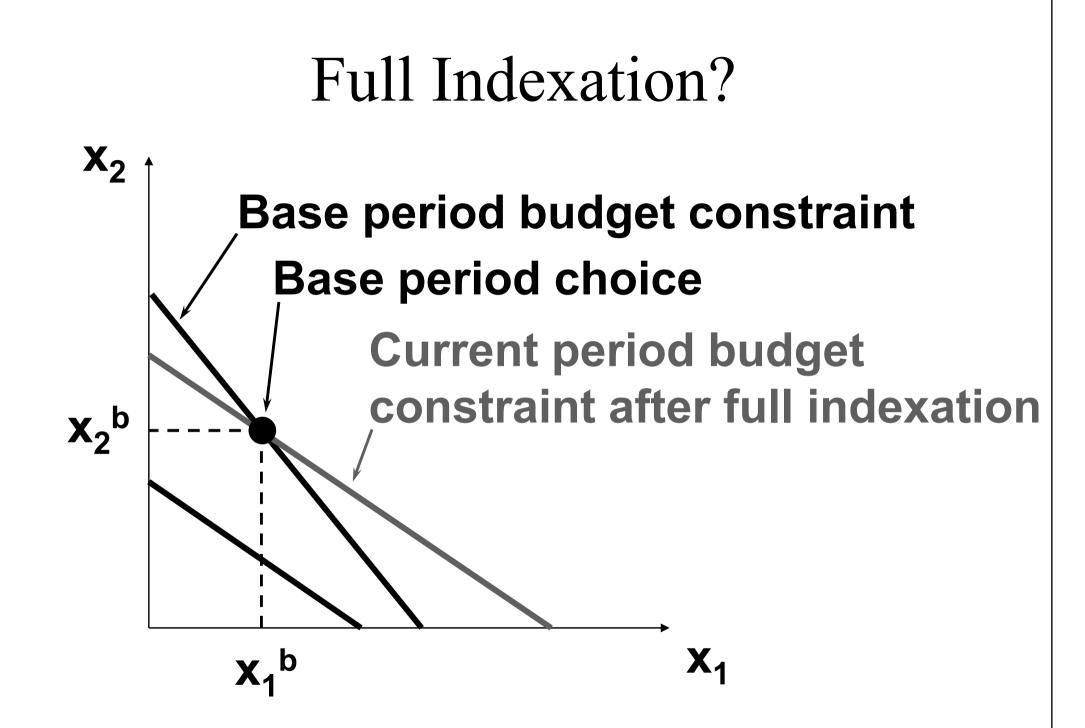
 The usual price index proposed for indexation is the Paasche quantity index (the Consumers' Price Index).
 What will be the consequence?

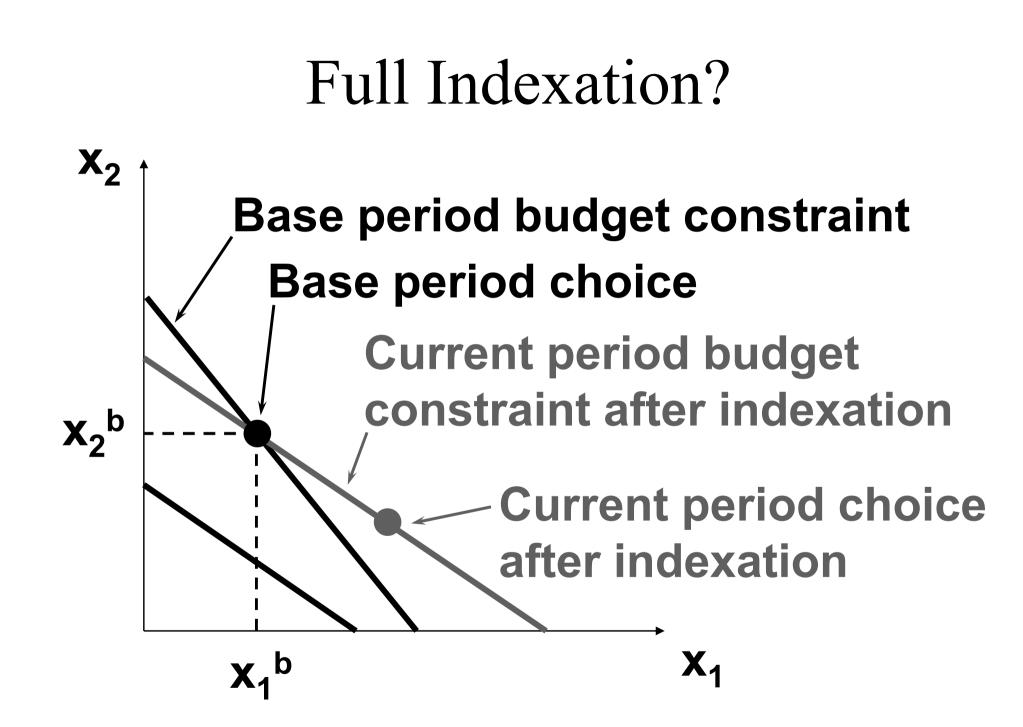
$$P_{q} = \frac{p_{1}^{t}x_{1}^{t} + p_{2}^{t}x_{2}^{t}}{p_{1}^{t}x_{1}^{b} + p_{2}^{t}x_{2}^{b}}$$

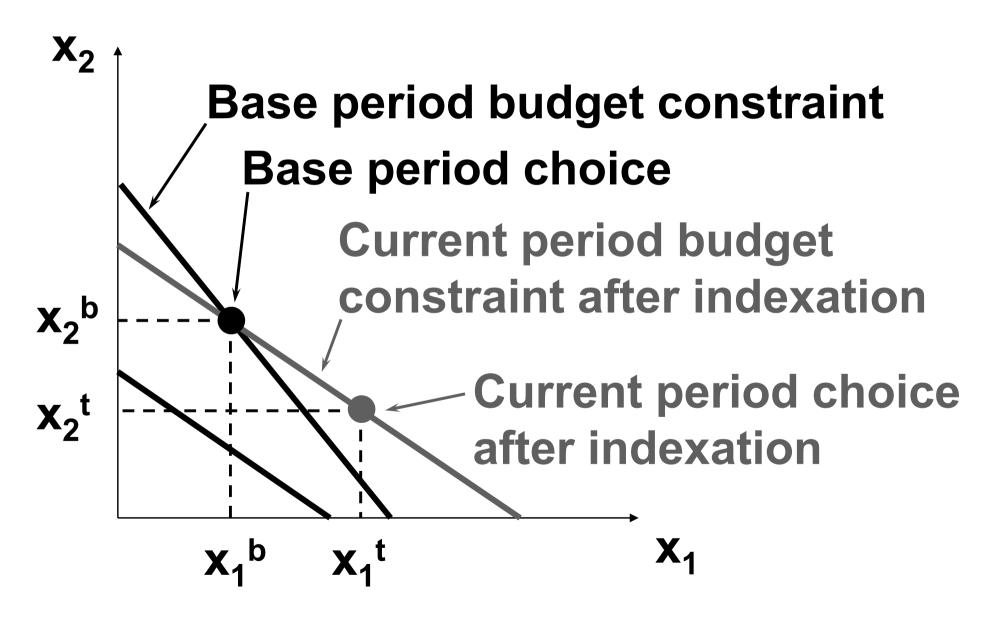
Notice that this index uses current period prices to weight both base and current period consumptions.

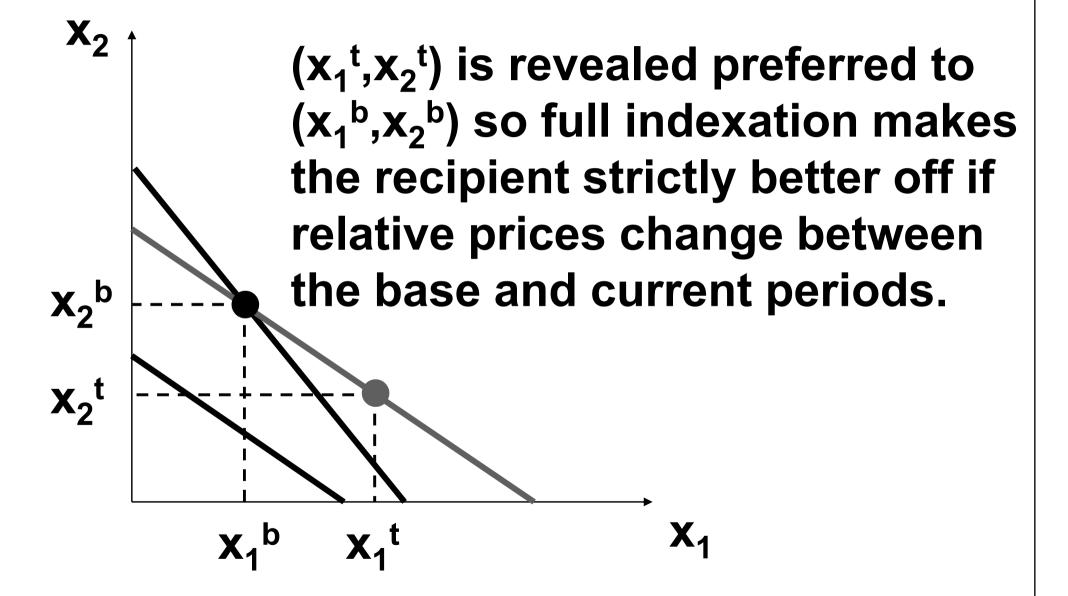












- So how large is this "bias" in the US CPI?
- A table of recent estimates of the bias is given in the Journal of Economic Perspectives, Volume 10, No. 4, p. 160 (1996). Some of this list of point and interval estimates are as follows:

Full Indexation?		
Author	Point Est.	Int. Est.
<b>Adv. Commission to</b>	1.0%	0.7 - 2.0%
<b>Study the CPI (1995)</b>		
Congressional		0.2 - 0.8%
<b>Budget Office (1995)</b>		
Alan Greenspan		0.5 - 1.5%
(1995)		
Shapiro & Wilcox	1.0%	0.6 - 1.5%
(1996)		

- So suppose a social security recipient gained by 1% per year for 20 years.
- Q: How large would the bias have become at the end of the period?

- So suppose a social security recipient gained by 1% per year for 20 years.
- Q: How large would the bias have become at the end of the period?
- ♦ A:  $(1+0.01)^{20} = 1.01^{20} = 1.22$  so after 20 years social security payments would be about 22% "too large".