INTERNATIONAL PARITY CONDITIONS

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1. The Theory of Covered Interest Rate Parity

- The interest rate differential between two currencies approximately equals the percentage spread between the currencies' forward and spot rate.
- The intuition behind interest rate parity
  - **Future value of one unit of currency depends on interest rate for that currency**
  - Interest rate parity
    - Covered interest rate parity refers to a theoretical condition in which the relationship between interest rates and the spot and forward currency values of two countries are in equilibrium.
    - The covered interest rate parity condition says that the relationship between interest rates and spot and forward currency values of two countries are in equilibrium.
    - Relationship between forward/spot rates and the interest rate differential between two countries
      - $F(h/f)/S(h/f) = (1 + i_f)/(1 + i_h)$
  - Why there must be interest rate parity
    - If not, arbitrage possibilities would exist (borrowing any government controls)

1. The Theory of Covered Interest Rate Parity

- $10M to invest, iUS = 8%; iUK = 12%; S = $1.60/£; F1-year = $1.53/£
- *Buy 1year US bond or 1year UK bond? (**US investor** wants to have the highest US return)*
- Steps:
  - Convert using spot rate: $\$10M \div (\$1.60/£) = £6.25M$
  - Invest at foreign interest rate: $£6.25M \times 1.12 = £7M$
  - Convert back at forward rate: $£7M \times (\$1.53/£) = \$10.71M$
  - Compare to what you could have earned by just investing in your home nation:
    - $\$10M \times (1 + 0.08) = \$10.8M$
  - Investing at home (U.S.) is more profitable for Kevin.
- **But what if he could borrow or lend? Is the answer still the same?**

1. The Theory of Covered Interest Rate Parity

- $10M to invest, iUS = 8%; iUK = 12%; S = $1.43/£; F1-year = $1.53/£
- Steps:
  - Borrow pounds: $£1M \times 1.12 = £1.12M$ (what Kevin owes at end of investment term)
  - Convert pounds to dollars: $£1.12M \times (\$1.43/£) = \$1.6M$
  - Invest at U.S. interest rate: $£1.6M \times 1.08 = \$1.728M$
  - Convert back at forward rate: $\$1.728M/(\$1.53/£) = £1{,}129{,}411.76$
  - Profit: $£1{,}129{,}411.76$ - $£1{,}120{,}000 = £9{,}411.76$
- Kevin would make £9,411.76 (Step 4 – Step 1) profit for every £1M that is borrowed!
  - Think of doing the transaction the other way.
- This arbitrage activity would quickly eliminate the profit opportunity by rising pound IR and the sale of pounds for dollar would lower dollar–pound spot exchange rate (pound depreciates).

1. The Theory of Covered Interest Rate Parity

- Deriving interest rate parity
  - When the forward rate is priced correctly, an investor is indifferent between investing at home or abroad
  - General expression for interest rate parity
    - $[1 + i*] = [1/S] \times [1 + i] \times F$
    - *i ** the foreign currency interest rate appropriate for one period, *i* the domestic currency interest rate appropriate for one period, *S* the spot exchange rate (domestic currency per foreign currency), *F* the one-period forward exchange rate (domestic currency per foreign currency)
  - Interest rate parity and forward premiums and discounts
    - $(1 + i*)/(1 + i) = F/S$
  - Subtracting 1 from each side and simplifying we obtain
    - $(i* - i)/(1 + i) = (F - S)/S$
    - If this equation is (+), the forward is selling at a premium
    - If it is (-), the forward is selling at a discount
  - With continuously compounded interest rates
    - $(i - i*) = \ln(F) - \ln(S)$

Problem solving

- Suppose the 5-year interest rate on a dollar-denominated pure discount bond is 4.5% p.a., whereas in France, the euro interest rate is 7.5% p.a. on a similar pure discount bond denominated in euros. If the current spot rate is 0.9259 (EUR/USD), what is the value of the forward exchange rate that prevents covered interest arbitrage?

- F (t,EUR/USD) = S(t,EUR/USD) x $\frac{(1+i_f)}{(1+i_h)}$, i_h = home/base currency
- F (t,5,EUR/USD) = S(t, EUR/USD) x $\frac{(1+i(t,5,EUR)^5}{(1+i(t,5,USD)^5}$
- F (t,5,EUR/USD) = 0.9259 x $\frac{(1,045)^5}{(1,075)^5}$ = 0.868

Problem solving

USD 3 months (90 days) Term Deposit Interest Rate : 3.5%
JPY 3 months (90 days) Term Deposit Interest Rate : 0.1%
Spot Rate USD/JPY = 120.10

- F-S = S x $(\frac{i_{JPY} - iUSD}{1+i_{USD}})$
- F-S = 120.1 X {(0.001 - 0.035) X 90/360} / (1 + 0.035 X 90/360) = -1.02
- F = 120.1 - 1.02 = 119.08

Problem solving

- If the 30-day yen interest rate is 3% p.a., and the 30-day euro interest rate is 5% p.a., is there a forward premium or discount on the euro in terms of the yen? What is the magnitude of the forward premium or discount?
- We know that the high interest rate currency must sell at a forward discount when priced in the low interest rate currency to prevent a covered interest arbitrage. Therefore the euro is at a discount in the forward market.

$$\frac{F^{EUR}/_{JPY} - S^{EUR}/_{JPY}}{S^{EUR}/_{JPY}} = \frac{i_{JPY} - iEUR}{(1+iEUR)}$$

The **de-annualized interest rates** are 0.0025 = (3/100) x (30/360) for the yen and 0.004167 = (5/100) x (30/360) for the euro. The right-hand side of the above expression is therefore – 0.00166. The annualized value is -0.00166 x (100) x (360/30) = -1.99%. We therefore say that the euro sells at an annualized discount of 1.99%

1. The Theory of Covered Interest Rate Parity

- Spot exchange rate: ¥146.03/EUR
- 1-year forward exchange rate: ¥141.9021/EUR
- EUR interest rate: 3.5200% p.a.
- JPY interest rate: 0.5938% p.a.

- % change of FX rate: $\frac{F-S}{S} = \frac{141.9021 - 146.03}{146.03} = 2.82\%$

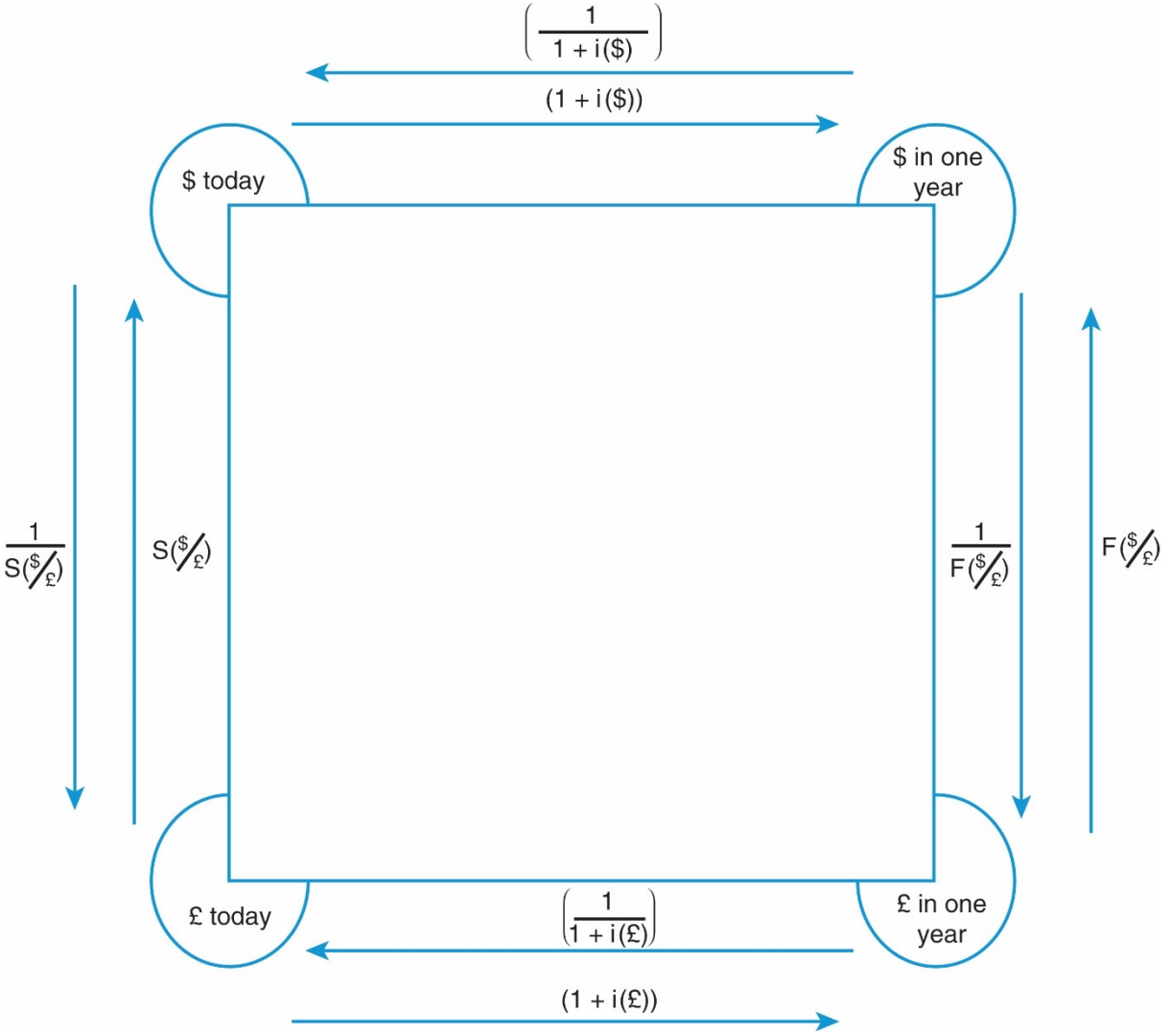
- Interest rate differential (Euro - Yen) is: 3.52% - 0.5938% = 2.9262%, which is approximately equal to the forward premium. $(1-i_{JPY})/(1+i_{EUR})$
- Premium compensates for the lower interest rate that yen investments offer.

Diagram of Covered Interest Arbitrage

For example:

If you are at the node representing pounds today and you move 1 pound to the future, the future pound revenue is (1 + i (£)).

If you place yourself at the dollar node in the future, and you move 1 dollar to the present, you receive 1 / (1 + i($)) dollars in the current period.



Interest Rates in the External Currency Market

- The external currency market is a bank market for deposits and loans that are denominated in currencies that are not the currency of the country in which the bank is operating.
- The first of these deposits and loans were called eurodollars because they were dollar denominated deposits at European banks.
- To determine the appropriate interest rate for a 3-month basis, we must "de-annualize" the quoted interest rates.
- Bid – deposit, Ask- loan
- Quated as % p.a
- $De - Annualised\ rate = Ann.rate \times \left(\frac{1}{100}\right) \times \left(\frac{num.\ of\ days}{360}\right)$

Exhibit 6.2 Interest rates in the external currency market

			Currency				
			USD	EUR	GBP	JPY	CHF
Maturity	1 Month	Bid	0.15	–0.18	0.45	–0.15	–
		Ask	0.25	–0.03	0.55	0.05	–0.82
	3 Month	Bid	0.23	–0.14	0.52	–0.05	–
		Ask	0.33	0.01	0.62	0.15	–0.78
	6 Month	Bid	0.35	–0.08	0.66	–0.05	–
		Ask	0.45	0.07	0.81	0.15	–0.70
	1 Year	Bid	0.62	0.06	0.98	0.00	–
		Ask	0.72	0.21	1.13	0.20	–0.58

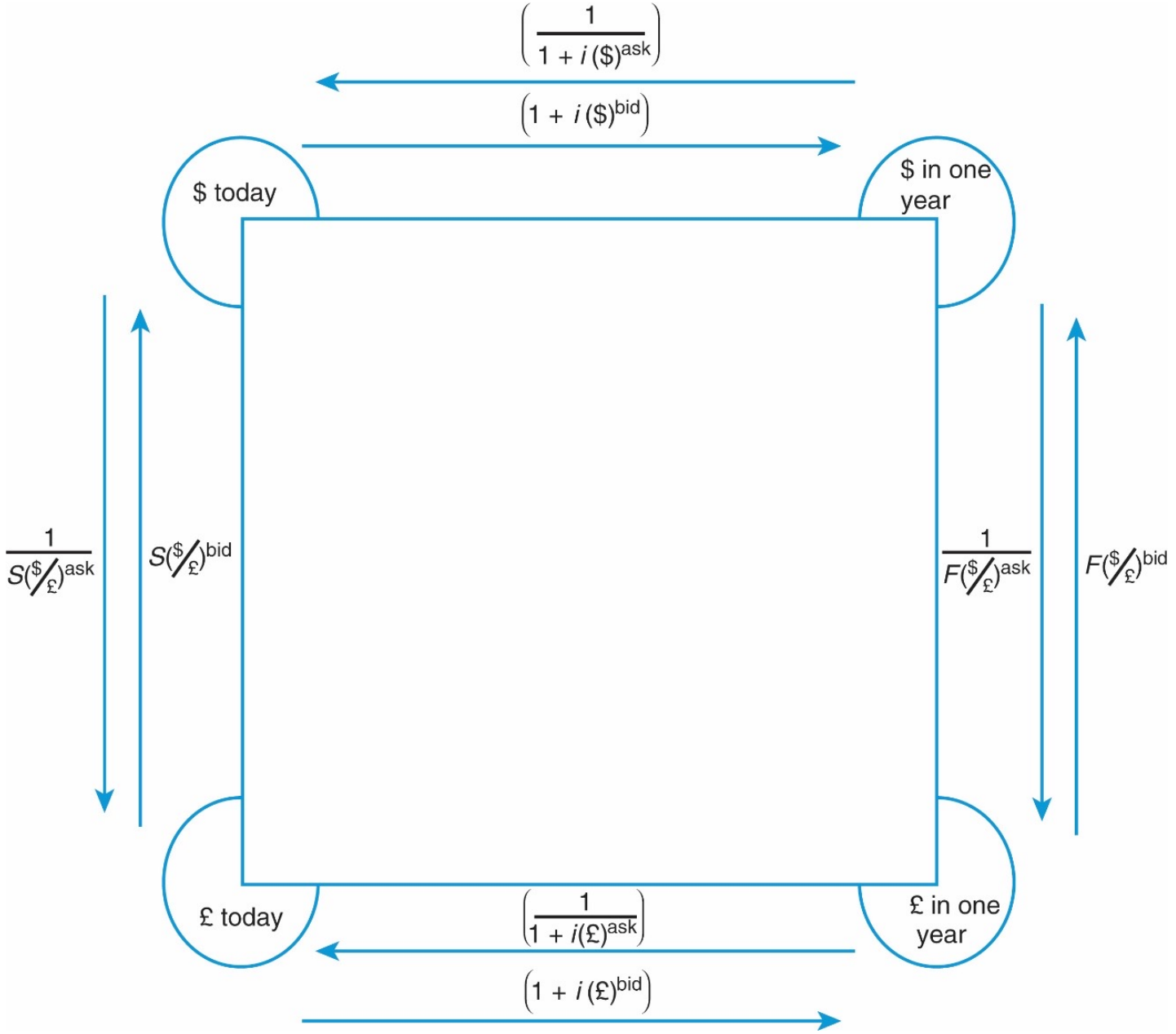
2. Covered Interest Rate Parity in Practice

- Cross currency market
  - External currency market influences rates elsewhere
  - Loans to investors/corporations are based on these interbank rates
  - Most important interbank reference rate is LIBOR. **LIBOR** is the average interbank interest rate at which a **selection of banks on the London money market are prepared to lend to one another**. Since the beginning of 2022, LIBOR comes in max 5 maturities (from overnight to 12 months) and in 3 different currencies. The official LIBOR interest rates are announced once per working day at around 11:45 a.m.
  - For actual Libor rates see: https://www.wsj.com/market-data/bonds

Covered Interest Rate Parity with Bid-Ask Rates (transaction costs)

Transaction costs

- Reduced regulatory burden and strong competition result in very small spreads



An Example with Transaction Costs

- Convert $10M to yen:
  - $\$10M \times ¥82.67/\$ = ¥826.7M$
- Invest for 3 months
  - $0.46 \times (1/100) \times (90/360) = 0.00115$
  - $¥826.7M \times 1.00115 = ¥827{,}650{,}705$
- Sell forward (enter into forward contract)
  - $(¥827{,}650{,}705)/(¥82.6495/\$) = \$10{,}013{,}983$
- Compare to what we would make in US
  - $\$10{,}013{,}983 - (\$10M \times 1.002275)$
  - 0.91x(1/100)x90/360) = 0.002275
- Lose money this way
  - No arbitrage, but borrowing yen also results in losses

	Bid	Ask
Spot exchange rates (¥ per $):	82.67	82.71
Forward exchange rates (¥ per $):	82.5895	82.6495
Dollar interest rates:	0.91	1.11
Yen interest rates:	0.46	0.58

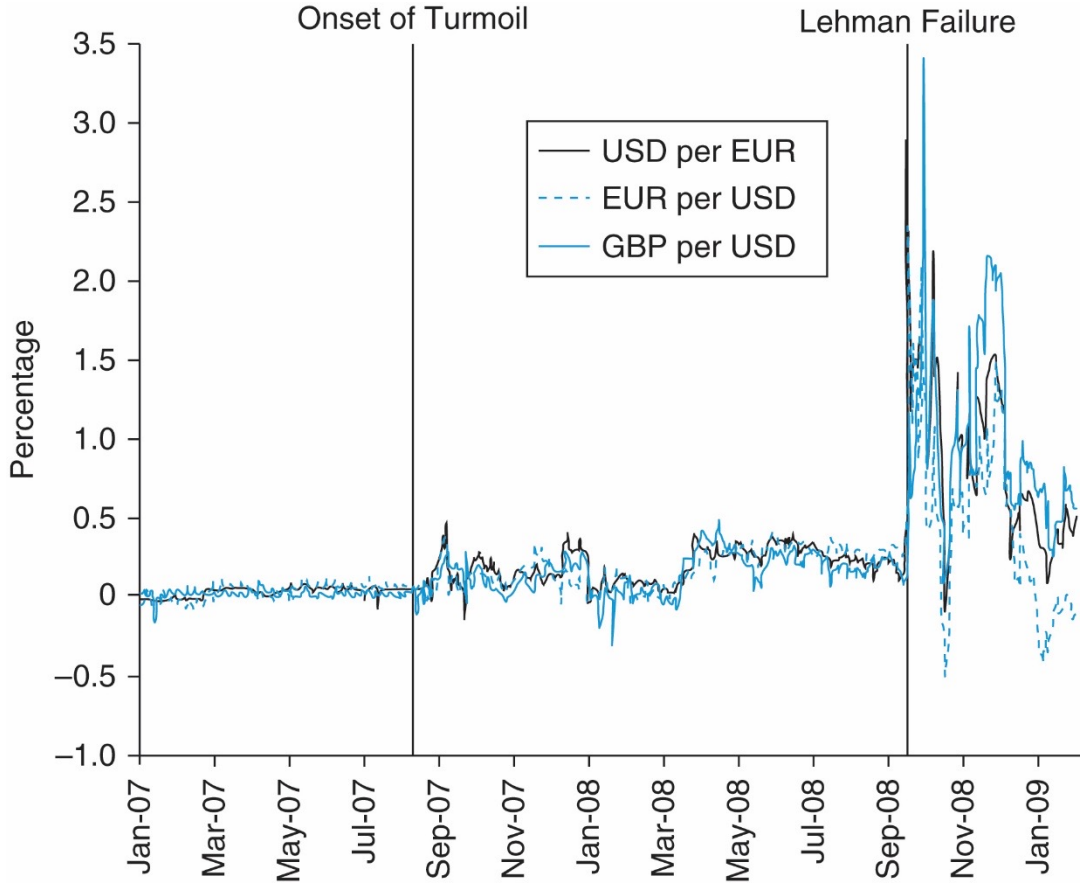
2. Covered Interest Rate Parity in Practice

- Does covered interest rate parity hold?
  - Prior to 2007, documented violations of interest rate parity were very rare
    - Akram, Rime, and Sarno (2008) – multiple short-lived deviations that persist for only a few minutes
  - Frequency, size and duration of apparent arbitrage opportunities do increase with market volatility
    - 2007-2009 financial crisis

3. Why Deviations from Interest Rate Parity May Seem to Exist

- Too good to be true?
  - Default risks
    - Risk that one of the counterparties may fail to honor its contract
  - Exchange controls
    - Limitations
    - Taxes (in interests)
  - Political risk
    - A crisis in a country could cause its government to restrict any exchange of the local currency for other currencies
    - Investors may also perceive a higher default risk on foreign investments
  - Liquidity

Covered Interest Parity Deviations During the Financial Crisis



4. Hedging Transaction Risk in the Money Market

- Zachy's: Importing wine for €4M, payable in 90 days
  - S: \$1.10/€; F(t+90): \$1.08/€; i(\$, t+90): 6.00% p.a; i(€, t+90): 13.519% p.a.
  - Choice #1: **Enter into a forward contract**
    - Cost in 90 days: $€4M \times (\$1.08/€) = \$4.32M$
  - Choice #2: **Money Market hedge**
    - You have euro liability, you must acquire euro asset
    - Invest X amount now that becomes what you owe in 90 days
    - $X = €4M/[1 + (\frac{13.519}{100})(\frac{90}{360})] = €3{,}869{,}229.71$ (PV of future euro payment $€4M$)
    - $X\ at\ spot\ rate = \$3{,}869{,}229.71 \times \$1.10/€ = \$4{,}256{,}152.68$ (I need to buy €3,869,229.71 at spot market)
    - PV of forward hedge (1st option)
      - $\$4.32M/\left[1 + \left(\frac{6.00}{100}\right)\left(\frac{90}{360}\right)\right] = \$4{,}256{,}157.64$
    - Forward contract is more expensive by \$4.96

4. Hedging Transaction Risk in the Money Market

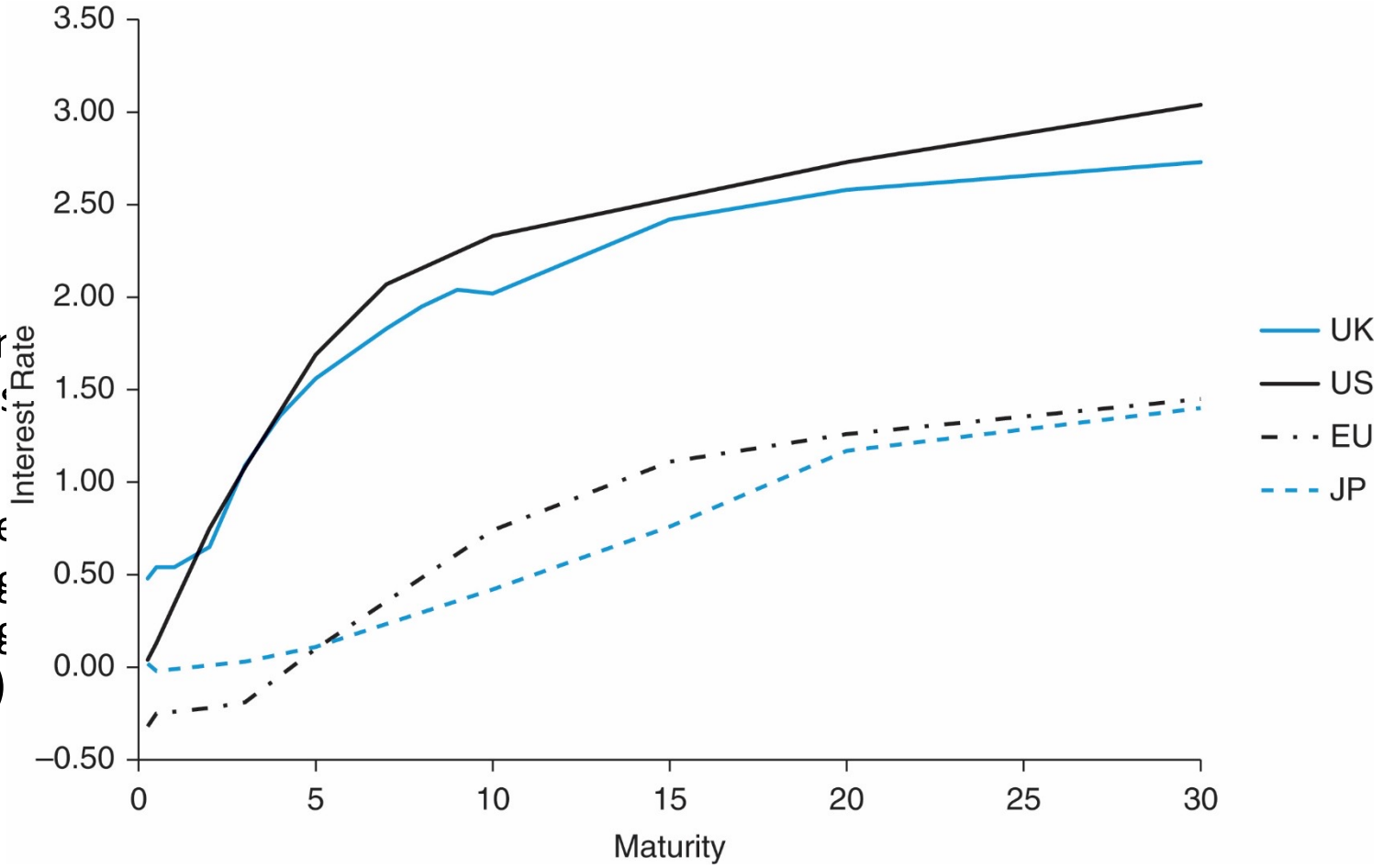
- Shetlant: Receive ¥500M in 30 days, you are British manufacturer
  - S: ¥179.5/£; F(t+30): ¥180/£; i(£, t+30): 2.70% p.a; i(¥, t+30): 6.01% p.a.
  - Choice #1: Sell yen forward
    - Earn: $¥500M/(¥180/£) = £2{,}777{,}778$
  - Choice #2: Money Market hedge
    - Borrow PV of ¥500M, and sell at spot
      - $PV = ¥500M/[1 + (\frac{6.01}{100})(\frac{30}{360})] = ¥497{,}508{,}313$
      - $£\,revenue = ¥497{,}508{,}313/(¥179.5/£) = £2{,}771{,}634$
    - FV of forward hedge
      - $£2{,}771{,}634 \times \left[1 + \left(\frac{2.70}{100}\right)\left(\frac{30}{360}\right)\right] = £2{,}777{,}785$
    - Forward contract is more expensive by £6,151

5. The Term Structure of Forward Premiums and Discounts

- The term structure of interest rates
  - Description of different spot interest rates for various maturities into the future
  - Rates derived by:
    - For **shorter maturities**, spot interest rates are directly **observable** because they are widely **quoted** by banks.
    - For **longer maturities**, the spot interest rates **derived from the market prices of coupon-paying** bonds.
    - Recent empirical evidence suggests that covered interest rate parity does not hold perfectly at longer horizons.

Yield Curves for Four Currencies

- The slope of the yield curve gives ar idea of future interest rate change and economic activity
- Three main shapes of yield curve shapes: normal (upward sloping curve), inverted (downward sloping curve - point to economic recession) and flat.



- Watch video

5. The Term Structure of Forward Premiums and Discounts

- A review of bond pricing
  - **Pure discount bonds** (simplest bonds) promise a single payment of, say, $1,000 at the maturity of the bond.
  - Price of a 10-year pure discount bond with a face value of $1,000 is $463.19
    - What is the spot interest rate for the 10-year maturity expressed in percentage per annum?
      - $\$463.19 \times [1 + i(10)]^{10} = \$1{,}000$
      - $i = 8\%$
  - The **spot interest rate** as the market interest rate that equates the price of a pure discount bond to the present value of the face value of the bond.

5. The Term Structure of Forward Premiums and Discounts

- Yield to maturity
  - The **discount rate** that equates **the present value of the n coupon payments plus the final principal** payment to the current market price
  - A 2-year bond with face value of $1,000, an annual coupon of $60, and a market price of $980
    - If the 1-year spot rate is 5.5%, the 2-year spot rate is found by solving:
      - $\$980 = (\$60/1.055) + (\$1060/1 + i(2)^2)$
      - $i(2) = 7.1574\%$

5. The Term Structure of Forward Premiums and Discounts

- Long-term forward rates and premiums
  - Let $i(2, ¥)$ and $i(2, \$)$ denote the spot interest rates for yen and dollar investments with 2-year maturities
    - If no arbitrage opportunities exist, then the rate of yen per dollar for the 2-year maturity must be:
      - $F(2) = S \times [1 + i(2, ¥)]^2 / [1 + i(2, \$)]^2$
  - Spot: ¥110/$; i(2,$) = 5% p.a.; i(2, ¥) = 4% p.a.; ¥10M to invest
    - Investing in Japan: (you invest 10,000,000 JPY in a 2-year yen pure discount bond. At the end of 2 years, your investment will grow to:
      - $¥10M \times (1.04)^2 = ¥10.816M$
      - $90,909.09 at current spot the dollar cost of $¥10M$
    - Investing at home:
      - $\$90{,}909.09 \times (1.05)^2 = \$100{,}227.27$
    - You would be indifferent between the two investments if the forward sale of $¥10.816M$ for dollars provides you with the same dollar return as investing directly in dollars $¥10.816M / F(2) = \$100{,}227.27$

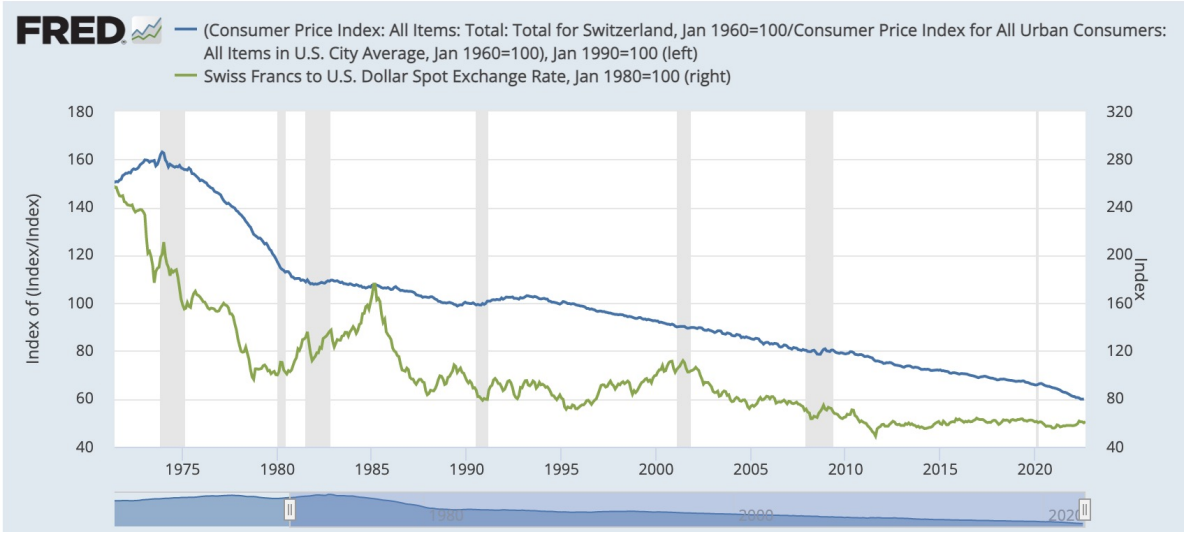
    F(2) = ¥107.9147/$

6. Relative Purchasing Power Parity

- Relative Purchasing Power Parity
  - Exchange rates adjust in response to differences in inflation across countries
  - Logic is that inflation lowers the purchasing power of money, so a change in the nominal exchange rate to compensate for different levels of inflation should occur
  - Purchasing power is the power of money expressed by the number of goods or services that one unit can buy, and which can be reduced by inflation. RPPP suggests that countries with higher rates of inflation will have a devalued currency.
  - The Big Mac Index

Relative PPP

- Relative PPP, which states that the percentage change in the exchange rate is equal to the difference in the percentage changes in average prices—that is, the inflation rate. (Formally: $(E_t-E_{t-1})/E_{t-1}= \pi(CH)_t-\pi(USA)_t$, where $\pi(x)_t$ is the inflation rate in country x at time t.)
- Source and more reading about RPPP



7. The Real Exchange Rate

- The real exchange rate
  - Adjusted for inflation:
    - $RS(t, \$/€) = \frac{S(t,\$/€) \times P(t,€)}{P(t,\$)}$
- Real appreciations and real depreciations
  - Three basic movements:
    - An increase in the nominal exchange rate ($/£), holding $ prices and £ prices constant
    - An increase in the £ prices of goods holding the $ prices of goods constant
    - An increase in the $ prices of goods holding the £ prices of goods constant
- Trade-weighted real exchange rates
  - Useful when looking at how forex changes will affect trade balance

8. Parity Conditions and Exchange Rate Forecasts

- The International Parity Conditions
  - CIRP – Covered Interest Rate Parity
    - Links forward rates, spot rates, and interest rate differentials
  - UIRP or Unbiasedness – Uncovered Interest Rate Parity
    - Sometimes called International Fisher Effect / Relationship
    - Links expected exchange rate changes and interest rate differentials
  - PPP
    - Links inflation rates and rates of changes in forex rates

8. Parity Conditions and Exchange Rate Forecasts

- The Fisher Hypothesis long-run link between inflation and interest rates the nominal interest rate is the sum of the expected real interest rate and the expected rate of inflation.

  - Real rates of return – measures how much your purchasing power has increased over time
  - The ex post real interest rate
  - The ex ante real interest rate
    - Expected real interest rate
    - Expected rate of inflation
    - Fisher hypothesis – decomposition of nominal int. rates (real interest rates + inflation)

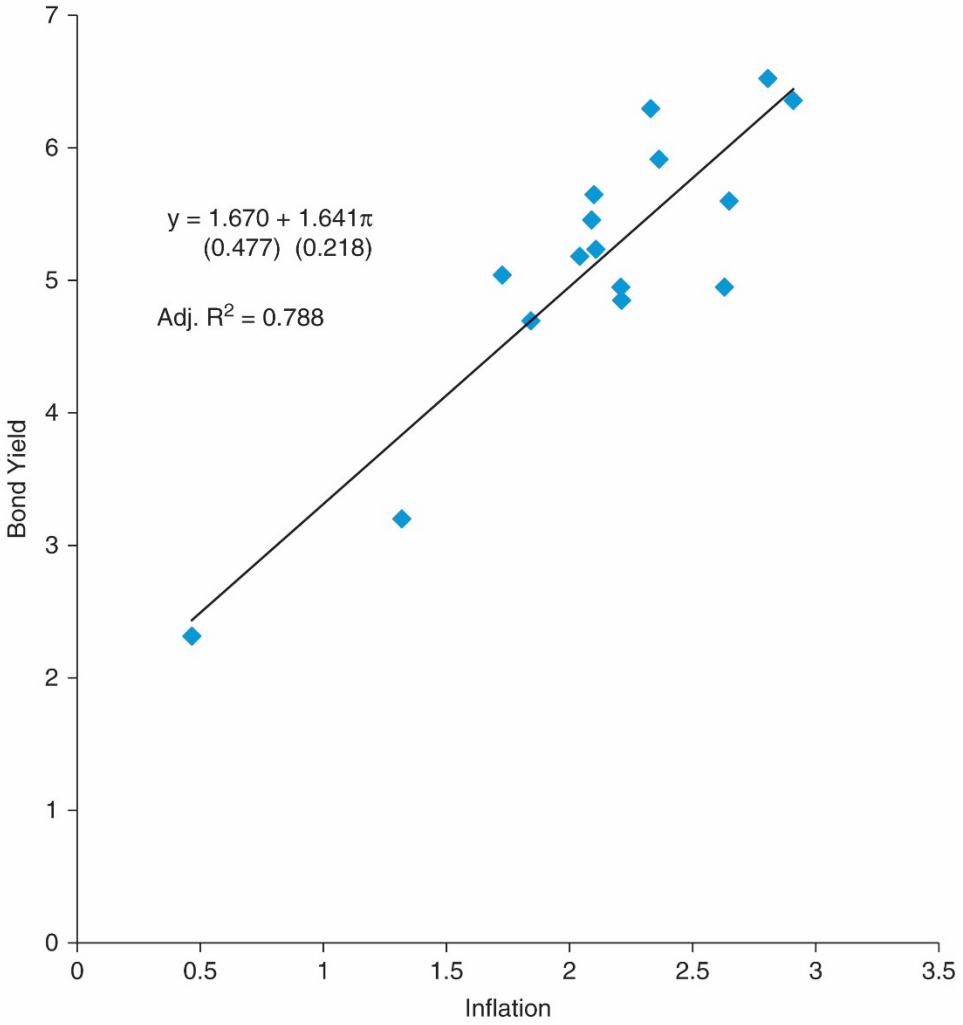
Problem solving:

- Suppose the nominal interest rate in Mexico is 10%, and the expected rate of inflation in Mexico is 7%.
  - What is the expected real rate of return in Mexico?

$$r^e = \frac{i - \pi^e}{1 + \pi^e}.$$

  - Simplified: r^e = 10% - 7% = 3%
  - By investing pesos at a nominal interest rate of 10% when the expected rate of inflation is 7%, the investor expects to earn a 3% real rate of return. The investor expects to have 3% more purchasing power over goods and services at the end of the year for every peso invested.
  - The real interest rate is important because it influences investment decisions.

Average Long-Term Government Bond Yields and Inflation Rates



An Example of International Parity Conditions: covered interest rate parity (CIRP), uncovered interest rate parity (UIRP), and purchasing power parity (PPP), the Fisher hypothesis The United Kingdom and Switzerland

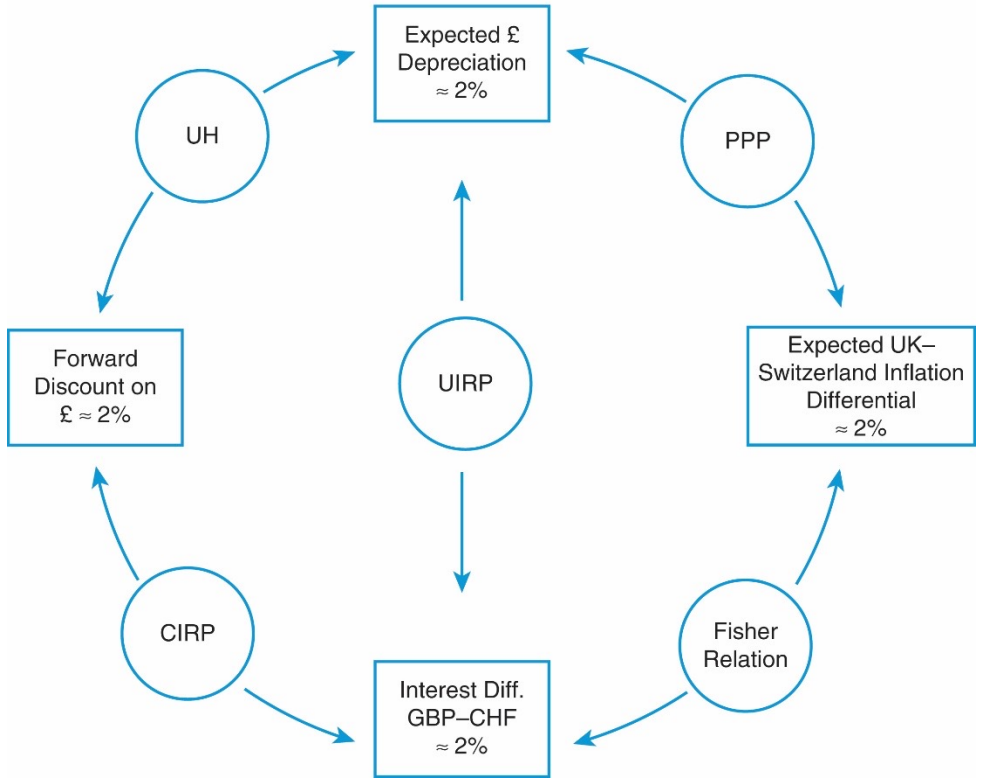
Nominal Interest Rates:
UK: $i^{GBP} = 5.00\%$. Switzerland: $i^{CHF} = 3.00\%$

Exchange Rates:
Spot Rate: S = CHF2.50/GBP Forward Rate: F = CHF2.4524/GBP

Forward Premium : $\left[\frac{2.4524}{2.50} - 1\right] = -1.90\%$

Expected Exchange Rate Change: −1.90%

Expected Inflation:
UK: 2.94% Switzerland: 0.98%



If the parity conditions all hold simultaneously, real interest rates are equal across countries.
If uncovered interest rate
parity and PPP hold, the nominal interest rate differential between the United Kingdom and
Switzerland reflects only an expected inflation differential.

8. Parity Conditions and Exchange Rate Forecasts

- International parity conditions
  - CIRP, UIRP (or Unbiasedness) and PPP
    - If they hold, real interest rates are the same everywhere
  - Empirical studies suggest that beyond CIRP, none hold in either long-run or short-run (CIRP may hold in the short run)
    - If International Fisher Relationship holds, the interest rates below would be equal – closer in long-run, but why not equal?
      - Significant deviations in PPP values
      - Returns in different currencies can have different risk premiums
      - Political risk and threat of capital control

Questions:

- If interest rate parity is satisfied, there are no opportunities for covered interest arbitrage. What does this imply about the relationship between spot and forward exchange rates when the foreign currency money market investment offers a higher return than the domestic money market investment?
- Describe the sequence of transactions required to do a covered interest arbitrage out of Japanese yen and into U.S. dollars.
- Suppose you are the French representative of a company selling soap in Canada. Describe your foreign exchange risk and how you might hedge it with a money market hedge.
- What do economists mean by the external currency market?
- What is the term structure of interest rates?