## BKM_DATS: Databázové systémy 9. Query Processing and Relational Algebra

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## Query Processing

ロ Overview
$\square$ Evaluation of Expressions

- Measures of Query Cost

■ Evaluation algorithms
$\square$ Sorting
$\square$ Join Operation

## Basic Steps in Query Processing

1. Parsing and translation
2. Optimization
3. Evaluation


## Basic Steps in Query Processing (Cont.)

- Parsing and translation
$\square$ Translate the SQL query into its internal form.
$\square$ This is then translated into relational algebra.
$\square$ Parser checks syntax, verifies relations
- Optimization
$\square$ Generate a query-evaluation plan and choose algorithms for evaluating individual operations
- Evaluation
$\square$ The query-execution engine takes a query-evaluation plan, executes that plan, and returns the answers to the query.


## Basic Steps in Query Processing (Cont.)

- Example of query:
$\square$ List salary of all instructors that earn less than $\$ 75,000$.
- SQL query
$\square$ SELECT salary FROM instructor WHERE salary < 75000
- Conversion to rel. algebra
$\square \prod_{\text {salary }}\left(\sigma_{\text {salary }}>5000(\right.$ instructor) $)$


## Basic Steps: Optimization

- A relational-algebra expression may have many equivalent expressions:
$\square \prod_{\text {salary }}\left(\sigma_{\text {salary<75000 }}(\right.$ instructor $)$ )
$\square \quad \sigma_{\text {salary }<75000}\left(\prod_{\text {salary }}\right.$ (instructor))
- For a relational-algebra expression, an expression tree is created



## Basic Steps: Optimization (Cont.)

- Each relational algebra operation can be evaluated using one of several different algorithms
$\square$ Correspondingly, a relational-algebra expression can be evaluated in many ways.
$\square$ Annotated expression specifying detailed evaluation strategy is called an execution-plan or evaluation-plan.
$\square$ E.g., to find instructors with salary < 75000
- use an index on salary, or
- perform complete relation scan and discard instructors with salary $\geq 75000$


## Basic Steps: Optimization (Cont.)

- Example of an evaluation-plan



## Basic Steps: Optimization (Cont.)

- Query Optimization
$\square$ Amongst all equivalent evaluation plans choose the one with lowest cost.
$\square$ Cost is estimated using statistical information from the database catalog
- E.g., number of tuples in each relation, size of tuples, etc.
$\square$ There is a huge number of possible evaluation plans
$\square$ Optimization uses some heuristics

1. Perform selection early
reduce the number of tuples (by using an index, e.g.)
2. Perform projection early
$\square$ reduce the number of attributes
3. Perform most restrictive operations early

- such as join and selection.


## Evaluation of Expressions

- Alternatives for evaluating an entire expression tree
$\square$ Materialization
- Evaluate one operation at a time, starting at the lowest-level.
- Use intermediate results materialized into temporary relations to evaluate next-level operations.
$\square$ Pipelining
- pass on tuples to parent operations even as an operation is being executed


## Evaluation of Expressions (Cont.)

- Materialized evaluation
$\square$ Compute $\sigma_{\text {buiding }}=$ Watson'(department) ) and store it
$\square$ Then read from stored intermediate result and compute its join with instructor, store it
$\square$ Finally read it and compute the projection on name and output it.
- This step can be conveniently evaluated using pipelining on join result.



## Measures of Query Cost

— Cost is generally measured as total elapsed time for answering query
$\square$ Many factors contribute to time cost

- disk accesses, CPU, or even network communication
$\square$ Typically disk access is the predominant cost and is also relatively easy to estimate. Measured by taking into account
$\square$ Number of seeks * average-seek-cost
$\square$ Number of blocks read * average-block-read-cost
$\square$ Number of blocks written * average-block-write-cost
- Cost to write a block is greater than cost to read a block

Data is read back after being written to ensure that the write was successful

## Measures of Query Cost (Cont.)

- For simplicity we just use the number of block transfers from disk and the number of seeks as the cost measures
$\square t_{T}$ - time to transfer one block
$\square t_{S}$ - time for one seek
$\square$ Cost for $b$ block transfers plus $S$ seeks

$$
b^{*} t_{T}+S^{*} t_{S}
$$

$\square$ We ignore CPU costs for simplicity
$\square$ Real systems do take CPU cost into account

- We do not include cost to writing output to disk in our cost formulae


## Measures of Query Cost (Cont.)

— Several algorithms can reduce disk I/O by using extra buffer space
$\square$ Amount of real memory available to buffer depends on other concurrent queries and OS processes, known only during execution
$\square$ We often use worst case estimates, assuming only the minimum amount of memory needed for the operation is available
$\square$ Required data may be buffer resident already, avoiding disk I/O
$\square$ But hard to take into account for cost estimation

## Relational Algebra

$\square$ Procedural language
$\square$ Six basic operations
$\square$ Select: $\sigma$

- Project: П
$\square$ Union: $\cup$
- Set difference: -
$\square$ Cartesian product: $\times$
$\square$ Rename: $\rho$
$\square$ Principle:
$\square$ An operation takes one or two relations as input and produce a new relation as a result.
- So, another operation can be applied to this result.
$\square$ Note: SQL is a declarative language.


## Select and Project Operations: Example

- Relation $r$

| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | $\alpha$ | 1 | 7 |
| $\alpha$ | $\beta$ | 5 | 7 |
| $\beta$ | $\beta$ | 12 | 3 |
| $\beta$ | $\beta$ | 23 | 10 |

- $\sigma_{A=B \wedge D>5}(r)$

| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | $\alpha$ | 1 | 7 |
| $\beta$ | $\beta$ | 23 | 10 |

$\square \prod_{\mathrm{A}, \mathrm{C}}(r)$

| $A$ | $C$ |
| :---: | :---: |
| $\alpha$ | 1 |
| $\alpha$ | 5 |
| $\beta$ | 12 |
| $\beta$ | 23 |

## Select Operation

$\square$ Select operation is defined as:

$$
\sigma_{p}(\boldsymbol{r})=\{t \mid t \in r \wedge p(t)\}
$$

where $p$ is a formula in propositional calculus:

$$
\left.\begin{array}{ll}
\text { formula := } & \text { term } \\
& \text { formula <conj> formula } \\
& \neg \text { formula } \\
& (\text { formula })
\end{array}\right] \begin{array}{ll}
\text { term }:= & \text { expr <cmp> expr } \\
\text { expr }:= & \text { attribute_name } \\
& \text { constant }
\end{array}
$$

— Project operation is defined as: $\prod_{A_{1}, \ldots, A_{k}}(r)$

$$
=\left\{t \mid \exists q \in r: t\left[A_{1}\right]=q\left[A_{1}\right] \wedge \cdots \wedge t\left[A_{k}\right]=q\left[A_{k}\right]\right\}
$$

where $A_{i}$ are attribute names and $r$ is a relation name.

## Union, Set Difference, Intersect Operations

( Relations r, s:

| $A$ | $B$ |
| :---: | :---: |
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$ | 1 |


$\square \quad \mathrm{r} \cup \mathrm{s}$

| $A$ | $B$ |
| :---: | :---: |
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$ | 1 |
| $\beta$ | 3 |

प $r-s$

| $A$ | $B$ |
| :---: | :---: |
| $\alpha$ | 1 |
| $\beta$ | 1 |

$\square \quad r \cap s$


## Cartesian product and Operation Composition

■ Can build complex expressions using multiple operations
$\square$ Example: $\sigma_{A=C}(r \times s)$
$\square$ Relations:


$r \times s:$| $A$ | $B$ | $C$ | $D$ | $E$ |
| :--- | :--- | :--- | :--- | :--- |
| $\alpha$ | 1 | $\alpha$ | 10 | $a$ |
| $\alpha$ | 1 | $\beta$ | 10 | $a$ |
| $\alpha$ | 1 | $\beta$ | 20 | $b$ |
| $\alpha$ | 1 | $\gamma$ | 10 | $b$ |
| $\beta$ | 2 | $\alpha$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 20 | $b$ |
| $\beta$ | 2 | $\gamma$ | 10 | $b$ |


$\sigma_{\mathrm{A}=\mathrm{C}}(r \times s)$| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 20 | $b$ |

## Example Queries

$\square$ Relations
$\square$ customer (customer name, customer_street, customer_city)
$\square$ loan (loan number, branch_name, amount)
$\square$ borrower (customer name, loan number)

■ Find the names of all customers who have a loan at the Perryridge branch.
$\square \prod_{\text {customer_name }}\left(\sigma_{\text {loan.loan_number }}=\right.$ borrower.loan_number $($ $\left(\sigma_{\text {branch_name }}=\right.$ 'Perryridge' $($ loan $\left.)\right) \times$ borrower $\left.)\right)$
$\square$ Alternatively, as:

- $\prod_{\text {customer_name }}$ ( $\sigma_{\text {branch_name }}=$ 'Perryridge' $($ $\sigma_{\text {borrower.loan_number }=\text { loan.loan_number }}($ borrower $\times$ loan $\left.)\right)$ )


## Natural-Join Operation: Example

- Relations $\mathrm{r}, \mathrm{s}$ :

$r$| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | a |
| $\beta$ | 2 | $\gamma$ | a |
| $\gamma$ | 4 | $\beta$ | b |
| $\alpha$ | 1 | $\gamma$ | a |
| $\delta$ | 2 | $\beta$ | b |


$s$| $B$ | $D$ | $E$ |
| :---: | :---: | :---: |
| 1 | a | $\alpha$ |
| 3 | a | $\beta$ |
| 1 | a | $\gamma$ |
| 2 | b | $\delta$ |
| 3 | b | $\in$ |

$\square \quad \mathrm{r} \bowtie \mathrm{s}$

| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | a | $\alpha$ |
| $\alpha$ | 1 | $\alpha$ | a | $\gamma$ |
| $\alpha$ | 1 | $\gamma$ | a | $\alpha$ |
| $\alpha$ | 1 | $\gamma$ | a | $\gamma$ |
| $\delta$ | 2 | $\beta$ | b | $\delta$ |

■ $r \bowtie s$ is defined as:

$$
\prod_{r . A, r . B, r . C, r . D, s . E}\left(\sigma_{r . B=s . B \wedge r . D=s . D}(r \times s)\right)
$$

## Bank Example Queries

- Relations:
$\square$ loan (loan number, branch_name, amount)
$\square$ depositor (customer name, account number)
$\square$ borrower (customer name, loan number)
- Find the names of all customers who have a loan and an account at the bank.
$\square \prod_{\text {customer_name }}$ (borrower) $\cap \prod_{\text {customer_name }}$ (depositor)
$\square$ Find the names of all customers who have a loan at the Perryridge branch.
$\square \prod_{\text {customer_name }}\left(\sigma_{\text {branch_name }}=\right.$ 'Perryridge' $($ loan $\bowtie$ borrower $\left.)\right)$
- Find all customers who have a loan at the bank and return his/her name, loan number and the loan amount.
$\square \quad \Pi_{\text {customer_name, loan_number, amount }}$ (borrower $\bowtie$ loan)


## Aggregate Operation: Example

- Relation account

| branch_name | account_number | balance |
| :--- | :---: | :---: |
| Perryridge | A-102 | 400 |
| Perryridge | A-201 | 900 |
| Brighton | A-217 | 750 |
| Brighton | A-215 | 750 |
| Redwood | A-222 | 700 |

$G_{\text {sum(balance) }}$ (account)
branch_name $G$ sum(balance) (account)

| sum |
| :---: |
| 3500 |


| branch_name | sum |
| :--- | :---: |
| Perryridge | 1300 |
| Brighton | 1500 |
| Redwood | 700 |

# Left Outer Join: Example <br> loan 

| loan_number | branch_name | amount |
| :--- | :--- | :---: |
| L-170 | Downtown | 3000 |
| L-230 | Redwood | 4000 |
| L-260 | Perryridge | 1700 |

borrower

| Customer_name | loan_number |
| :--- | :--- |
| Jones | $\mathrm{L}-170$ |
| Smith | $\mathrm{L}-230$ |
| Hayes | $\mathrm{L}-155$ |

- Left Outer Join loan $\searrow \checkmark$ borrower

| loan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-260 | Perryridge | 1700 | null |

- Right Outer Join loan $\bowtie \triangleleft$ borrower

| Ioan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-155 | null | null | Hayes |

- Full Outer Join loan $\mathbb{I}$ b borrower

| loan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-260 | Perryridge | 1700 | null |
| L-155 | null | null | Hayes |

## Query Processing Operators

- Selection or projection
$\square$ Table scan vs Index scan
■ Sorting for ORDER BY or table joins
$\square$ In-memory $\rightarrow$ quick sort, ...
$\square$ On-disk $\rightarrow$ external merge sort
- Joining tables
$\square$ Nested-loop join
$\square$ Merge-join
$\square$ Hash-join


## Selection Operation

- File scan (table / sequential scan) - no index structure is necessary
$\square$ Scan each file block and test all records to see whether they satisfy the selection condition.
$\square$ Cost estimate $=b_{r}$ block transfers +1 seek
- $b_{r}$ denotes number of blocks containing records from relation $r$
$\square$ If selection is on a key attribute, can stop on finding matching record
- cost $=\left(b_{r} / 2\right)$ block transfers +1 seek
$\square$ Linear search can be applied regardless of
- selection condition or
- ordering of records in the file, or
- availability of indices
- Note: binary search generally does not make sense since data is not stored consecutively
$\square$ except when there is an index available,
$\square$ and binary search requires more seeks than index search


## Selections Using Indices

- Index scan - search algorithms that use an index
$\square$ selection condition must be on search-key of index
— Now, assume the sequential file is ordered by this key:
$\square$ Algorithm for primary index \& equality on primary key
$\square$ Retrieve a single record that satisfies the corresponding equality condition
- Cost $=\left(h_{i}+1\right)$ * $\left(t_{S}+t_{T}\right)$

व $h_{i}$ - height of index $i$ (for hashing $h_{i}=1$ )
$0+1$ - for reading the actual record
$\square$ Algorithm for primary index \& equality on non-primary key
$\square$ Retrieve multiple records.
$\square$ Records will be on consecutive blocks

- Let $b=$ number of blocks containing all $n$ matching records
- Cost $=h_{i}{ }^{*}\left(t_{S}+t_{T}\right)+t_{S}+t_{T}{ }^{*} b$


## Selections Using Indices

- Algorithm for secondary index \& equality on non-primary key
$\square$ Sequential file is not ordered by this search key!
$\square$ Retrieve a single record if the search-key is a candidate key
- Cost $=\left(h_{i}+1\right){ }^{*}\left(t_{S}+t_{T}\right)$
$\square$ Retrieve multiple records if search-key is not a candidate key
- Each of $n$ matching records may be on a different block.
- Cost $=\left(h_{i}+n\right)^{*}\left(t_{S}+t_{T}\right)$
- Can be very expensive!


## Sorting Relations

$\square$ We may build an index on the relation, and then use the index to read the relation in the sorted order.
$\square$ May lead to one disk block access for each tuple.

- Use a sorting algorithm
$\square$ For relations that fit in memory, techniques like quick-sort can be used.
$\square$ For relations that don't fit in memory, external sort-merge is a good choice.


## External Sort-Merge

Let $M$ denote memory size (in pages/blocks):

1. Create sorted runs. Let $i$ be 0 initially.

Repeatedly do the following till the end of the relation:
(a) Read $M$ blocks of relation into memory
(b) Sort the in-memory blocks
(c) Write sorted data to run $R_{i}$; increment $i$.

Let the final value of $i$ be $N$
2. Merge the runs. (next slide)

## External Sort-Merge (Cont.)

2. Merge the runs (N-way merge).

We assume (for now) that $N<M$.

1. Use $N$ blocks of memory to buffer input runs, and 1 block to buffer output.
2. Read the first block of each run into its buffer page
3. repeat
4. Select the first record (in sort order) among all buffer pages
5. Write the record to the output buffer.

- If the output buffer is full write it to disk.

3. Delete the record from its input buffer page.

- If the buffer page becomes empty then read the next block (if any) of the run into the buffer.

4. until all input buffer pages are empty.

## External Sort-Merge (Cont.)

— If $N \geq M$, several merge passes are required.
$\square$ In each pass, continuous groups of $M-1$ runs are merged.
$\square$ A pass reduces the number of runs by a factor of $M-1$, and creates runs longer by the same factor.

- E.g. If $\mathrm{M}=11$, and there are 90 runs, one pass reduces the number of runs to 9 , each 10 times the size of the initial runs
$\square$ Repeated passes are performed till all runs have been merged into one.


## Example: External Sorting Using Sort-Merge



## Example: External Sorting Using Sort-Merge (2)



## External Sort-Merge (Cont.)

- Cost analysis:
$\square$ Total number of merge passes required: $\left\lceil\log _{M-1}\left(b_{r} / M\right)\right\rceil$.
$\square$ Block transfers for initial run creation as well as in each pass is $2 b_{r}$
- for final pass, we don't count write cost
- we ignore final write cost for all operations since the output of an operation may be sent to the parent operation without being written to disk
- Thus total number of block transfers for external sorting:

$$
b_{r}\left(2\left\lceil\log _{M-1}\left(b_{r} / M\right)\right\rceil+1\right)
$$

$\square$ Seeks: next slide

## External Sort-Merge (Cont.)

- Cost in seeks
$\square$ During run generation: one seek to read each run and one seek to write each run
- $2\left\lceil b_{r} / M\right\rceil$
$\square$ During the merge phase
- Buffer size: $b_{b}$ (read/write $b_{b}$ blocks at a time)
- cannot be larger than ( $M-1$ ) / "number of runs"
- Need $2\left\lceil b_{r} / b_{b}\right\rceil$ seeks for each merge pass
except the final one which does not require a write
- Total number of seeks:
- $2\left\lceil b_{r} / M\right\rceil+\left\lceil b_{r} / b_{b}\right\rceil\left(2\left\lceil\log _{M-1}\left(b_{r} / M\right)\right\rceil-1\right)$


## Join Operation

- Several different algorithms to implement joins
$\square$ Nested-loop join
$\square$ Block nested-loop join
- Improved nested-loop join by reading records in blocks
$\square$ Indexed nested-loop join
- Improved by using an index to look up equal records
$\square$ Merge-join
- Hash-join
- Choice based on cost estimate
$\square$ For each of the variants a cost estimation can be stated.


## Nested-Loop Join

$\square$ To compute the join $r \bowtie s$
$\square$ for each tuple $t_{r}$ in $r$ do begin for each tuple $t_{s}$ in $s$ do begin
test pair $\left(t_{r}, t_{s}\right)$ to see
if they satisfy the equality on shared attributes if they do, add $t_{r} \cdot t_{s}$ to the result. end
end
$\square r$ is called the outer relation and $s$ the inner relation of the join.
$\square$ Requires no indices and can be used with any kind of join condition.
$\square$ Expensive since it examines every pair of tuples in the two relations.
$\square$ Cost $=n_{r}{ }^{*}\left(t_{S}+t_{T}\right){ }^{*}\left(n_{s}{ }^{*}\left(t_{S}+t_{T}\right)\right)$

- where $n_{r}=$ number of tuples in $r$


## Nested-Loop Join (Cont.)

- In the worst case, if there is enough memory only to hold one block of each relation, the estimated cost is

$$
\begin{array}{ll}
n_{r} * b_{s}+b_{r} & \text { block transfers, plus } \\
n_{r}+b_{r} & \text { seeks }
\end{array}
$$

$\square$ Example on student and takes

- student (the smaller one) as the outer relation:
- $5000 * 400+100=2,000,100$ block transfers,
- $5000+100=5,100$ seeks

$$
\begin{aligned}
& \mathrm{n}_{\text {student }}=5,000 \\
& \mathrm{~b}_{\text {student }}=100 \\
& \mathrm{n}_{\text {takes }}=10,000 \\
& \mathrm{~b}_{\text {takes }}=400
\end{aligned}
$$

- takes (the larger one) as the outer relation
- $10000 * 100+400=1,000,400$ block transfers and 10,400 seeks
- If the smaller relation fits entirely in memory, use that as the inner relation.
$\square$ Reduces cost to $b_{r}+b_{s}$ block transfers and 2 seeks
$\square$ Example: student fits entirely in memory
a the cost estimate is 500 block transfers.
- Block nested-loops algorithm (next slide) is preferable.


## Block Nested-Loop Join

- Variant of nested-loop join in which every block of inner relation is paired with every block of outer relation.
for each block $B_{r}$ of $r$ do begin for each block $B_{s}$ of $\boldsymbol{s}$ do begin for each tuple $t_{r}$ in $B_{r}$ do begin for each tuple $t_{s}$ in $B_{s}$ do begin

Check if $\left(t_{r}, t_{s}\right)$ satisfy the join condition if they do, add $t_{r} \cdot t_{s}$ to the result. end end end
end

- Cost: $b_{r}{ }^{*}\left(1+b_{s}\right)$ blocks; $b_{r}{ }^{*}(1+1)$ seeks
$\square$ For student (outer) and takes (inner):
口 $100+100$ * $400=40,100$ block transfers
- $100+100$ seeks

$$
\begin{aligned}
& \mathrm{n}_{\text {student }}=5,000 \\
& \mathrm{~b}_{\text {student }}=100 \\
& \mathrm{n}_{\text {takes }}=10,000 \\
& \mathrm{~b}_{\text {takes }}=400
\end{aligned}
$$

## Merge-Join

1. Sort both relations on their join attributes
$\square$ If not already sorted.
2. Merge the sorted relations to join them
$\square$ Join step is similar to the merge stage of the sort-merge algorithm.
$\square$ Main difference is handling of duplicate values in join attribute

- Every pair with same value on join attribute must be matched

| $\xrightarrow{\mathrm{pr}}$ | $a 1 \quad a 2$ |  |
| :---: | :---: | :---: |
|  | a | 3 |
|  | b | 1 |
|  | d | 8 |
|  | d | 13 |
|  | f | 7 |
|  | m | 5 |
|  | q | 6 |
|  |  | $r$ |


$\xrightarrow{ } \stackrel{y y}{c}$| $a 1$ |
| :---: |
| $p s$ |
|  |
| a A <br> b G <br> c L <br> d N <br> m B |

## Hash-Join

- A hash function $h$ is used to partition tuples of both relations
$\square$ JoinAttrs are the common attributes of $r$ and $s$ used in $r \bowtie s$
- $h$ maps JoinAttrs values to $\{0,1, \ldots, n\}$
$\square r_{0}, r_{1}, \ldots, r_{n}$ denote buckets of $r$
- Each tuple $t_{r} \in r$ is put in bucket $r_{i}$
- where $i=h\left(t_{r}\right.$ [JoinAttrs]).
$\square s_{0}, s_{1}, \ldots, s_{n}$ denotes buckets of $s$
- Each tuple $t_{S} \in s$ is put in bucket $s_{i}$,
- where $i=h\left(t_{S}\right.$ [JoinAttrs]).


## Hash-Join (Cont.)


buckets $r_{i}$ of $r$ buckets $s_{i}$ of $s$

## Hash-Join (Cont.)

$\square$ Tuples in $r_{i}$ need only to be compared with tuples in $s_{i}$
$\square$ Need not be compared with $s$ tuples in any other bucket, since:
$\square$ a tuple of $r$ and a tuple of $s$ that satisfy the join condition will have the same value for the join attributes.
$\square$ If that value is hashed to some value $i$, the tuple of $r$ has to be in $r_{i}$ and the tuple of $s$ in $s_{i}$.

- Cost of hash join is $3\left(b_{r}+b_{s}\right)$ block transfers
$\square 3^{*}(100+400)$ for student $\bowtie$ takes

$$
\begin{aligned}
& n_{\text {student }}=5,000 \\
& b_{\text {student }}=100 \\
& n_{\text {takes }}=10,000 \\
& b_{\text {takes }}=400 \\
& \hline
\end{aligned}
$$

