Assignment 5

(Solution)

Financial Investments
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1. **A Bond is described as follow**

    - **Annual coupon 8%**
    - **Coupon payment frequency: quarterly**
    - **Interest payment dates: March 1, June 1, September 1 and December 1.**
    - **Maturity: December 1, 2024**
    - **Day count convention: 30/360**
    - **Market rate: 4% (quarterly compounded).**

    (a) **What is the full price of the bond today?**
    Sol:

    - The next payments days, from today (Oct 25, 2023), are: 1Dec23, 1Mar24, 1Jun24, 1Sep24, and 1Dec24. Each coupon pays 2%. The rate per quarter is 1%.
    - The bond valuation for a par value of 100 at Sep 1, 2023, immediately after the coupon payment, is given by:

    $$\begin{aligned} B_{\text{1Sep23}} &= \frac{2}{(1.01)^1} + \frac{2}{(1.01)^2} + \frac{2}{(1.01)^3} + \frac{2}{(1.01)^4} + \frac{102}{(1.01)^5} \\ &= 104.85 \end{aligned}$$

    - According to the 30/360 convention one-quarter has 90 days and from Sep 1, 2023, to Oct 25, 2023, there are roughly 55 days, Then the full price for the bond is:

    $$B_{\text{25Oct23}} = B_{\text{1Sep23}} (1.01)^{\frac{55}{90}} = 105.49$$

(b) **What is the clean price for bond today?**
Sol:

- The accrued interest from the last coupon payment is:

$$AI_{25\text{Oct}23} = 2 * \frac{55}{90} = 1.22$$

- Thus, the clean price for the bond:

$$B^{\text{Clean}}_{25\text{Oct}23} = B_{25\text{Oct}23} - AI_{25\text{Oct}23} = 104.27$$

(c) **What is the full and clean price of the bond on January 1, 2024.**
Sol:

- We can repeat the procedure but this time evaluating the bond on Dec 1, 2023:

$$\begin{aligned} B_{1\text{Dec}23} &= \frac{2}{(1.01)^1} + \frac{2}{(1.01)^2} + \frac{2}{(1.01)^3} + \frac{102}{(1.01)^4} \\ &= 103.90 \end{aligned}$$

$$B_{1\text{Jan}24} = B_{1\text{Dec}23}\,(1.01)^{\frac{30}{90}} = 104.54$$

$$AI_{1\text{Jan}24} = 2 * \frac{30}{90} = 0.67$$

$$B^{\text{Clean}}_{1\text{Jan}24} = B_{1\text{Jan}24} - AI_{1\text{Jan}24} = 103.87$$

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2. **A Bond is described as follow**

- **Annual coupon 8%**
- **Coupon payment frequency: semiannual**
- **Interest payment dates: June 1 and December 1.**
- **Maturity: December 1, 2024**
- **Day count convention: 21/252**
- **YTM: 4%**

(a) **What is the full price of the bond today?**
Sol:

- The next payments days, from today (Oct 25, 2023), are: 1Dec23, 1Jun24, and 1Dec24. Each coupon pays 4%. The rate per semester is 2%.

- The bond valuation for a par value of 100 at Jun 1, 2023, immediately after the coupon payment, is given by:

$$
\begin{aligned}
B_{\text{1Jun23}} &= \frac{4}{(1.02)^1} + \frac{4}{(1.02)^2} + \frac{104}{(1.02)^3} \\
&= 105.77
\end{aligned}
$$

- According to the 21/252 convention (working days only) one semester has 126 days and from Jun 1, 2023, to Oct 25, 2023, there are roughly 100 days, Then the full price for the bond is:

$$B_{\text{25Oct23}} = B_{\text{1Jun23}} (1.01)^{\frac{100}{126}} = 107.44$$

(b) **What is the clean price for bond today?**
Sol:

- The accrued interest from the last coupon payment is:

$$AI_{\text{25Oct23}} = 4 * \frac{100}{126} = 3.17$$

- Thus, the clean price for the bond:

$$B^{\text{Clean}}_{\text{25Oct23}} = B_{\text{25Oct23}} - AI_{\text{25Oct23}} = 104.27$$

(c) **What is the full and clean price of the bond on January 1, 2023.**
Sol:

- We can repeat the procedure but this time evaluating the bond on Dec 1, 2023:

$$
\begin{aligned}
B_{\text{1Dec23}} &= \frac{4}{(1.02)^1} + \frac{104}{(1.02)^2} \\
&= 103.88
\end{aligned}
$$

$$B_{\text{1Jan24}} = B_{\text{1Dec23}} (1.01)^{\frac{1}{12}} = 104.52$$

$$AI_{\text{1Jan24}} = 4 * \frac{1}{12} = 0.33$$

$$B^{\text{Clean}}_{\text{1Jan24}} = B_{\text{1Jan24}} - AI_{\text{1Jan24}} = 103.72$$

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3. **Valuate a FRN with 2-years of maturity and a fixed annual payments of coupon rate (5%), where the 1 year market discount rate is equal to 3% and the 1y2y implicit forward rate of 1%.**

   Sol:

   - The cash flows should be discounted in the following way:
     - From year 2 to year 1, we use $r_{2\to1} = 1\%$ (1y2y implicit forward rate).
     - From year 1 to the inception time, we use the one-year spot rate $r_{1\to0} = 3\%$.
   - Then, the bond FRN valuation per 100 of par value is computed as:

$$\begin{aligned} \mathrm{FRN}_0 &= \frac{5}{1+r_{1\to0}} + \frac{105}{(1+r_{2\to1})(1+r_{1\to0})} \\ &= 105.79 \end{aligned}$$

   - Alternatively, one can compute directly the two-year spot rate $r_{2\to0}$:

$$(1+r_{2\to0})^2 = (1+r_{1\to0})(1+r_{2\to1}) \Longleftrightarrow r_{2\to0} \approx 2\%$$

     - And then:

$$\mathrm{FRN}_0 = \frac{5}{1+r_{1\to0}} + \frac{105}{(1+r_{2\to0})^2} = 105.78$$