MUNI ECON

## Financial Investment

# Class 5: Fixed-income markets

Lecturer: Axel A. Araneda, Ph.D.

# Bond valuation

- Bond pricing is an application of discounted cash flow analysis.

  **Bond price** should be equal to the value of all discounted future cash flows.

- On an option-free fixed-rate bond, the promised future cash flows are a series of coupon interest payments and repayment of the full principal at maturity.
- The market discount rate is used to obtain the present value.

  The **market discount rate** is the rate of return required by investors given the risk of the investment in the bond.

## Formula for calculating the bond price given the market discount rate:

$$\text{PV} = \frac{\text{PMT}}{(1+r)^1} + \frac{\text{PMT}}{(1+r)^2} + \cdots + \frac{\text{PMT} + \text{FV}}{(1+r)^N}$$

where

- **PV** is the present value (price) of the bond
- **PMT** is the coupon payment per period
- **FV** is the future value paid at maturity, or the bond's par value
- ***r*** is the required rate of return per period
- ***N*** is the number of evenly spaced periods to maturity

- The price of a fixed-rate bond, relative to par value, depends on the relationship of the coupon rate to the market discount rate.

| | |
|---|---|
| If the bond price is higher than par value, the bond is said to be traded **at a premium.** | • This happens when the coupon rate is greater than the market discount rate. |
| If the bond price is lower than par value, the bond is said to be traded **at a discount.** | • This happens when the coupon rate is less than the market discount rate. |
| If the bond price is equal to par value, the bond is said to be traded **at par.** | • This happens when the coupon rate is equal to the market discount rate. |

- The price of a fixed-rate bond will change whenever the market discount rate changes.

The bond price is inversely related to the market discount rate. When the market discount rate increases, the bond price decreases (the inverse effect).

For the same coupon rate and time-to-maturity, the percentage price change is greater when the market discount rate goes down than when it goes up (the convexity effect).

For the same time-to-maturity, a lower-coupon bond has a greater percentage price change than a higher-coupon bond when their market discount rates change by the same amount (the coupon effect).

For the same coupon rate, a longer-term bond has a greater percentage price change than a shorter-term bond when their market discount rates change by the same amount (the maturity effect).

# Relationships between Bond Prices and Bond Characteristics

| Bond | Coupon Rate | Maturity | Price at 20% | Discount Rates Go Down | | Discount Rates Go Up | |
|---|---|---|---|---|---|---|---|
| | | | | Price at 19% | % Change | Price at 21% | % Change |
| A | 10% | 10 | 58.075 | 60.950 | 4.95% | 55.405 | –4.60% |
| B | 20% | 10 | 100.000 | 104.339 | 4.34% | 95.946 | –4.05% |
| C | 30% | 10 | 141.925 | 147.728 | 4.09% | 136.487 | –3.83% |
| D | 10% | 20 | 51.304 | 54.092 | 5.43% | 48.776 | –4.93% |
| E | 20% | 20 | 100.000 | 105.101 | 5.10% | 95.343 | –4.66% |
| F | 30% | 20 | 148.696 | 156.109 | 4.99% | 141.910 | –4.56% |

The **yield-to-maturity** is the internal rate of return on a bond’s cash flows. It is the implied market discount rate.

The **yield-to-maturity (YTM)** is the rate of return on the bond to an investor provided three conditions are met:

- The investor holds the bond to maturity.
- The issuer does not default on coupon or principal payments.
- The investor is able to reinvest coupon payments at that same yield.

Therefore, the yield-to-maturity is the promised yield.

- Because the market discount rates for the cash flows with different maturities are rarely the same, it is fundamentally better to calculate the price of a bond by using a sequence of market discount rates that correspond to the cash flow dates.

These market discount rates are called "spot rates."

Spot rates are yields-to-maturity on zero-coupon bonds maturing at the date of each cash flow.

**General formula for calculating a bond price given the sequence of spot rates:**

$$\mathrm{PV} = \frac{\mathrm{PMT}}{(1+Z_1)^1} + \frac{\mathrm{PMT}}{(1+Z_2)^2} + \cdots + \frac{\mathrm{PMT}+\mathrm{FV}}{(1+Z_N)^N}$$

where $Z_1$, $Z_2$, *and* $Z_N$ are spot rates for period 1, 2, and $N$, respectively.

**Example.** Suppose that the one-year spot rate is 2%, the two-year spot rate is 3%, and the three-year spot rate is 4%. Calculate the price of a three-year 5% annual coupon paying bond:

$$\frac{5}{(1.02)^1} + \frac{5}{(1.03)^2} + \frac{105}{(1.04)^3} =$$

$$4.902 + 4.713 + 93.345 = \mathbf{102.960}$$

**The bond price is 102.960.**

- The present values of the individual cash flows discounted using spot rates differ from those using yield-to-maturity, but the sum of the present values is the same. Thus, the same price is obtained using either approach.

# Prices and yields: conventions for quotes and calculations

Bond price consists of two components

Flat (clean) price ($P_c$) + Accrued interest (AI)

The sum of flat price and accrued interest is the full (dirty) price ($P_f$).

Bond dealers usually quote the flat price.

$$P_f = P_c + \mathrm{AI}$$

Buyers pay the full price for the bond on the settlement date.

- Accrued interest is the proportional share of the next coupon payment:

$$\mathrm{AI} = \frac{t}{T} \times \mathrm{PMT}$$

where ***t*** is the number of days from the last coupon payment to the settlement date; ***T*** is the number of days in the coupon period; ***t*/*T*** is the fraction of the coupon period that has gone by since the last payment; and **PMT** is the coupon payment per period.

**30/360** is common for corporate bonds.

The two most common conventions to count days in bond markets:

(*Days in the month/Days in a year*)

**Actual/actual** is common for government bonds.

- The full price of a fixed-rate bond between coupon payments given the market discount rate per period (*r*) can be calculated as:

$$P_f = \frac{\text{PMT}}{(1+r)^{1-t/T}} + \frac{\text{PMT}}{(1+r)^{2-t/T}} + \cdots + \frac{\text{PMT} + \text{FV}}{(1+r)^{N-t/T}}$$

where ***N* – *t*/*T*** represents the time before the appropriate payment is made and **FV** is the face value of the bond.

- The above formula can be simplified to:

$$P_f = \text{PV} \times (1+r)^{t/T}$$

where **PV** is the value of the bond on the most recent coupon payment date and can be calculated using the standard bond price formula.

**Example.** A 6% German corporate bond is priced for settlement on 18 June 2015. The bond makes semiannual coupon payments on 19 March and 19 September of each year and matures on 19 September 2026. Using the 30/360 day-count convention, calculate the **full price**, the **accrued interest**, and the **flat price** per EUR100 of par value if the YTM is 5.80% (2.90% per six months):

- The value of the bond after the latest coupon (19 March) is

$$\mathrm{PV} = \frac{3}{(1.0290)^1} + \frac{3}{(1.0290)^2} + \cdots + \frac{103}{(1.0290)^{23}} = 101.6616$$

The present value of the bond is EUR101.6616.

## Example (continued):

- The **full price** on 18 June 2015 is

$$P_f = 101.66 \times (1.0290)^{89/180} = \mathbf{EUR103.1088}$$

- The **accrued interest** is

$$\text{AI} = \frac{89}{180} \times 3 = \mathbf{EUR1.4833}$$

- The **clean/flat price** is

$$P_c = 103.1088 - 1.4833 = \mathbf{EUR101.6254}$$

**Matrix pricing** is an estimation process used for bonds that are not actively traded.

In matrix pricing, market discount rates are extracted from comparable bonds (i.e., bonds with similar time-to-maturity, coupon rate, and credit quality).

**Example.** An analyst is pricing a three-year, 4% semiannual coupon corporate bond with no active market to derive the appropriate YTM. He finds two bonds with a similar credit quality: A two-year bond is traded at a YTM of 3.8035%, and a five-year bond is traded at a YTM of 4.1885%. Using linear interpolation, the estimated YTM of a three-year bond will be 3.9318%:

$$0.038035 + \left(\frac{3-2}{5-2}\right) \times (0.041885 - 0.038035) = 0.039318$$

Matrix pricing is also used in underwriting new bonds to get an estimate of the **required yield spread** over the **benchmark rate**.

- The benchmark rate is typically the yield-to-maturity on a government bond having the same, or close to the same, time-to-maturity.

The **spread** is the difference between the yield-to-maturity on the new bond and the benchmark rate.

- The yield spread is the additional compensation required by investors for the difference in the credit risk, liquidity risk, and tax status of the bond relative to the government bond. This spread is sometimes called the "**spread over the benchmark**."

# Yield Measures for Fixed-Rate Bonds

- Investors use standardized yield measures to allow for comparison between bonds with varying maturities.

**For bonds maturing in more than one year:**

- An **annualized and compounded** yield-to-maturity is used.

**For money market instruments of less than one year to maturity:**

- These are **annualized but not compounded**.

- An annualized and compounded yield on a fixed-rate bond depends on the **periodicity** of the annual rate.

The periodicity of the annual market discount rate for a zero-coupon bond is arbitrary because there are no coupon payments.

The **effective annual rate** helps to overcome the problem of varying periodicity. It assumes there is just one compounding period per year.

- Another way to overcome a problem of varying periodicities is to calculate a **semiannual bond equivalent yield** (i.e., a YTM based on a periodicity of two).

**General formula to convert yields based on different periodicities:**

$$\left(1+\frac{\mathrm{APR}_m}{m}\right)^m=\left(1+\frac{\mathrm{APR}_n}{n}\right)^n$$

where **APR** is the annual percentage rate and ***m*** and ***n*** are the number of payments/compounding periods per year, respectively.

- For example, converting a YTM of 4.96% from a semiannual periodicity to a quarterly periodicity gives a YTM of 4.93%:

$$\left(1+\frac{0.0496}{2}\right)^2=\left(1+\frac{APR_4}{4}\right)^4, APR_4=0.0493$$

# Other yield measures

| Street convention yield-to-maturity: | True yield-to-maturity: | Government equivalent yield: | Current yield: | Simple yield: |
|---|---|---|---|---|
| The internal rate of return on the cash flows, assuming the payments are made on the scheduled dates (no weekends or holidays) | The internal rate of return on the cash flows using the actual calendar of weekends and bank holidays | Restatement of a yield-to-maturity based on a 30/360 day-count to one based on actual/actual | The sum of coupon payments received over the year divided by the flat price | The sum of coupon payments plus the straight-line amortized share of the gain or loss, divided by the flat price |