Assignment 8

(Solution)

Financial Investments
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Autumn 2023

1. Consider assets A and B. Asset A is featured by a return of 5% per year and risk (measured by annualized standard deviation) of 10%. On the other hand, Asset B has 3% of return and 9% of volatility. The correlation among them is $\rho_{AB} = -0.5$. The risk-free rate of return is 2%.

   (a) Compute the Sharpe ratio for the portolio composed of 60% (of the initial wealth) in asset A and 40% in asset B.

   Ans.

   - $R_A = 5\%$, $R_B = 3\%$, $\sigma_A = 10\%$, $\sigma_B = 9\%$, $\rho_{AB} = -0.5$, $w_A = 0.6$, and $w_B = 0.4$.
   - In order to compute the Sharpe ratio, we need to find both, risk and return of the portfolio.
   - Return:

   $$R_P = w_A \cdot R_A + w_B * R_B = 4.2\%$$

   - Risk:

   $$\begin{aligned}
   \sigma_P^2 &= \text{var}\left(R_p\right) \\
   &= \text{var}\left(w_A \cdot R_A + w_B * R_B\right) \\
   &= \text{var}\left(w_A R_A\right) + \text{var}\left(w_B R_B\right) + 2\rho_{AB}\sqrt{\text{var}\left(w_A R_A\right) \cdot \text{var}\left(w_B R_B\right)} \\
   &= w_A^2 \text{var}\left(R_A\right) + w_B^2 \text{var}\left(R_B\right) + 2\rho_{AB}\sqrt{w_A^2 \text{var}\left(R_A\right) \cdot w_B^2 \text{var}\left(w_B R_B\right)} \\
   &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2\rho_{AB} w_1 w_2 \sigma_A \sigma_B \\
   &= 0.002736
   \end{aligned}$$

   $$\longrightarrow \sigma_P = \sqrt{\sigma_P^2} = 5.23\%$$

   - Sharpe ratio:

$$SR_P = \frac{R_P - R_f}{\sigma_P} = 0.42$$

(b) How the results changes if $\rho_{AB} = 0$

Ans.

- $\rho_{AB} = 0 \rightarrow \sigma_P^2 = 0.004896 \rightarrow \sigma_P \approx 7\% \rightarrow SR_P = 0.31$

(c) How the results changes if $\rho_{AB} = 0.5$

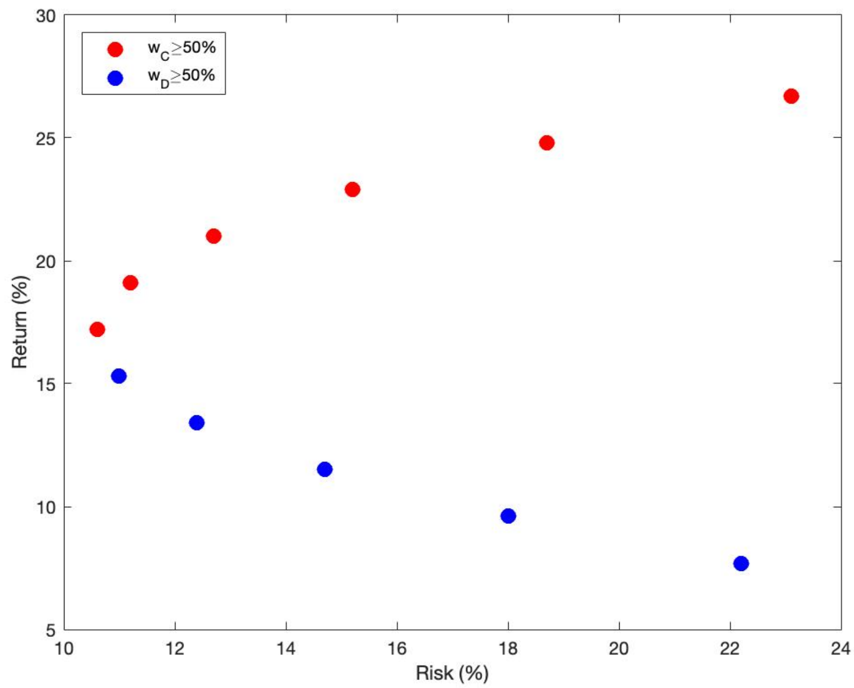
Ans.

- $\rho_{AB} = 0. \rightarrow \sigma_P^2 = 0.007056 \rightarrow \sigma_P = 8.4\% \rightarrow SR_P = 0.26$

2. Consider the following data of a portfolio composed by asset C and D:

Weight asset C (%)	Weight asset D (%)	Portfolio risk (%)	Portfolio return (%)
0	100	22.2	7.7
10	90	18.0	9.6
20	80	14.7	11.5
30	70	12.4	13.4
40	60	11.0	15.3
50	50	10.6	17.2
60	40	11.2	19.1
70	30	12.7	21.0
80	20	15.2	22.9
90	10	18.7	24.8
100	0	23.1	26.7

(a) Plot the risk-return relationship for all the weight possibilities described in the above table.



(b) What conclusion can you state?

Ans.

- An optimal investment is achieved when the asset C has a weight greather or equal than 50%.