Test 1

Financial Investments
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Solution

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1. A trader has purchased on margin 1000 shares of a company at \$30 per share, paying a total purchase commision of \$10, with a leverage ratio of 1.25 and a call money rate of 10% annually. After 1 year, the trader recieves a dividend 1\$ per share and inmediatly after that goes takes a short position in asset at \$40/share. Assuming that the sale and buy comissions are equal. What is the total rate of return for the investor after 1 year. How this result is compared for a non-margin purchase (without leverage)?

   - $Q = 1000$, $P_0 = 30$, $C_{buy} = 10$, $L = 1.2$, $r = 10\%$.
   - $D = 1$, $P_1 = 40$, $C_{sale} = 10$.

   $$L = \frac{1}{\text{Equity}} \longrightarrow \text{Equity} = 0.8 \longrightarrow \text{Debt} = 0.2$$

   - Total Initial investment: $\text{Equity} * P_0 * Q + C_{buy} = 24000 + 10 = 24010$.
   - End of period:
     - Proceed on sales: $P_1 * Q = 40000$
     - Loan payment=$\text{Debt} * P_0 * Q = 6000$
     - Margin interest paid: $r * \text{Debt} * P_0 * Q = 600$
     - Income from dividends: $D * Q = 1000$
     - Sale commission: $C_{sale} = 10$
     - Remaining equity: 40000-6000-600+1000-10=34390
   - Return of the investment

   $$R = \frac{34390 - 24010}{24010} = \frac{10380}{24010} = 43.23\%$$

   - Without leverage:
     - Alternative 1: Same initial investment (24010).
       * 10 on buy comission ($C_{buy} = 10$) and \$24000 in the stock (24000/30=800 shares).
       * Proceed on sales: $P_1 * 800 = 32000$.
       * Income from dividends: $D * 800 = 800$
       * $C_{sale} = 10$.
       * Return:

       $$R = \frac{32790 - 24010}{24010} = \frac{7980}{24010} = 36.57\%$$

     - Alternative 2: Purchase 1000 shares without debt
       * 10 on buy comission ($C_{buy} = 10$) and \$30000 in the stock
       * Proceed on sales: $P_1 * 1000 = 40000$.
       * Income from dividends: $D * 1000 = 1000$
       * $C_{sale} = 10$.

* Return:

$$R = \frac{40990 - 30010}{30010} = \frac{10980}{30010} = 36.59\%$$

– In both cases, the return is lower. Then, the leverage enhances the rate of return.

2. An analyst gathers the folllowing information for a market-cap-weighted index comprised of assets MNO, QRS, and XYZ.

Security	Beginning of Period Price (¥)	End of Period Price (¥)	Dividends Per Share (¥)	Shares Outstanding
MNO	2,500	2,700	100	5,000
QRS	3,500	2,500	150	7,500
XYZ	1,500	1,600	100	10,000

(a) Compute the price return of the index.

- Alternative 1:
  - Beginning of period (BOP):

$$\begin{aligned} I_0 &= \frac{2500 \cdot 5000 + 3500 \cdot 7500 + 1500 \cdot 10000}{D} \\ &= \frac{12500000 + 26250000 + 15000000}{D} \\ &= \frac{53750000}{D} \end{aligned}$$

  - End of period (EOP):

$$\begin{aligned} I_1 &= \frac{2700 \cdot 5000 + 2500 \cdot 7500 + 1600 \cdot 10000}{D} \\ &= \frac{13500000 + 18750000 + 16000000}{D} \\ &= \frac{48250000}{D} \end{aligned}$$

  - Price return of the index:

$$R = \frac{I_1 - I_0}{I_0} = \frac{\frac{48250000}{D} - \frac{53750000}{D}}{\frac{53750000}{D}} = \frac{48250000 - 53750000}{53750000} = -10.23\%$$

- Alternative 2
  - $MC_1 = 5000 * 2500 = 12500000$
  - $MC_2 = 7500 * 3500 = 26250000$
  - $MC_3 = 10000 * 1500 = 15000000$
  - Total market capitalization:$TMC = \sum_i MC_i = 53750000$
  - Weights: $w_1 = \frac{12500000}{53750000} = 0.2325$; $w_2 = \frac{26250000}{53750000} = 0.4883$; $w_3 = \frac{15000000}{53750000} = 0.2791$
  - Individual returns:

$$R^1 = \frac{2700 - 2500}{2500} = 8\%$$

$$R^2 = \frac{2500 - 3500}{3500} = -28.57\%$$

$$R^3 = \frac{1600 - 1500}{1500} = 6.67\%$$

– Price return of the index:

$$R = w_1 R^1 + w_2 R^2 + w_3 R^3 = 0.0186 - 0.1395 + 0.0186 = -10.23\%$$

(b) Compute the total return of the index.

- The total return also includes the income/dividend gains along the period.
- Total dividend gains:

$$\frac{\text{Inc}_1}{D} = \frac{(5000 * 100) + (7500 * 150) + (10000 * 100)}{D} = \frac{2625000}{D}$$

- Total return of the index:

$$TR = \frac{I_1 - I_0 + \text{Inc}_1}{I_0} = -5.35\%$$

- Alternatively:

$$TR = w_1 \left(R^1 + 100\right) + w_2 \left(R^2 + 150\right) + w_3 \left(R^3 + 100\right) = -5.35\%$$

(c) How the results change if we consider a price-weighted scheme.

- Price-weighted value of the index (BOP):

$$I_0 = \frac{2500 + 3500 + 1500}{D} = \frac{7500}{D}$$

- Price-weighted value of the index (EOP):

$$I_1 = \frac{2700 + 2500 + 1600}{D} = \frac{6800}{D}$$

- Return of the index (price-weighted):

$$PR_I = \frac{I_1 - I_0}{I_0} = -9.33\%$$

- Total dividend gains (price-weighted):

$$\frac{\text{Inc}_1}{D} = \frac{100 + 150 + 100}{D} = \frac{350}{D}$$

- Total return of the index (price-weighted):

$$TR_I = \frac{I_1 - I_0 + \text{Inc}_1}{I_0} = -4.67\%$$

3. An investor who owns a bond with a 4% coupon rate that pays interest quarterly and matures in two years is considering its sale. If the required market discount rate is 12%, compute the present value of the bond per 100 of par value. Is this bond traded at premium, discount or par?

   - Bond valuation: sum of the discounted cash flows.

$$\begin{aligned} PV &= \sum_{n=1}^{8} \frac{C/4}{(1+r/4)^n} + \frac{FV}{(1+r/4)^8} \\ &= \frac{1}{1.03} + \frac{1}{(1.03)^2} + \frac{1}{(1.03)^3} + \frac{1}{(1.03)^4} \\ &\quad + \frac{1}{(1.03)^5} + \frac{1}{(1.03)^6} + \frac{1}{(1.03)^7} + \frac{101}{(1.03)^8} \\ &= 85.96 \end{aligned}$$

   - The bond is traded at discount. $PV < FV$. $r > C$.

4. A bond with a 4% coupon and maturity on Sep 1, 2025, pays interest in the following calendar days: 1 Mar, 1 June, 1 Sep, 1 Dec. If the market discount rate is 12%, What is the full and clean price of the bond today (Nov 1, 2023) per 100 of par value?

- Today: 01.11.23. 2 months ($\sim$ 60 days) after the last coupon payment.
- Coupon rate: 4% annual; i.e., 1% quarterly.
- Discount rate: 12% annual, i.e., 3% quarterly.
- Next payment dates: 1Dec23, 1Mar24, 1Jun24, 1Sep24, 1Dec24, 1Mar24, 1Jun24, and 1Sep24. 8 periods.
- The bond price V per 100 of par value on Sep 1, inmediatly after the coupon payment is given by (same equation as in exercise 4):

$$\begin{aligned} V_{1\text{Sep23}} &= \sum_{n=1}^{8} \frac{1}{(1.03)^n} + \frac{100}{(1.03)^8} \\ &= \frac{1}{1.03} + \frac{1}{(1.03)^2} + \frac{1}{(1.03)^3} + \frac{1}{(1.03)^4} \\ &\quad + \frac{1}{(1.03)^5} + \frac{1}{(1.03)^6} + \frac{1}{(1.03)^7} + \frac{101}{(1.03)^8} \\ &= 85.96 \end{aligned}$$

- Then, the full price of the bond today is:

$$\begin{aligned} V_{1\text{Nov23}} &= V_{1\text{Sep23}} (1 + 0.03)^{\frac{2}{3}} \\ &= 87.67 \end{aligned}$$

- The accrued interest from the last coupon payment is:

$$AI_{1\text{Nov23}} = 1 * \frac{2}{3} = 0.67$$

- Thus, the clean price for the bond:

$$V_{1\text{Nov23}}^{\text{Clean}} = V_{1\text{Nov23}} - AI = 87.00$$

5. A bond traded at 105 per 100 of par value has a maturity equal to 2 years and semiannual couponds of 5 percent. Find the yield to maturity.

    - Bond valuation equation:

$$105 = \frac{2.5}{\left(1 + \frac{r}{2}\right)} + \frac{2.5}{\left(1 + \frac{r}{2}\right)^2} + \frac{2.5}{\left(1 + \frac{r}{2}\right)^3} + \frac{102.5}{\left(1 + \frac{r}{2}\right)^4}$$

    - To find r, we assigns values to r in a way that the partial sum of the right-hand side matches the left-hand side (105). Since the bond is traded at premium (105>100) is clear that that the coupon rate (5%) is higuer than the discount rate. In particular we obtain:

$$r \approx 2.42\%$$