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Financial Investment

Introduction to portfolio theory

Lecturer: Axel A. Araneda, Ph.D.

3 Assets example

- Suposse (☹) you have some money, and you want to invest it taking a long position in a set of assets.

- By the sake of simplicity we consider that we are able to purchase AAPL, CAT, KO.

- Otherwise, we can invest directly in both the market as a whole (SP500) or just take a risk-free asset.

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3 Assets example

- For the risk-free asset, we have plenty certain about its performance (namely, 3% per year).

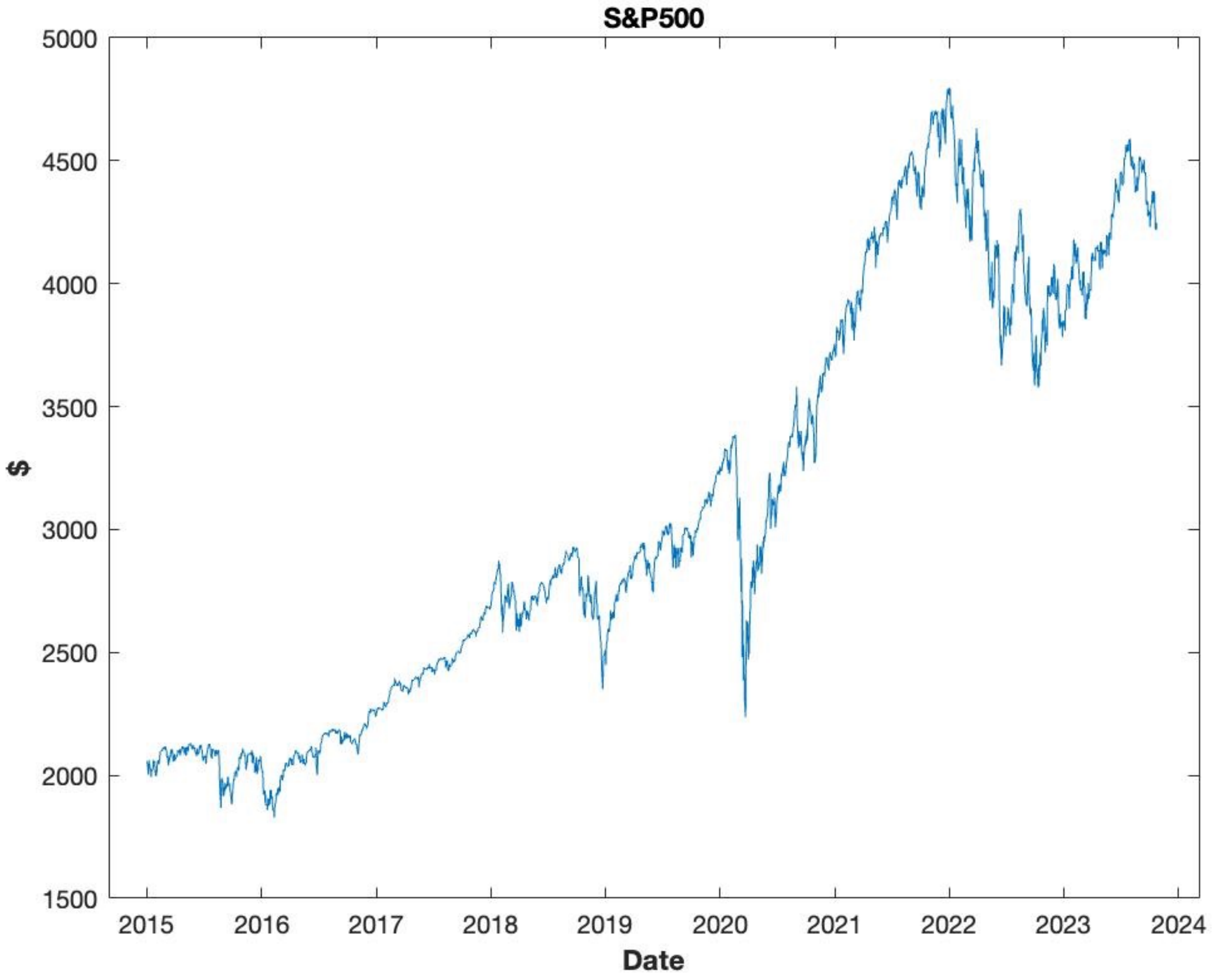
- In the case of the risky assets (index, KO, CAT, AAPL), we don't know how they will behave.

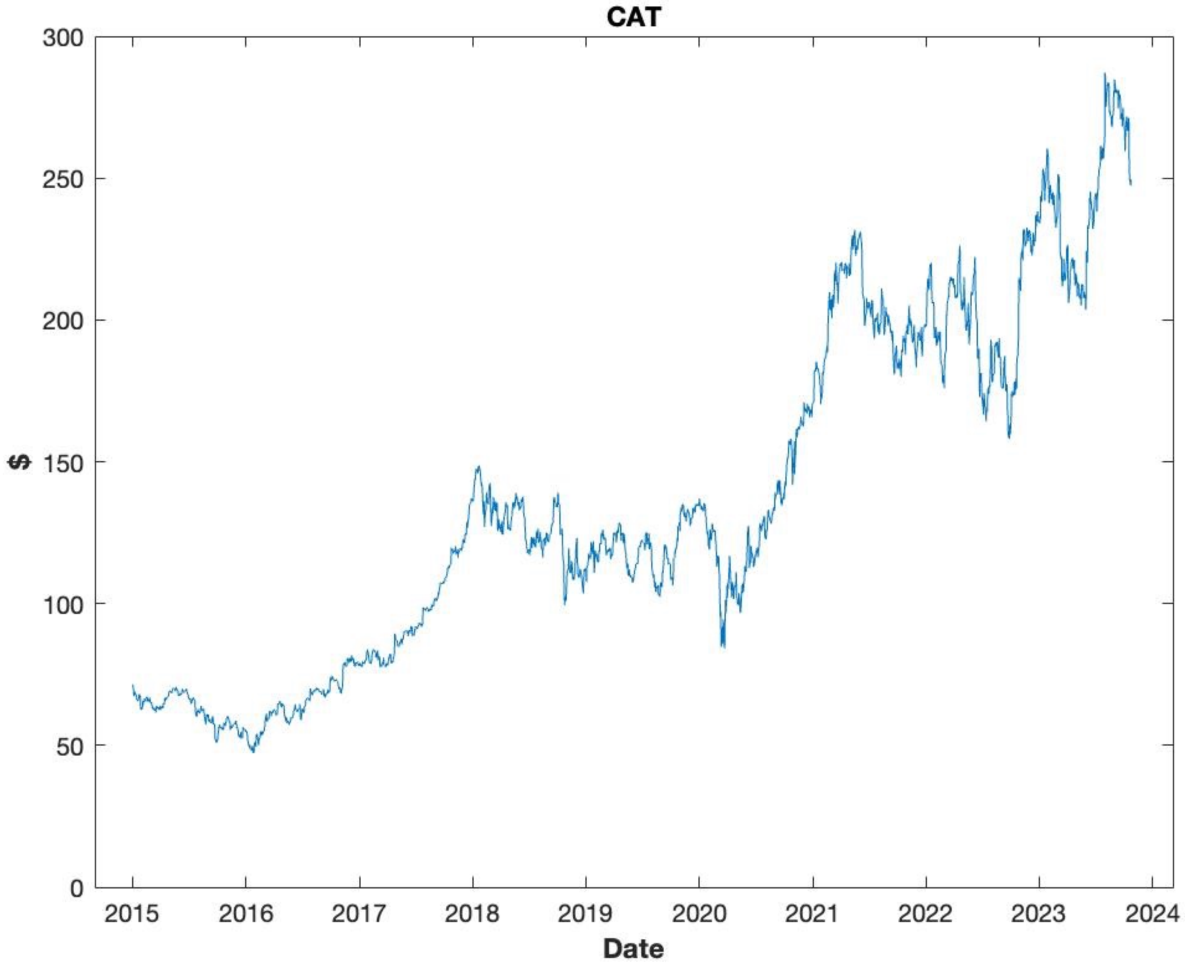
- However, on average, we can expect that the return of stocks is higher than risk-free assets.

The investment purpose

- Utility maximization.
- Maximize returns over investment.
- Prices are random: subject to uncerntainty.
- What about the risk exposure?

Historical performance





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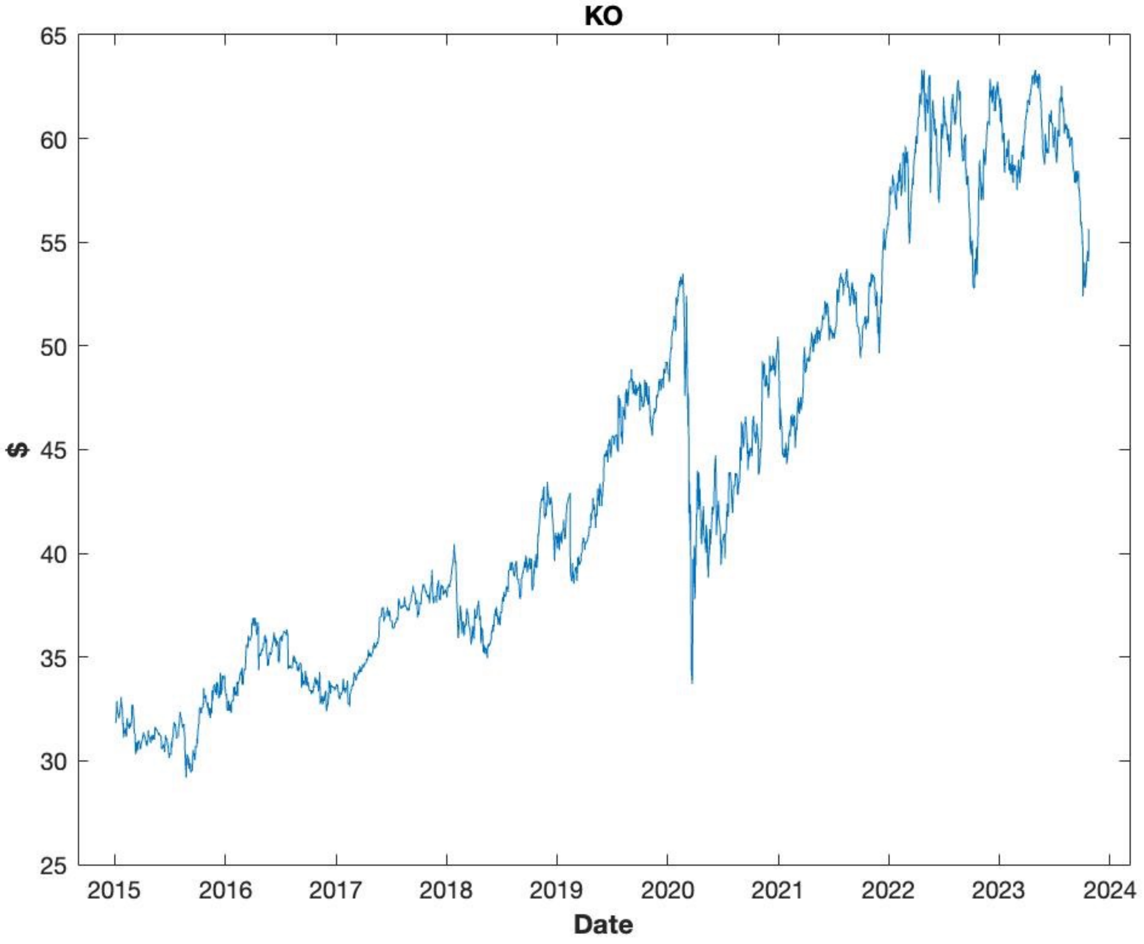
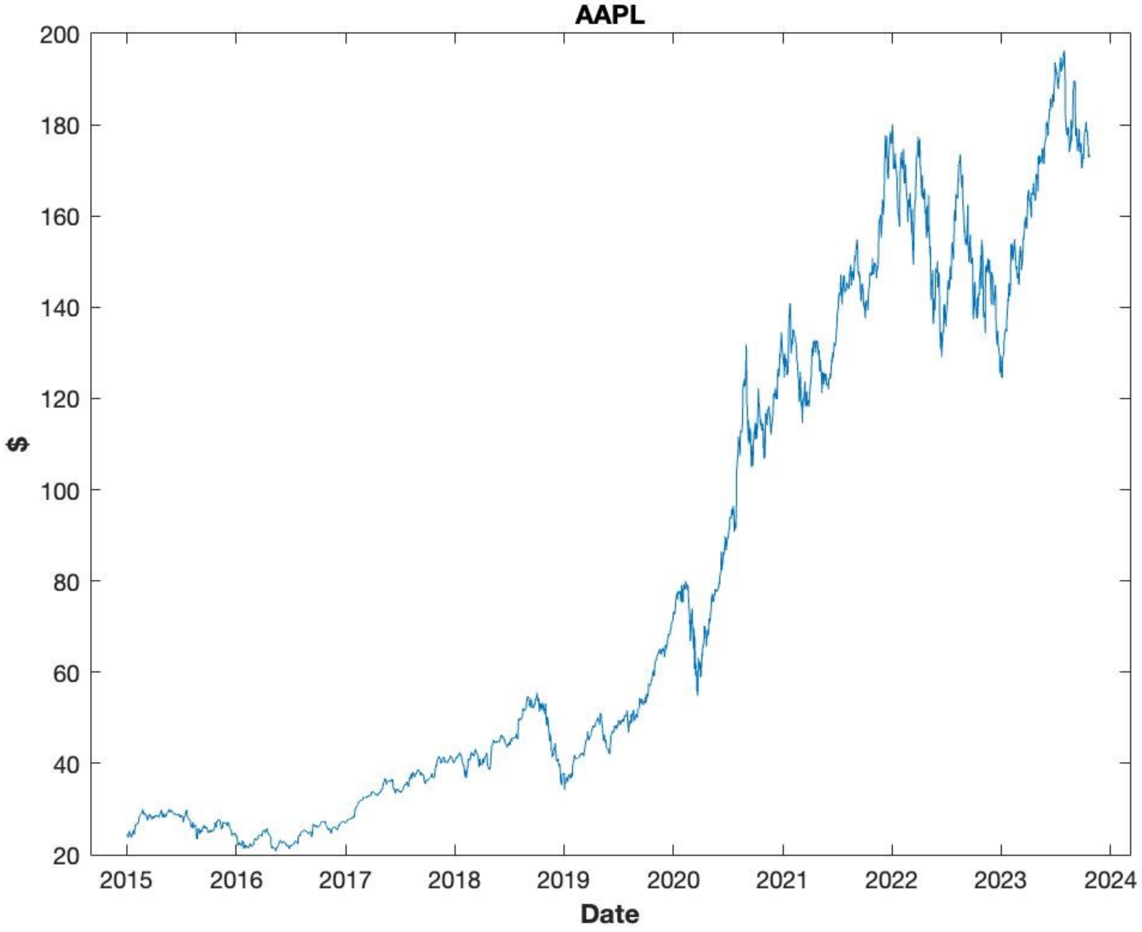
Historical performance





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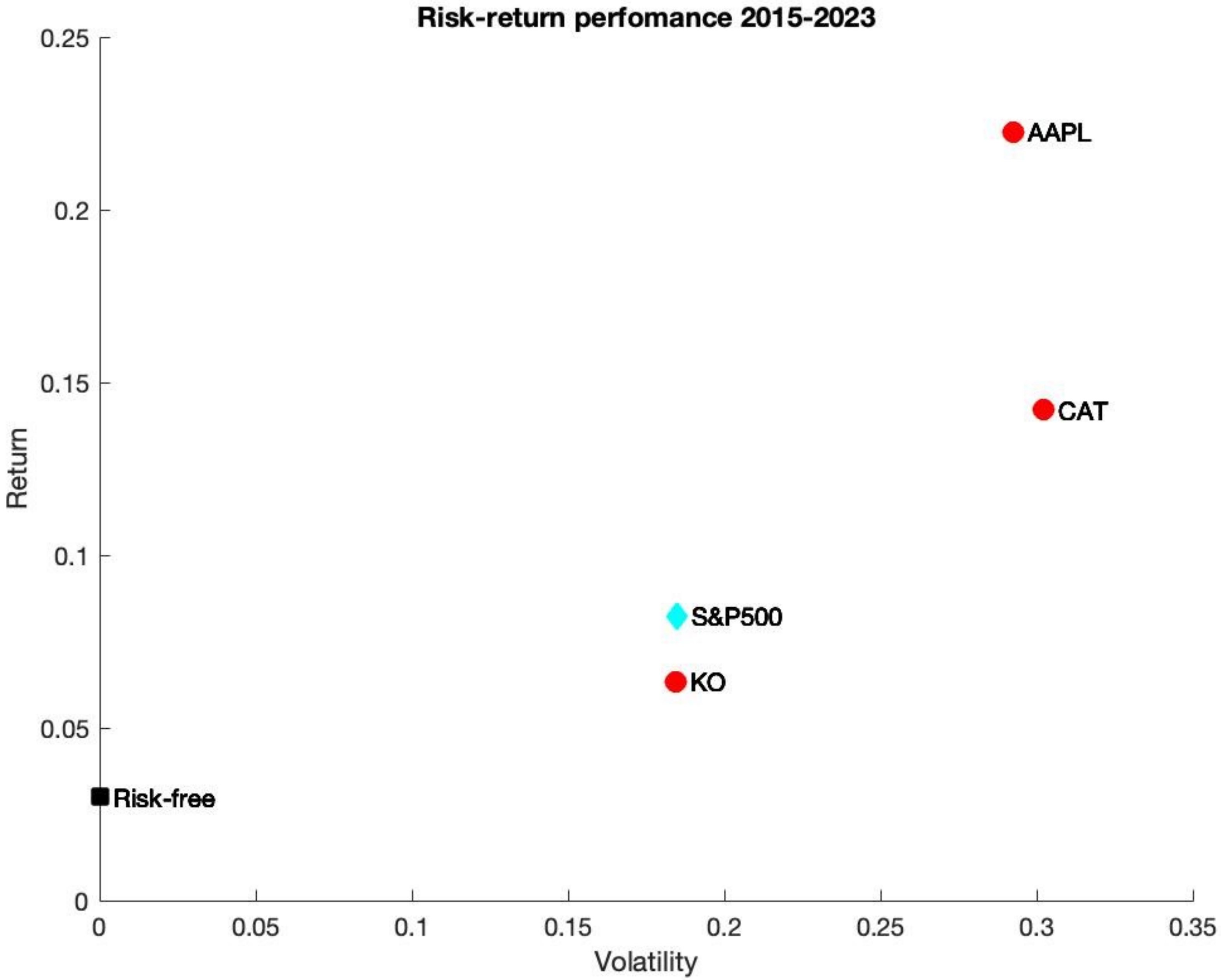
What about risk-return relationship?

- Risk proxy: returns SD (variance).
- How much return I had and what was the risk for each asset in the period 2015-today?


Risk-return perfomance 2015-2023
Return
Volatility
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
AAPL
CAT
S&P500
KO
Risk-free

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What about portfolio performance?

- Combination of KO, AAPL, and CAT.
- Divide the initial wealth to purchase some shares of the above stocks.
- For example: investing the half of the wealth in AAPL, one-quarter in KO, and the remaining quarter in CAT.
- In principle we have many combinations as we want.
- Restrictions: investing the whole wealth in long positions only.
- How we measure the risk for 2 or more assets: covariance.

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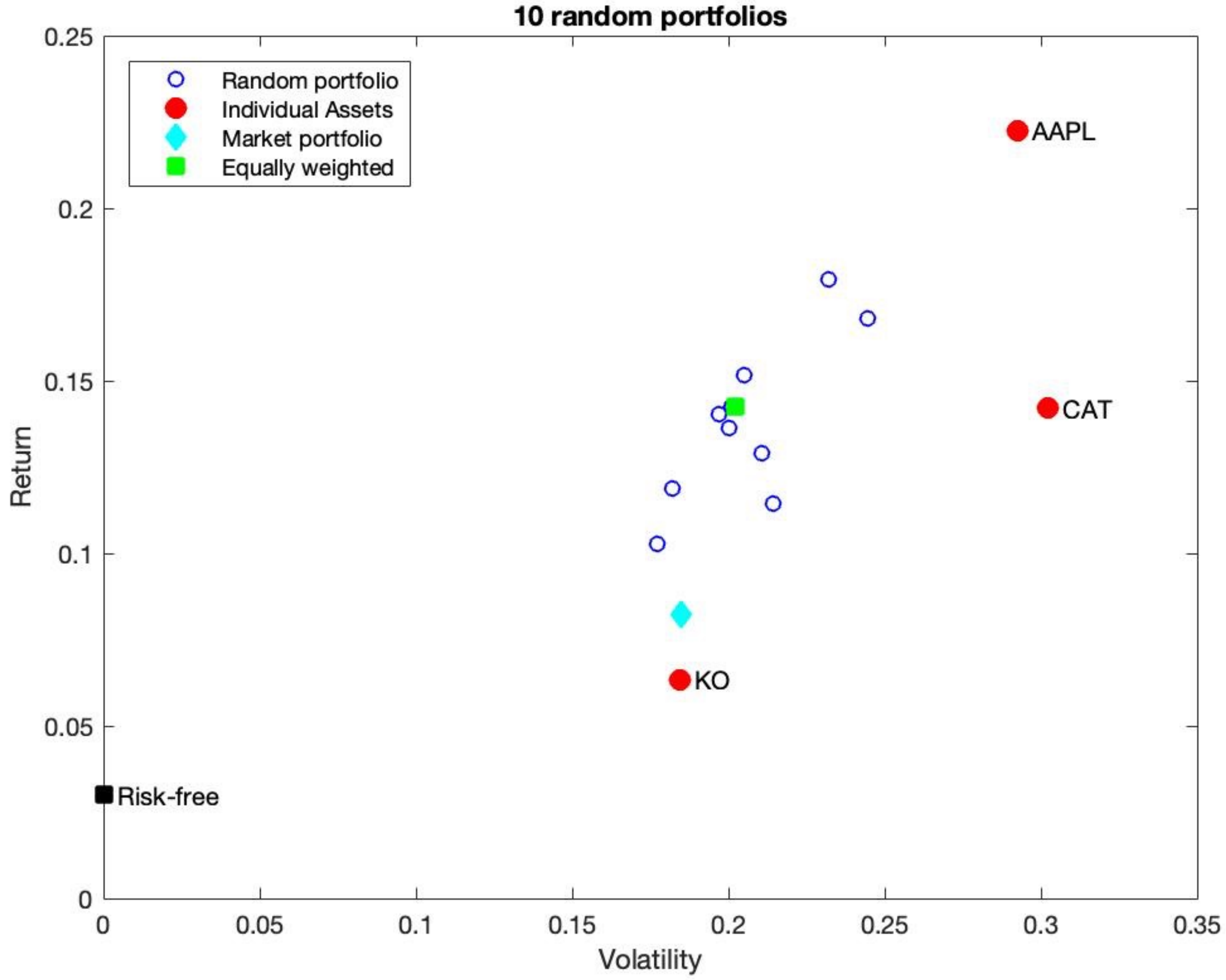
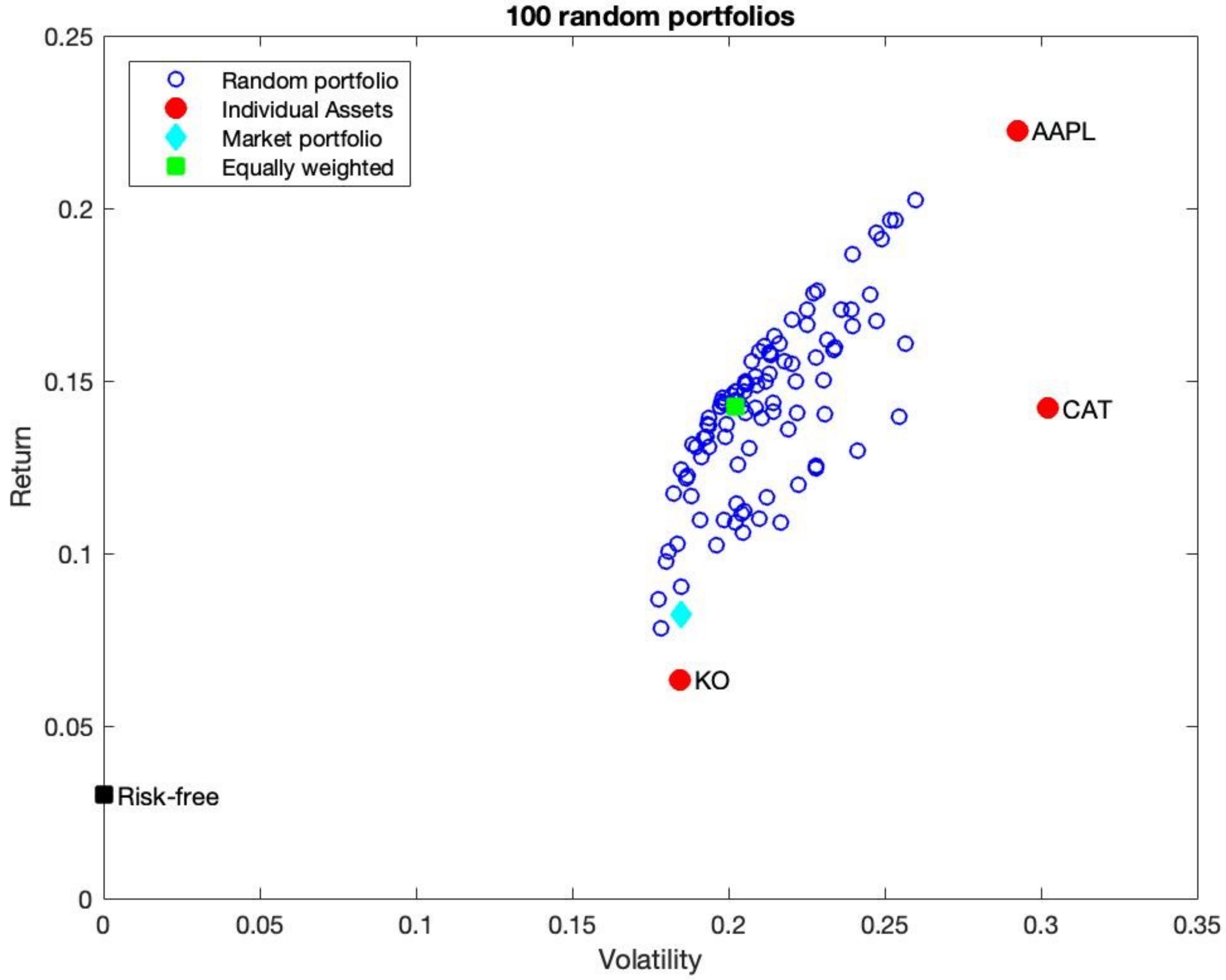
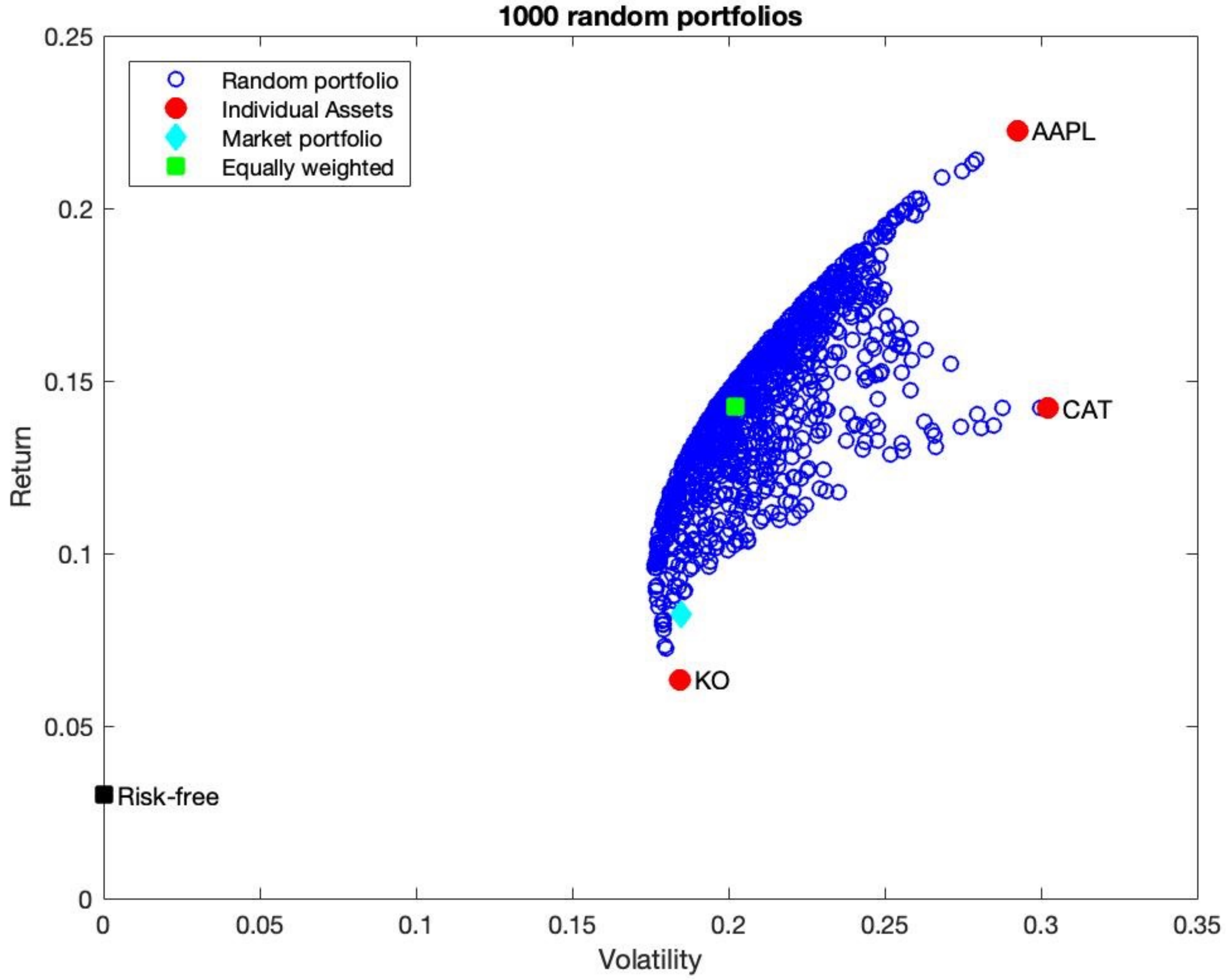

10 random portfolios
Random portfolio
Individual Assets
Market portfolio
Equally weighted
AAPL
CAT
KO
Risk-free
Return
Volatility
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35


100 random portfolios
Random portfolio
Individual Assets
Market portfolio
Equally weighted
AAPL
CAT
KO
Risk-free
Return
Volatility
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35


1000 random portfolios
Random portfolio
Individual Assets
Market portfolio
Equally weighted
AAPL
CAT
KO
Risk-free
Return
Volatility
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

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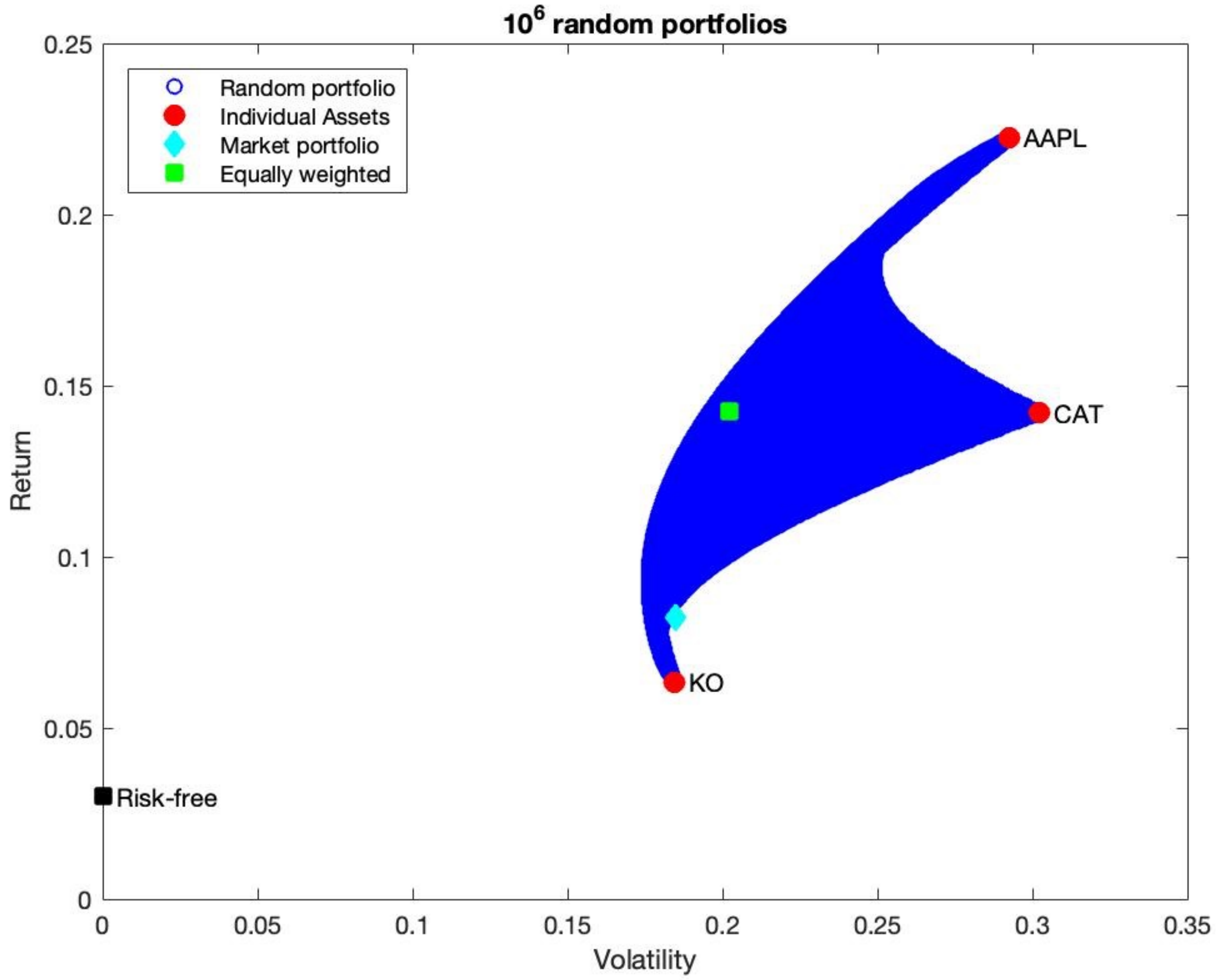
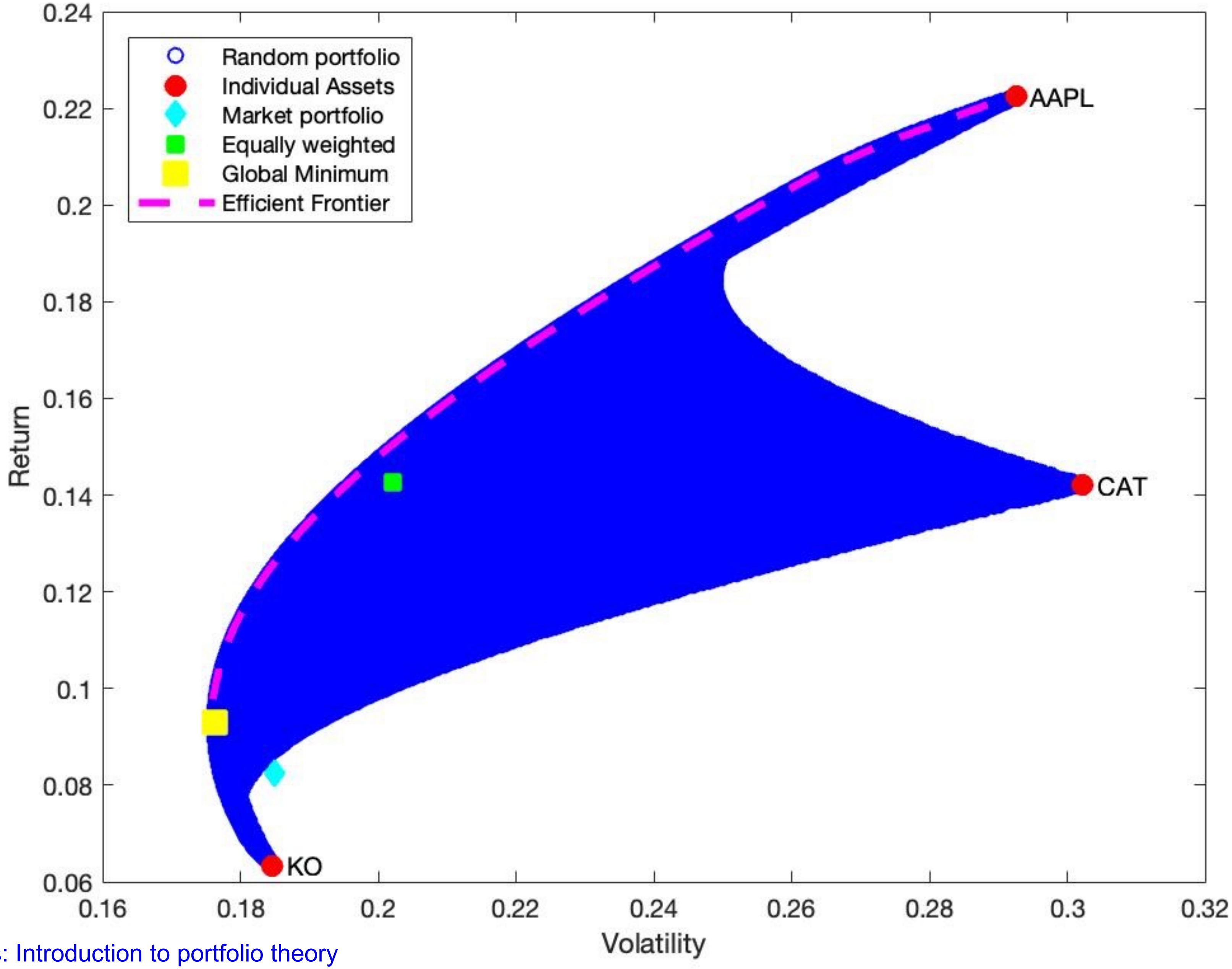

10^6 random portfolios
Random portfolio
Individual Assets
Market portfolio
Equally weighted
AAPL
CAT
KO
Risk-free
Return
Volatility
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35


Random portfolio
Individual Assets
Market portfolio
Equally weighted
Global Minimum
Efficient Frontier
AAPL
CAT
KO
Return
Volatility
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.16
0.18
0.2
0.22
0.24
0.26
0.28
0.3
0.32

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Can we compute the efficient frontier?

- Optimization process.
- Maximize portfolio returns for a given level of portfolio risk
- Alternatively (dual problem): Minimize risk (square root of the portfolio covariance) for a given level of return.
- We can use covariance directly instead of square-root of covariance (monotonic function).
- We need to consider the restrictions of long-only portfolio problem:
  - Weights must sum 1.
  - Weight should non-negatives and the maximum allowed value is one.

Mean-variance optimization

- Lets consider a portfolio of N-assets, with weights $w=\{w_1,w_2,\dots w_N\}$.
- Each asset has a return R_i, and risk σ_i, with $i=\{1,\dots,N\}$.
- The portfolio return is given by: $$R_P = \sum_{i=1}^{N} w_1 R_i$$
- The long-only restrictions are mathematically defined as:

$$\sum_{i=1}^{N} w_1 = 1, \quad 0 \leq w_i \leq 1$$

Mean-variance optimization

- The risk of the portfolio is measured by the square-root of portfolio return variance:

$$\begin{aligned}\operatorname{var}(R_P) &= \sum_{i=1}^{N} w_i^2 \operatorname{var}(R_i) + 2\sum_{i=1}^{N}\sum_{j>i}^{N} w_i w_j \operatorname{cov}(R_i, R_j) \\ &= \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}\end{aligned}$$

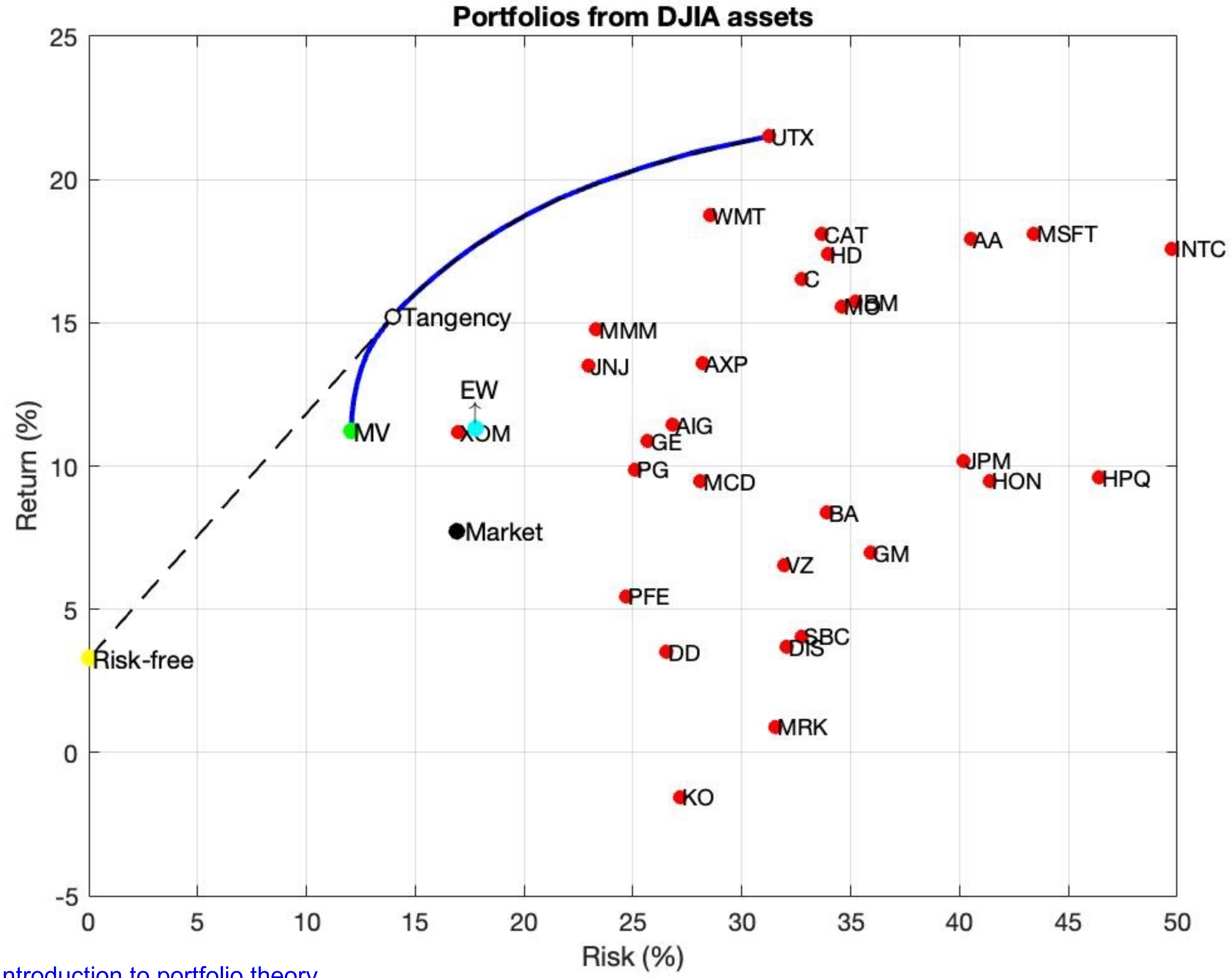
- ρ_{ij}: Correlation between asset i and j.

Measuring the investment (portfolio) performance

– Sharpe ratio:

$$SR_P = \frac{R_P - R_f}{\sigma_p}$$

– R_f: Risk-free rate of interest.
– It measure the excess of return adjusted by risk.
– The portfolio over the efficient frontier with the highest Sharpe ratio is the tangency portfolio.



Portfolios from DJIA assets
Return (%)
Risk (%)
25
20
15
10
5
0
-5
0
5
10
15
20
25
30
35
40
45
50
UTX
WMT
CAT
AA
MSFT
INTC
HD
C
IBM
MO
Tangency
MMM
AXP
JNJ
EW
AIG
MV
XOM
GE
PG
JPM
HON
HPQ
MCD
BA
Market
GM
VZ
PFE
SBC
DIS
DD
Risk-free
MRK
KO

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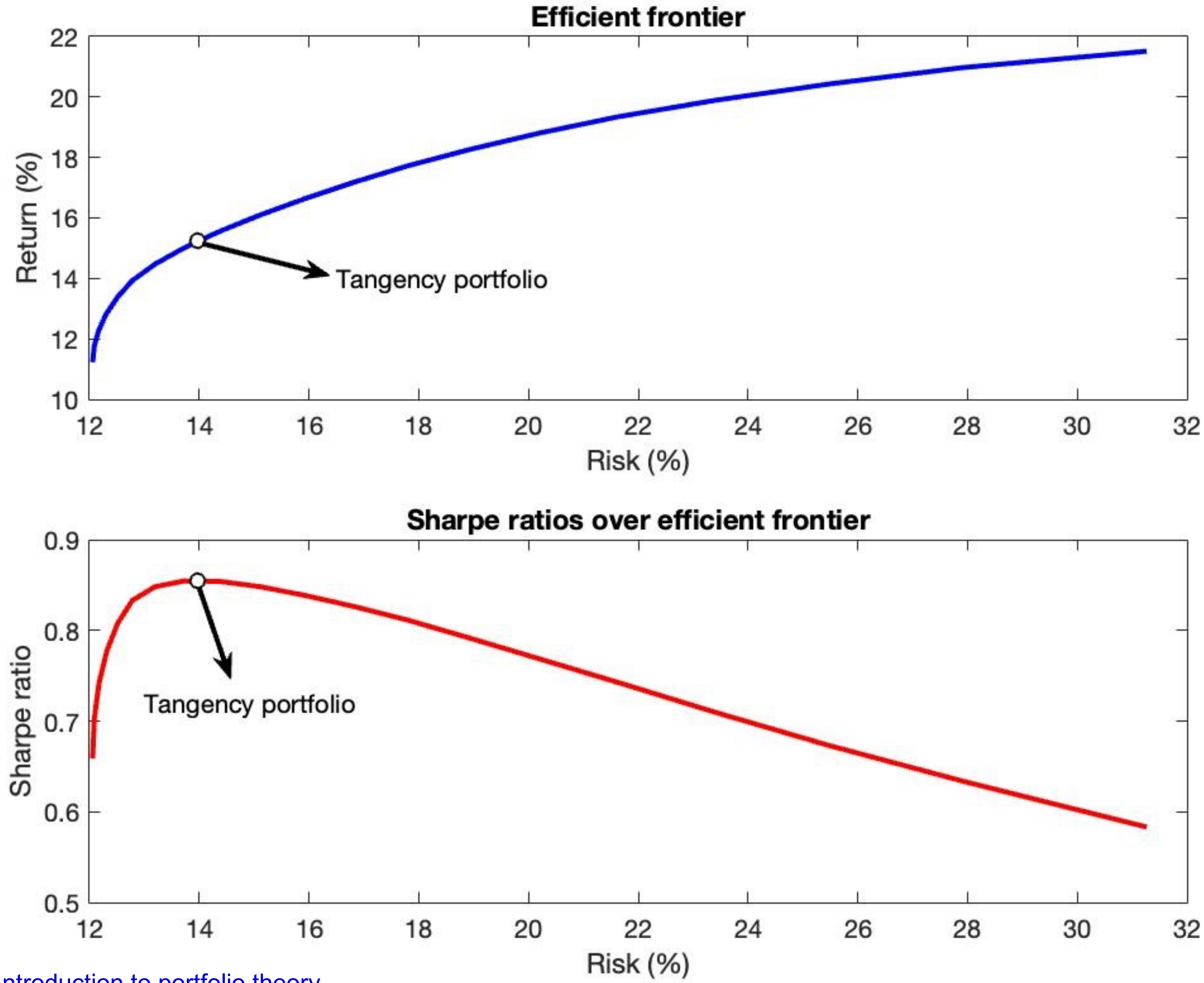
Efficient frontier
Return (%)
Risk (%)
Tangency portfolio
22
20
18
16
14
12
10
12
14
16
18
20
22
24
26
28
30
32
Sharpe ratios over efficient frontier
Sharpe ratio
Risk (%)
Tangency portfolio
0.9
0.8
0.7
0.6
0.5

Example: Computing Sharpe ratio (R_f=2%)

	Port. 1	Port. 2	Port. 3	Port. 4
Return (anualized)	7.3%	8.7%	12.3%	32.8%
Risk (annual SD)	20.2%	25.4%	18.1%	29.5%
Sharpe ratio	0.26	0.26	0.57	1.04

- The portfolio 4 is ranked first in terms of SR. Even though is the riskier one, It has the best reward per unit of risk.
- Port. 1 and 2 has similar SR. However, port. 1 has less risk exposure.