

Linear programming-introduction

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USE of Linear Programming approach

- **Slitting and Levelling of material (coils, bars, sheets)**-Cutting material, trimming,...
- **Blending** - blending, diet, feeding rations for animals, ..
- **Transport problems** - material flow from stock to the destination and route planning - shortest route
- **Assignment of resources with limited capacities** - **CCR**
- **Sources** : Operation Management, Quality and Competitiveness in a global environment, Russel and Taylor (can be found easily in ESF library)

CCR=Capacity Constraint Resource

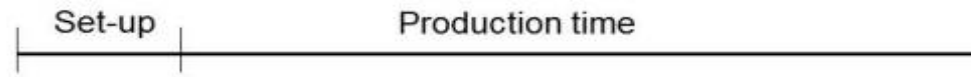
See next slide for explanantion of CCR



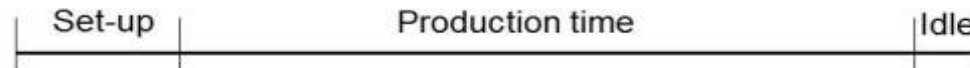
CCR –additional information

- There are **3 categories** of resources from the point of view of capacity:
- **Bottleneck**
- **CCR – Capacity Constraint Resource (closed to bottleneck)**
- **Non-CCR**

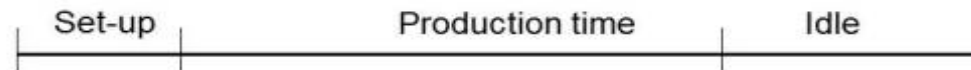
Bottleneck – demand on the machine **is higher than the available capacity**.
Works 24x7, the whole year around.





CCR (Capacity Constraint Resource) – according to the available time that you allow it to work, it becomes a trouble maker. The load bigger than 70%. The idle time is so little and unstable that in no time it can turn to Bottleneck.



Non-CCR – idle capacity includes some protective capacity.



Formulation of the simple model

| Product | Description | Work /hour | Material/pcs | Return/pcs |
|---|-------------|------------|--------------|------------|
|  | Dish x1 | 1 | 4 | 40 |
|  | Mug x2 | 2 | 3 | 50 |



Which combination of products will have the greatest return at the limits of maximum production capacity type = **40** hours and moreover, the amount of material that is limited to **120** kg of **clay**?

Note: A similar task in terms of flow was solved in the P&Q example (see Product Mixture I. File in study material), where the limitation resource was machine B and with a maximum capacity of 2400 minutes

Description x1 and x2 stands for variables.

Material means e.g. 4 kg for one piece (product) – in our example Dish

Basic structures and used terminology

- We minimize our **target function** in the form of:

Target function
 $Z = C * X$

$Z = c_1 * x_1 + c_2 * x_2 + \dots + c_n * x_n$ with respect to the matrix of restrictive conditions:
 (in our case $c_1=40$ and $c_2=50$ which means return/pc) -> see Return for both products

$$A_{11} * x_1 + A_{12} * x_2 + \dots + A_{1n} * x_n \quad (<=>) \quad B_1$$

$$A_{22} * x_1 + A_{22} * x_2 + \dots + A_{2n} * x_n \quad (<=>) \quad B_2$$

$$A_{m1} * x_1 + A_{m2} * x_2 + \dots + A_{mn} * x_n \quad (<=>) \quad B_m$$

- Solved problem can be described as a set of linear equations $A * x = B$
- The solving of such a linear equation system, e.g. by use of GAUSS-JORDAN algorithm is not required if we will use **Excel Solver Add-on (application)**.
- x_{ij} : decision variable = level of operation activity specified by this variable
- B_i : restrictive conditions , allowed deviations from the norm (in time and material)
- c_j : coefficient of the target function (in our case returns, meaning return **40** and **50**)
- A_{ij} : restrictive coefficients: work and material for one unit (pcs) of the product

Example I (introduction to the problem – practical demonstration)

| Product | Description | Work /hour | Material/pcs | Return/pcs |
|---------|-------------|------------|--------------|------------|
| Dish | x1 | 1 | 4 | 40 |
| Mug | x2 | 2 | 3 | 50 |

$$Z = c_1*x_1 + c_2*x_2 + \dots + c_n*x_n \text{ (classical equation from)}$$

Target function: $Z = 40*x_1 + 50*x_2$, which we must maximize

Maximal production capacity = 40 hours and Maximal quantity of material (clay)=120 kg (B1 and B2 in our mathematical expression) - **This is our constraint in this model**

Specifications of task restrictions by use of 2x2 matrix:

$$1*x_1 + 2*x_2 = 40 \text{ (work-no more than 40 hours)}$$

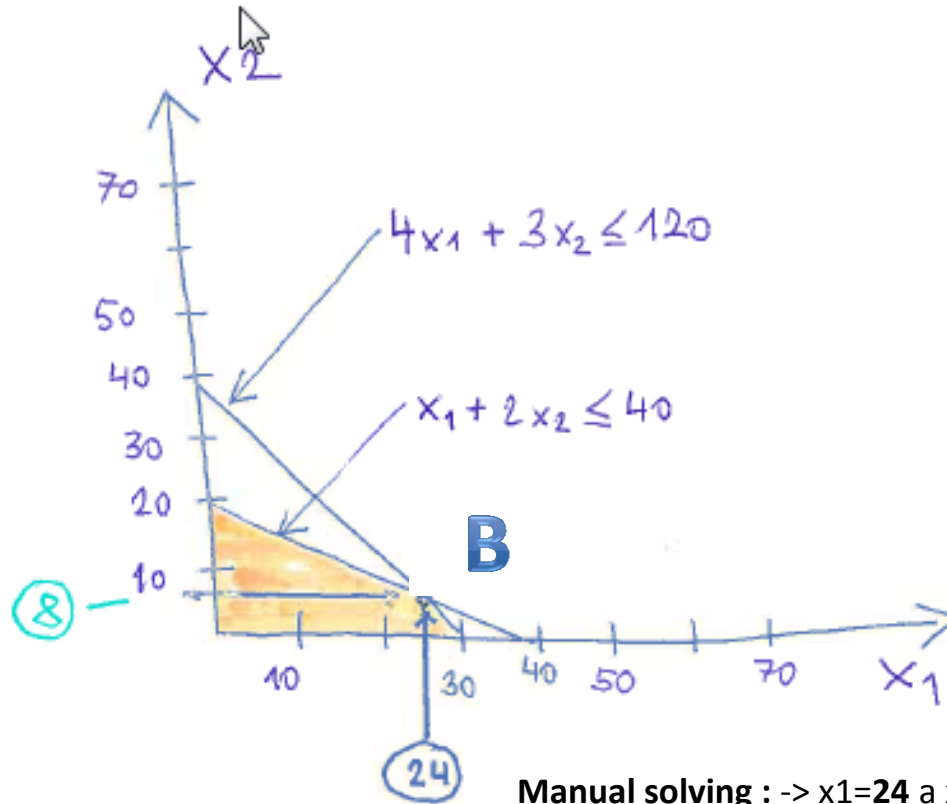
$$4*x_1 + 3*x_2 = 120 \text{ (material=kg of clay in our case)} \rightarrow x_1 = (40 - 2x_2) + 3x_2 = 120 \dots$$

Manual solving : $\rightarrow x_1 = 24$ a $x_2 = 8$ and after substitution of variables (24 pcs of Dish and 8 pcs of Mug) in target function we will get

$$Z = 40*24 + 50*8 = 1360$$

(optimal Return meets the point B – see next slide)

Graphical solution

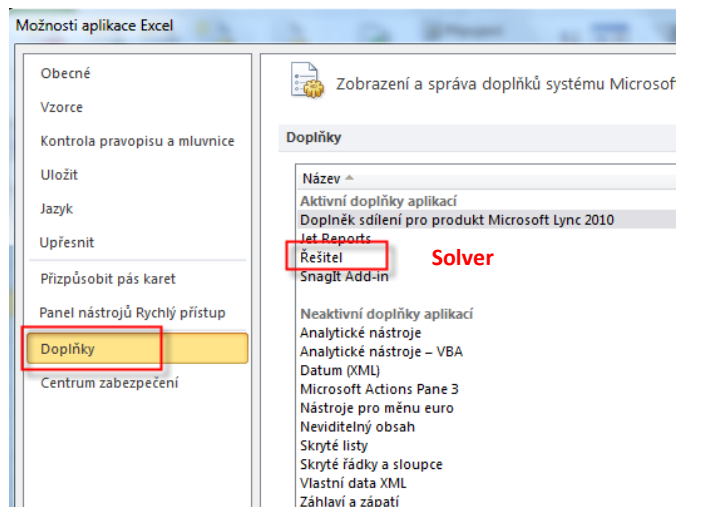


Manual solving : -> $x_1=24$ a $x_2=8$ and after substitution of variables (24 pcs of Dish and 8 pcs of Mug) in target function we will get

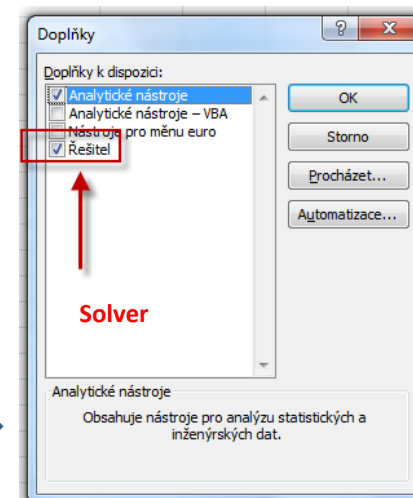
I apologize for the inappropriate graphic expression....

Use of Solver (Czech EXCEL) with some basic translations

Complements Supplement



Excel setup

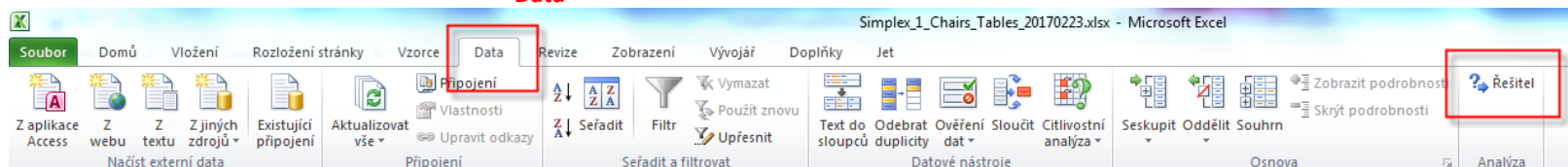


Doplněk: Analytické nástroje
Vydavatel: Microsoft Corporation
Kompatibilita: K dispozici nejsou žádné informace o kompatibilitě.
Umístění: C:\Program Files (x86)\Microsoft Office\Office14\Library\Analysis\ANALYS32.XLL
Popis: Obsahuje nástroje pro analýzu statistických a inženýrských dat.

Spravovat: Doplnění aplikace Excel

Data

Solver



Use o solver (see actual Excel formulas on one of the next slides)

| | Dish | Mug | Total | Capacity |
|--------------------|------|-----|-------|----------|
| Variables (x1, x2) | 0 | 0 | | |
| Return | 40 | 50 | 0 | |
| Material | 4 | 3 | 0 | 120 |
| Work | 1 | 2 | 0 | 40 |

Target function

Set of linear equations

$x1 = \text{Dish}$, $x2 = \text{Mug}$, max 40 hour (B1), max 120 kg (B2)

Assignment entered in table

Assignment

Excel formulas shown: $=D7*D6+E7*E6$, $=D10*D6+E10*E6$, $=D11*D6+E11*E6$

Target function $Z = x1 * c1 + x2 * c2 = 40 * x1 + 50 * x2$

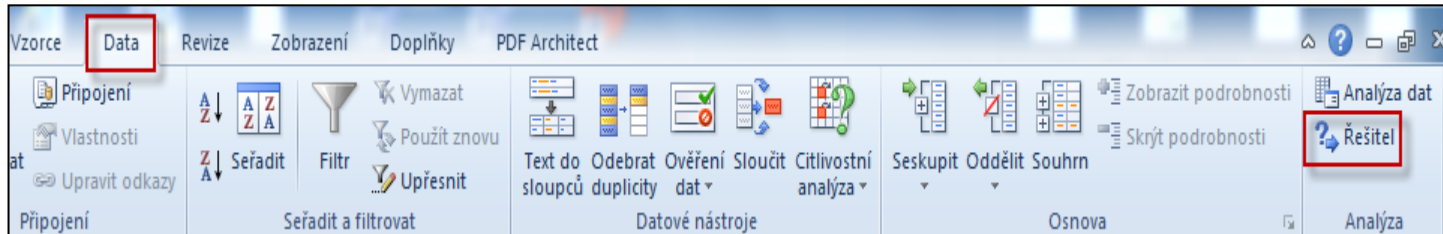
$4 * x1 + 3 * x2 = 120$ - capacity restrictions = max quantity of material = B1

$1 * x1 + 2 * x2 = 40$ - capacity restrictions by max work capacity = B2



| Product | Description | Work /hour | Material/pcs | Return/pcs |
|---------|-------------|------------|--------------|------------|
| Dish | x1 | 1 | 4 | 40 |
| Mug | x2 | 2 | 3 | 50 |

Solver start



The Excel file that is used for our example is stored both in the study materials and in MS TEAMS

When using Solver in Excel, just open this file
Name of the file : LP_EXCEL_SOLVER USE_20221124

Use of Solver (Czech- not for MHP_AOPR)

| | A | B | C | D | E | F | G |
|---|---|-----------------------|--------------|--------------|--------------|-----------------|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | Miska | Hrnek | Total | Kapacita | |
| 4 | | Proměnné x1,X2 | 0 | 0 | | | |
| 5 | | Přínos | 40 | 50 | 0 | | |
| 6 | | | | | | | |
| 7 | | Materiál | 4 | 3 | 0 | 120 | |
| 8 | | Práce | 1 | 2 | 0 | 40 | |
| 9 | | | | | | | |

$$E5 = =D7*D6+E7*E6$$

$$Z = x1 * c1 + x2 * c2 = 40 * x1 + 30 * x2$$

$$E7 = C7 * C4 + D7 * D4 = 4 * x1 + 3 * x2 = 120$$

$$E8 = C8 * C4 + D8 * D4 = x1 + 2 * x2 = 40$$

Parametry Řešitele

Účelová funkce:

Hledat: Max Min Hodnota:

Proměnné modelu:

Omezující podmínky:

$SE7 \leq SF7$
 $SE8 \leq SF8$

| | Miska | Hrnek | Total | Kapacita |
|-----------------------|-------|-------|-------|----------|
| Proměnné x1,X2 | 24 | 8 | | |
| Přínos | 40 | 50 | 1360 | |
| Materiál | 4 | 3 | 120 | 120 |
| Práce | 1 | 2 | 40 | 40 |

Use of solver (for MPH_AOPR)

F7 =

=D7*D6+E7*E6

| | A | B | C | D | E | F | G | H |
|----|---|---|------------------|------|-----|-------|----------|---|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | Dish | Mug | Total | Capacity | |
| 6 | | | Variables X1, X2 | 0 | 0 | | | |
| 7 | | | Return | 40 | 50 | 0 | | |
| 8 | | | | | | | | |
| 9 | | | | | | | | |
| 10 | | | Material | 4 | 3 | 0 | 120 | |
| 11 | | | Work | 1 | 2 | 0 | 40 | |
| 12 | | | | | | | | |

$$Z = x1 * c1 + x2 * c2 = 40 * x1 + 30 * x2$$

$$F10 = D10 * D6 + E10 * E6 = 4 * x1 + 3 * x2 = 120$$

$$F11 = D11 * D6 + E11 * D6 = x1 + 2 * x2 = 40$$

=D10*D6+E10*E6

=D11*D6+E11*E6

Parametry Řešitele

Nastavit cíl: **Target** \$F\$7

Na: Max Min Hodnota: 0

Na základě změny proměnných buněk: **Variables** \$D\$6:\$E\$6

Omezující podmínky: **Restrictions**

\$F\$10 <= \$G\$10
\$F\$11 <= \$G\$11

Nastavit proměnné bez omezujících podmínek jako nezáporné

Vyberte metodu řešení: GRG Nonlinear

Metoda řešení
Modul GRG Nonlinear vyberte pro hladké nelineární problémy Řešitele. Modul LP Simplex zvolte pro lineární problémy Řešitele a modul Evolutionary pro nehladké problémy Řešitele.

Solve it



| | Dish | Mug | Total | Capacity |
|------------------|------|-----|-------|----------|
| Variables X1, X2 | 24 | 8 | | |
| Return | 40 | 50 | 1360 | |
| Material | 4 | 3 | 120 | 120 |
| Work | 1 | 2 | 40 | 40 |

Využití Řešitele (Czech -use of Solver)

Microsoft Excel 15.0 Citlivostní sestava

List: [Simplex_1_Misky_Hrnky_Chairs_Tables_20170228.xlsx]List1

Sestava vytvořena: 9. 3. 2017 16:19:56

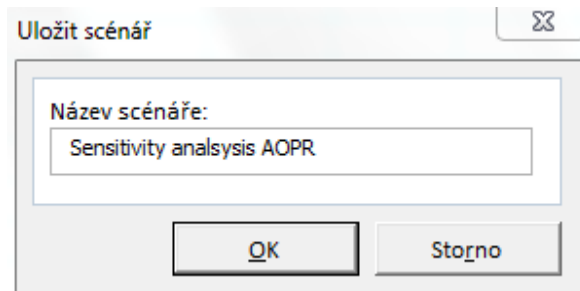
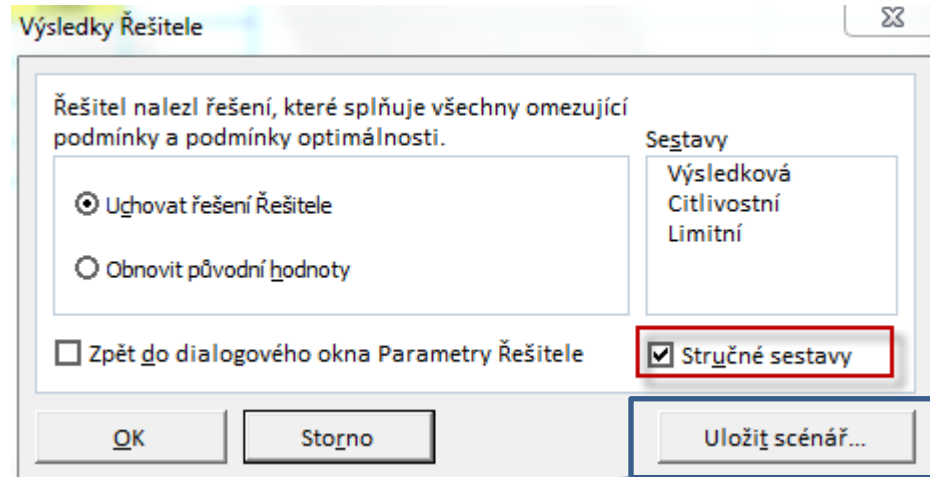
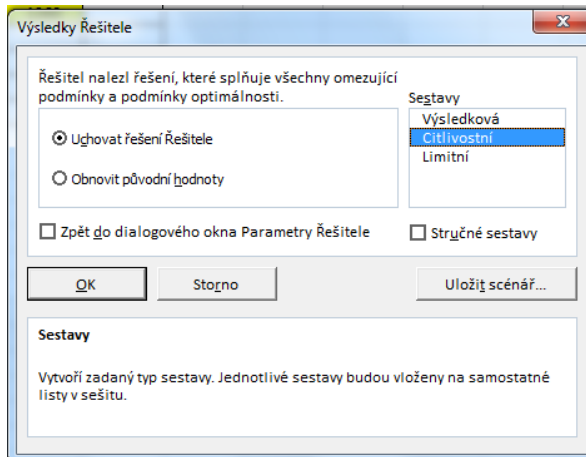
Proměnné

| Levá strana omezující podmínky | Název | Konečná Hodnota | Redukovaná náklady | Účelová funkce koeficient | Povolený nárůst | Povolený pokles |
|--------------------------------|----------------------|--------------------|-----------------------|------------------------------|--------------------|--------------------|
| \$C\$4 | Proměnné x1,X2 Miska | 24 | 0 | 40 | 26,66666667 | 15 |
| \$D\$4 | Proměnné x1,X2 Hrnek | 8 | 0 | 50 | 30 | 20 |

Omezující podmínky

| Levá strana omezující podmínky | Název | Konečná Hodnota | Stínová cena | Pravá strana omezující podmínky | Povolený nárůst | Povolený pokles |
|--------------------------------|----------------|--------------------|-----------------|------------------------------------|--------------------|--------------------|
| \$E\$7 | Materiál Total | 120 | 6 | 120 | 40 | 60 |
| \$E\$8 | Práce Total | 40 | 16 | 40 | 40 | 10 |

Use of Solver (MPH_AOPR)



New Excel List



Microsoft Excel 14.0 Citlivostní sestava
 List: [LP_EXCEL_SOLVER USE_20171101.xlsx]List1
 Sestava vytvořena: 2.11.2017 8:49:10

Proměnné buňky

| Buňka | Název | Konečná Hodnota | Snížené Gradient |
|--------|-----------------------|-----------------|------------------|
| \$D\$6 | Variables X1, X2 Dish | 24 | 0 |
| \$E\$6 | Variables X1, X2 Mug | 8 | 0 |

Omezující podmínky

| Buňka | Název | Konečná Hodnota | Lagrangeův multiplikátor |
|---------|----------------|-----------------|--------------------------|
| \$F\$10 | Material Total | 120 | 6 |
| \$F\$11 | Work Total | 40 | 16 |

Změna úlohy- jiné výnosy jiná omezení typu práce na dvou strojích a jejich kapacitní omezení

(Change of parameters- not necessary for MPH_AOPR !!!!!)

| | Miska | Hrnek | Total | Kapacita |
|----------------|-------|-------|-------|----------|
| Proměnné x1,x2 | 0 | 0 | | |
| Přínos | 40 | 50 | 0 | |
| Stroj 1 | 7 | 5 | 0 | 200 |
| Stroj 1 | 5 | 5 | 0 | 400 |



| | Miska | Hrnek | Total | Kapacita |
|----------------|-------|-------|-------|----------|
| Proměnné x1,x2 | 0 | 40 | | |
| Přínos | 40 | 50 | 2000 | |
| Stroj 1 | 7 | 5 | 200 | 200 |
| Stroj 1 | 5 | 5 | 200 | 400 |

Parametry Řešitele

Účejová funkce:

Hledat: Max Min Hodnota:

Proměnné modelu:

Omezující podmínky:

- \$E\$15 <= \$F\$15
- \$E\$16 <= \$F\$16



OK ?