Linear programming-introduction

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USE of Linear Programming approach

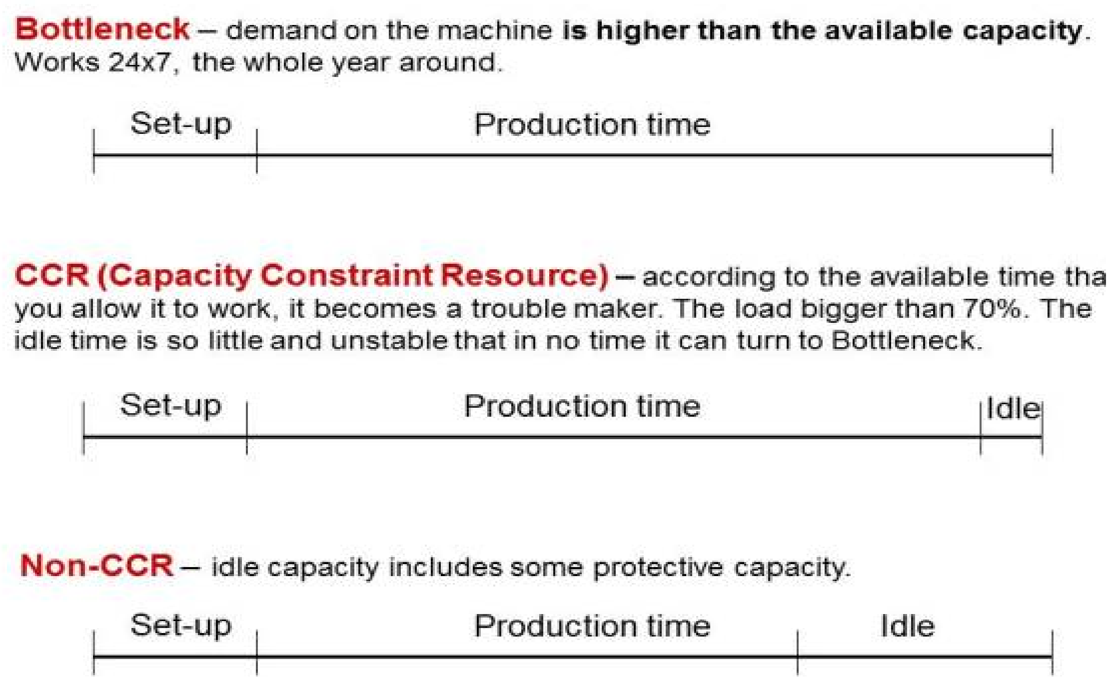
- **Slitting and Levelling of material (coils, bars, sheets)-**Cutting material, trimming,…
- **Blending -** blending, diet, feeding rations for animals, ..
- **Transport problems -** material flow from stock to the destination and route planning - shortest route
- **Assignment of resources with limited capacities - CCR**
- **Sources :** Operation Management, Quality and Competitiveness in a global environment, Russel and Taylor **(can be found easily in ESF library)**

CCR=Capacity Constraint Resource

See next slide for explanantion of CCR

CCR –additional information

- There are **3 categories** of resources from the point of view of capacity:
- **Bottleneck**
- **CCR – Capacity Constraint Resource (closed to bottlenek)**
- **Non-CCR**



Formulation of the simple model

Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	$x1$	1	4	40
Mug	$x2$	2	3	50

Which combination of products will have the greatest return
at the limits of maximum production capacity type = **40** hours
and moreover, the amount of material that is limited to **120** kg of clay?

Note: A similar task in terms of flow was solved in the P&Q example (see Product Mixture I. File in study material), where the limitation resource was machine B and with a maximum capacity of 2400 minutes

Description x1 and x2 stands for variables.
Material means e.g. 4 kg for one piece (product) – in our example Dish

Basic structures and used terminology

Target function
Z=C*X

- We minimize our **target function** in the form of:

Z = c1*x1+c2*x2+…..+cn*xn with respect to the matrix of restrictive conditions:
(in our case **c1=40** and **c2=50 which means return/pc**) -> see Return for both products

A11*x1 + A12*x2+ …+ A1n*xn (<>=) B1
A22*x1 + A22*x2+ …+ A2n*xn (<>=) B2
⋮
Am1*x1 + Am2*x2+ …+ Amn*xn (<>=)Bm

- Solved problem can be described as a set of linear equations **A*x=B**
- The solving of such a linear equation system, e.g. by use of GAUSS-JORDAN algorithm is not required if we will use **Excel Solver Add-on (application)**.
- **xij** : decision variable = level of operation activity specified by this variable
- **Bi** : restrictive conditions , allowed deviations from the norm (in time and material)
- **cj** : coefficient of the target function (in our case returns, meaning return **40** and **50**)
- **Aij** : restrictive coefficients: work and material for one unit (pcs) of the product

Example I (introduction to the problem – practical demonstration)

Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	x1	1	4	40
Mug	x2	2	3	50

Z = c1*x1+c2*x2+…..+cn*xn (classical equation from)

Target function: **Z**= 40*x1+50*x2, which we must maximize

Maximal production capacity = **40** hours and Maximal quantity of material (clay)=120 kg
(B1 and B2 in our mathematical expression) - **This is our constraint in this model**

Specifications of task restrictions by use of 2x2 matrix:

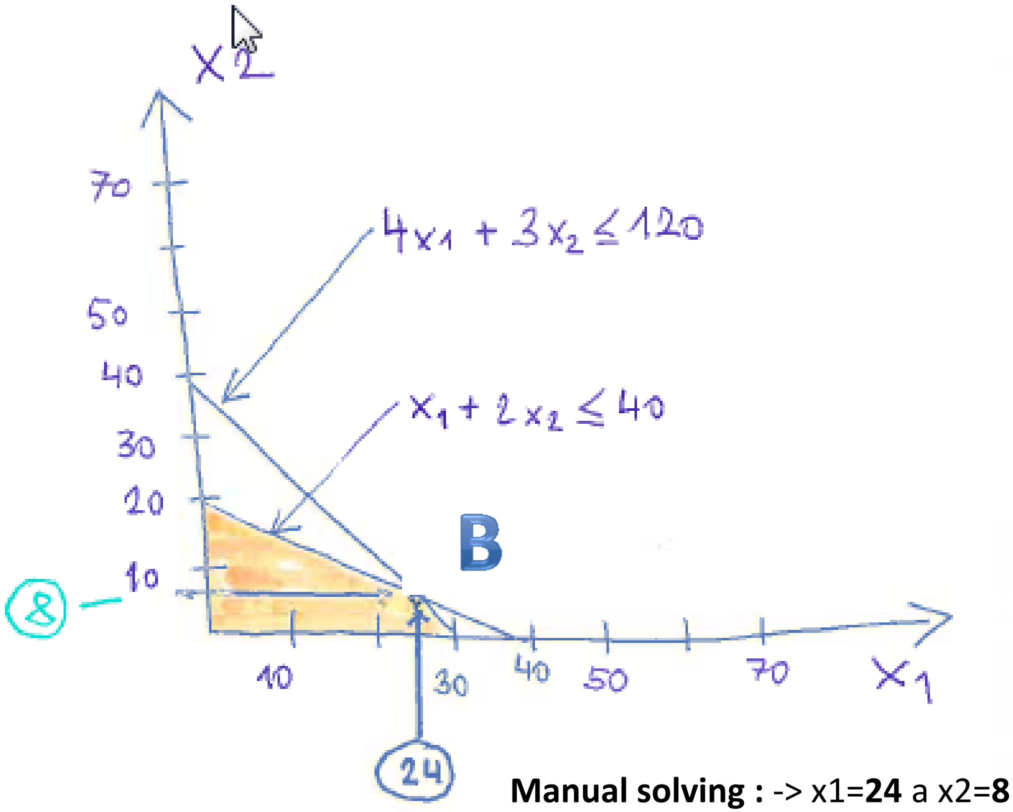
1*x1 + 2*x2 =**40** (work-no more than 40 hours)
4*x1 + 3*x2 =120 (material=kg of clay in our case)->x1=(40-2x2)+3x2=120….

Manual solving : -> x1=24 a x2=8 and after substitution od variables (24 pcs of Dish and 8 pcs of Mug) in target function we will get

Z=40*24+50*8=**1360**

(optimal Return meets the point B – see next slide)

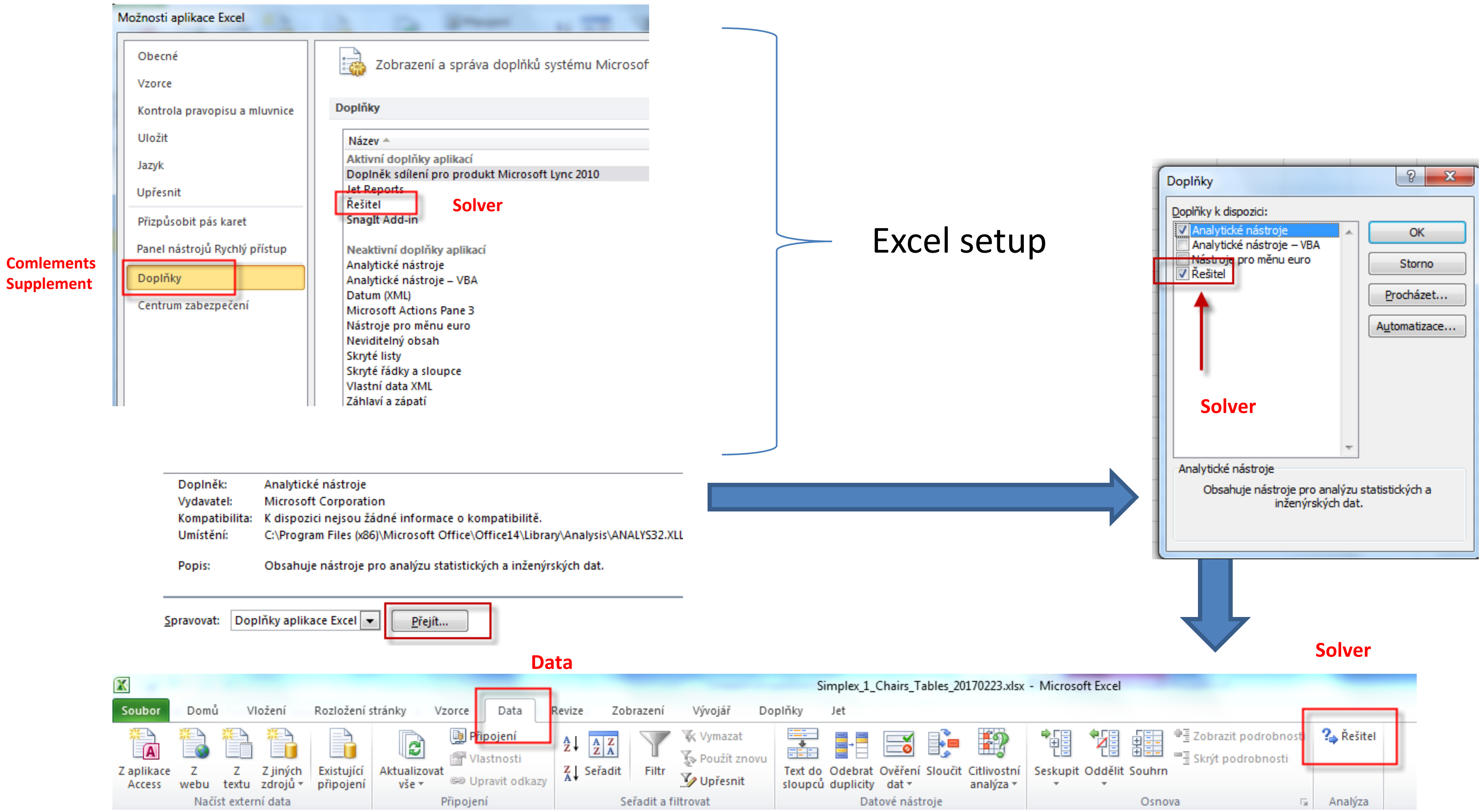
Graphical solution



Manual solving : -> x1=**24** a x2=**8** and after substitution od variables (24 pcs of Dish and 8 pcs of Mug)
in target function we will get

I apologize for the inappropriate graphic expression....

Use of Solver (Czech EXCEL) with some basic translations



Use o solver (see actual Excel formulas on one of the next slides)

Target function

x1 x2

	Dish	Mug	Total	Capacity
Variables (x1, x2)	0	0		
Return	40	50	0	
Material	4	3	0	120
Work	1	2	0	40

Set of linear equations

=D7*D6+E7*E6

Z

=D10*D6+E10*E6

=D11*D6+E11*E6

Assignment entered in table

x1=Dish , x2=Mug, max 40 hour (B1), max 120 kg (B2)

Assignment

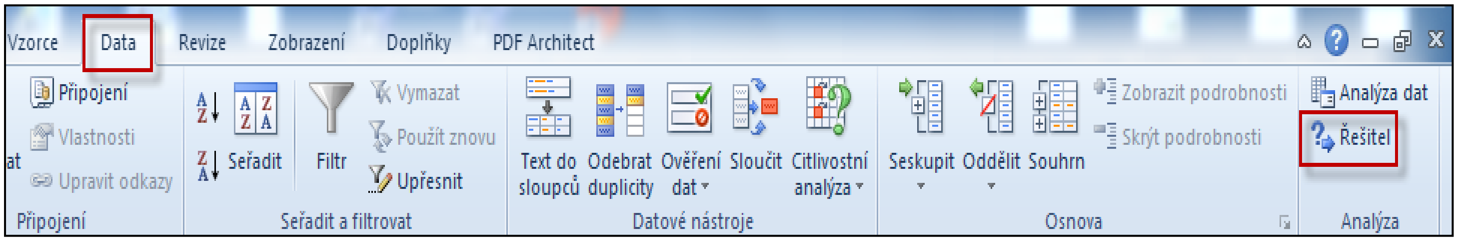
Target function **Z** = x1*c1 + x2*c2 = 40*x1+50*x2

4 * x1 + 3 *x2 =120 - capacity restrictions= max quantity of material =B1

1 * x1 + 2 *x2 = 40 -capacity restrictions by max work capacity=B2

Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	x1	1	4	**40**
Mug	x2	2	3	**50**

Solver start



The Excel file that is used for our example is stored both in the study materials and in MS TEAMS

When using Solver in Excel, just open this file
Name of the file : LP_EXCEL_SOLVER USE_20221124

Use of Solver (Czech- not for MHP_AOPR)

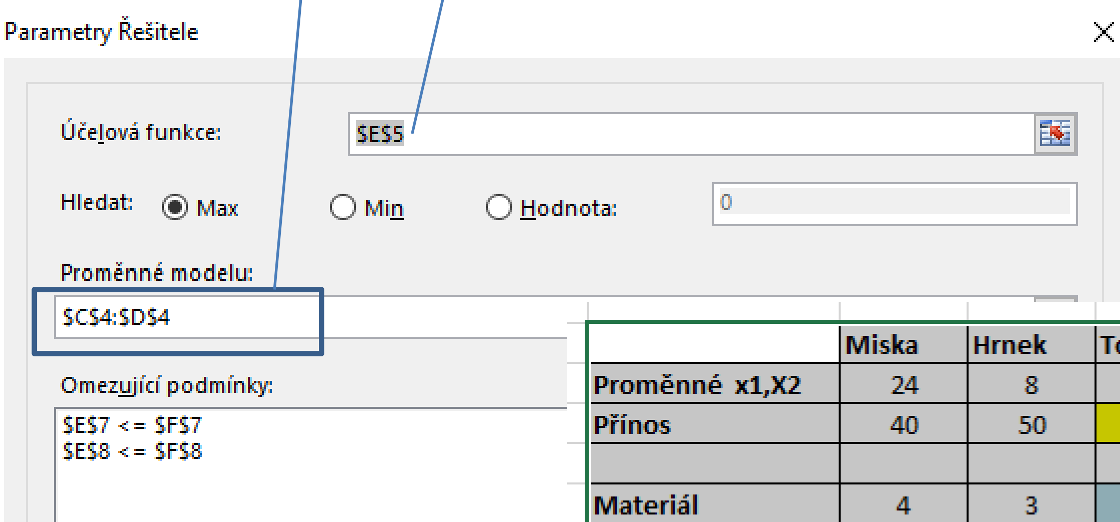
	Miska	Hrnek	Total	Kapacita
Proměnné x1,X2	0	0		
Přínos	40	50	0	
Materiál	4	3	0	120
Práce	1	2	0	40

E5 = =D7*D6+E7*E6

Z = x1*c1 + x2*c2 = 40*x1+30*x2

E7=C7*C4+D7*D4=4*x1+3*x2=120

E8=C8*C4+D8*D4=x1+2*x2=40



	Miska	Hrnek	Total	Kapacita
Proměnné x1,X2	24	8		
Přínos	40	50	1360	
Materiál	4	3	120	120
Práce	1	2	40	40

Use of solver (for MPH_AOPR)

F7 = =D7*D6+E7*E6

	Dish	Mug	Total	Capacity
Varaibles X1, X2	0	0		
Return	40	50	0	
Material	4	3	0	120
Work	1	2	0	40

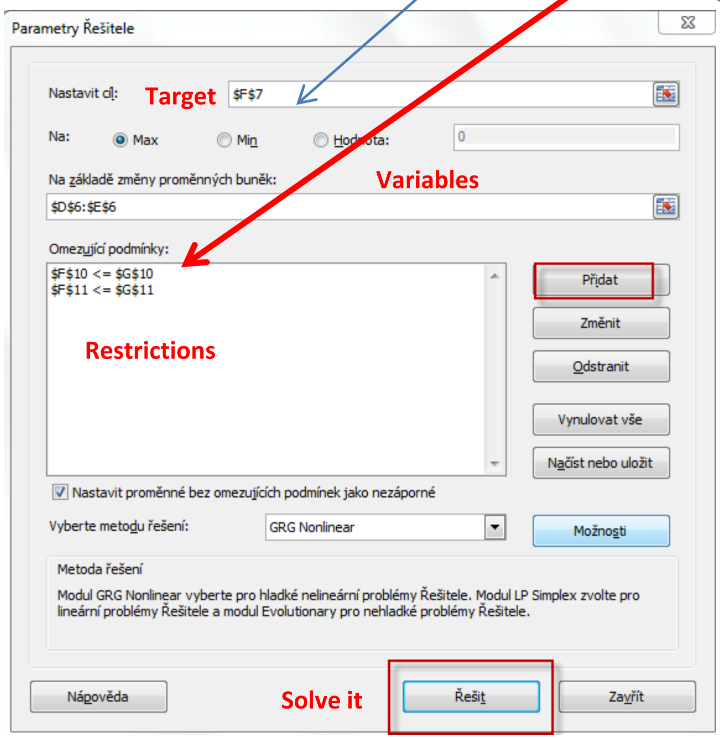
Z = x1*c1 + x2*c2 = 40*x1+30*x2

F10=D10*D6+E10*E6=4*x1+3*x2=120

F11=D11*D6+E11*D6=x1+2*x2=40

=D10*D6+E10*E6

=D11*D6+E11*E6



	Dish	Mug	Total	Capacity
Varaibles X1, X2	24	8		
Return	40	50	1360	
Material	4	3	120	120
Work	1	2	40	40

Využití Řešitele (Czech -use of Solver)

Microsoft Excel 15.0 Citlivostní sestava
List: [Simplex_1_Misky_Hrnky_Chairs_Tables_20170228.xlsx]List1
Sestava vytvořena: 9. 3. 2017 16:19:56

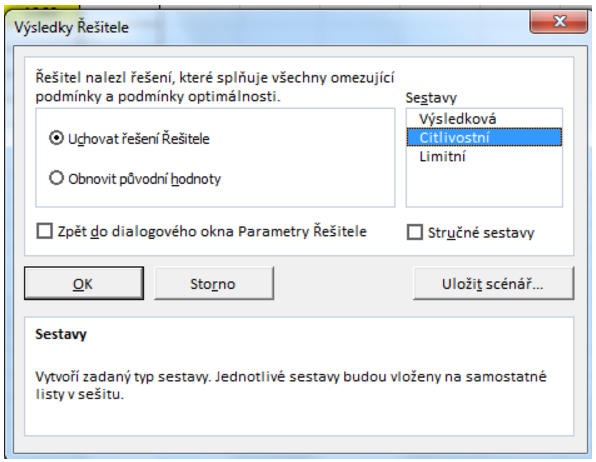
Proměnné

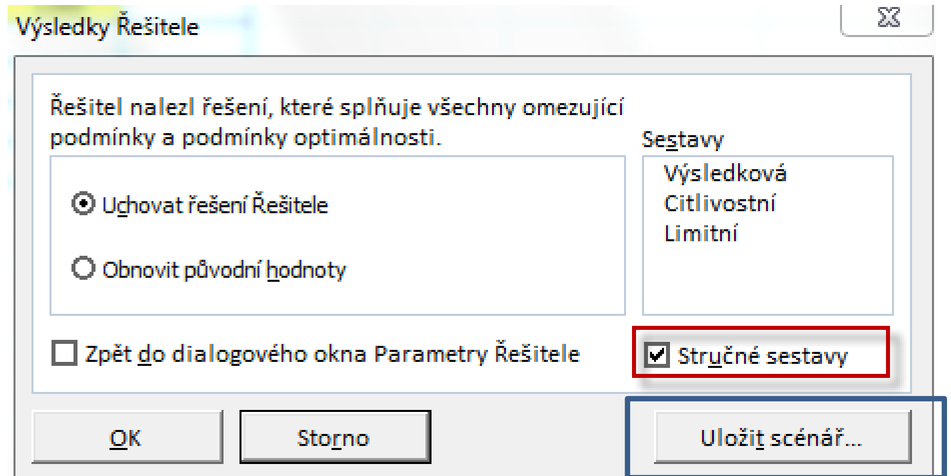
Levá strana omezující podmínky	Název	Konečná Hodnota	Redukovaná náklady	Účelová funkce koeficient	Povolený nárůst	Povolený pokles
C4	Proměnné x1,X2 Miska	24	0	40	26,66666667	15
D4	Proměnné x1,X2 Hrnek	8	0	50	30	20

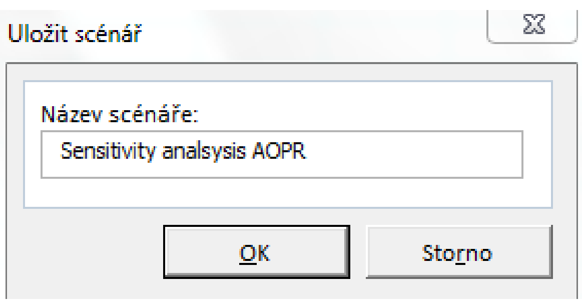
Omezující podmínky

Levá strana omezující podmínky	Název	Konečná Hodnota	Stínová cena	Pravá strana omezující podmínky	Povolený nárůst	Povolený pokles
E7	Materiál Total	120	6	120	40	60
E8	Práce Total	40	16	40	40	10

Use of Solver (MPH_AOPR)







Microsoft Excel 14.0 Citlivostní sestava
List: [LP_EXCEL_SOLVER USE_20171101.xlsx]List1
Sestava vytvořena: 2.11.2017 8:49:10

Proměnné buňky

Buňka	Název	Konečná Hodnota	Snížené Gradient
D6	Varaibles X1, X2 Dish	24	0
E6	Varaibles X1, X2 Mug	8	0

Omezující podmínky

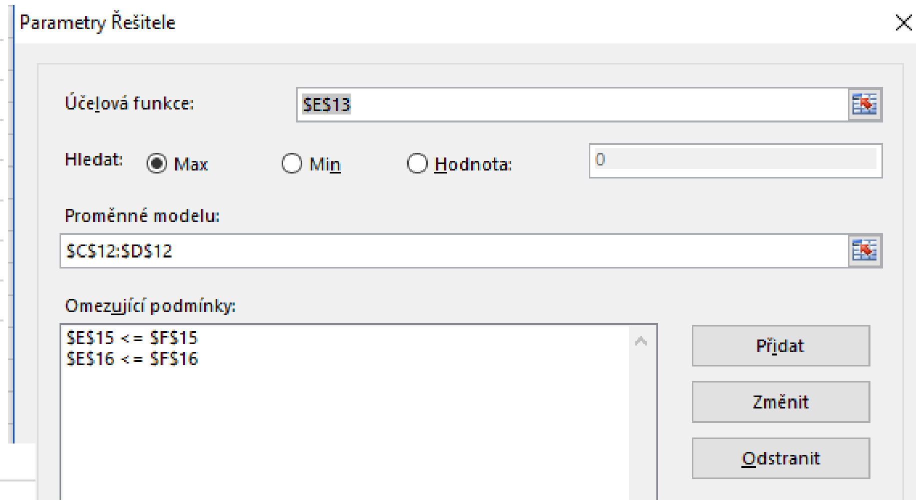
Buňka	Název	Konečná Hodnota	Lagrangeův multiplikátor
F10	Material Total	120	6
F11	Work Total	40	16

New Excel List

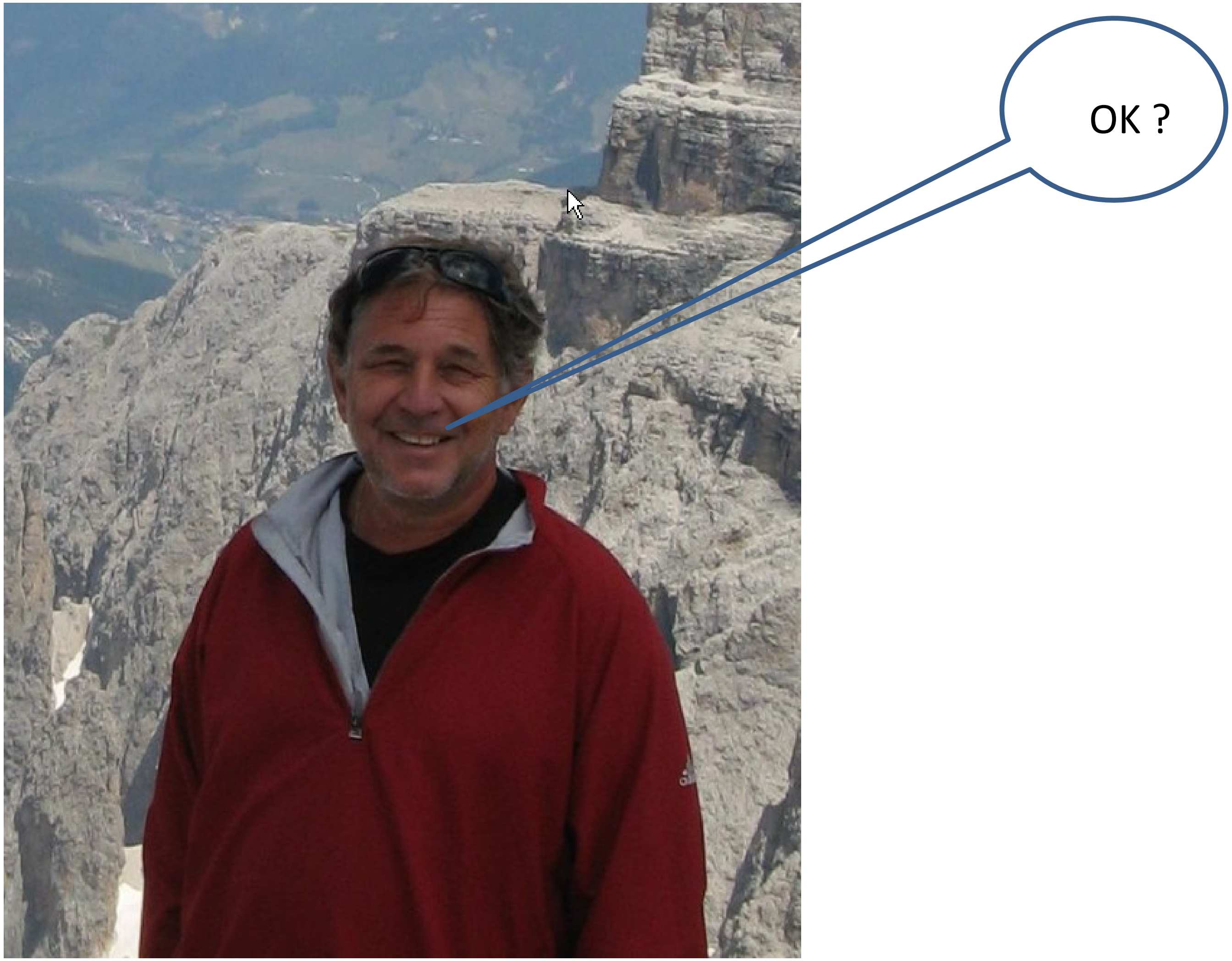
Změna úlohy- jiné výnosy jiná omezení typu práce na dvou strojích a jejich kapacitní omezení

(**Change of parameters- not necessary for MPH_AOPR !!!!!**)

	Miska	Hrnek	Total	Kapacita
Proměnné x1,x2	0	0		
Přínos	40	50	0	
Stroj 1	7	5	0	200
Stroj 1	5	5	0	400



	Miska	Hrnek	Total	Kapacita
Proměnné x1,x2	0	40		
Přínos	40	50	2000	
Stroj 1	7	5	200	200
Stroj 1	5	5	200	400


OK ?