Introductory Econometrics
Lecture 4: Hypothesis Testing
Suggested Solution

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1.

Consider an equation to explain salaries of CEOs in terms of annual firm sales return on equity (`roe` in percentage form) and return on the firm's stock (`ros` in percentage form):

$$\log(\text{salary}) = \beta_0 + \beta_1 \log(\text{sales}) + \beta_2 \text{roe} + \beta_3 \text{ros} + u$$

- In terms of the model parameters, state the null hypothesis that after controlling for sales and `roe` `ros` has no effect on CEO salary. State the alternative that better stock market performance increases a CEO's salary.

  Solution:
  $$H_0 : \beta_3 = 0 \quad \text{vs} \quad H_1 : \beta_3 > 0.$$

- Using the data in `CEOSAL1.RAW` the following equation was obtained by OLS:

  $$\log(\text{salary}) = 4.32 + 0.280 \log(\text{sales}) + 0.0174\text{roe} + 0.00024\text{ros}$$
  $$(0.32) \quad (0.035) \quad (0.0041) \quad (0.00054)$$
  $$n = 209 \quad R^2 = 0.283$$

  By what percentage is salary predicted to increase if `ros` increases by 50 points? Does `ros` have a practically large effect on salary?

  Solution: The proportionate effect on salary is $0.00024 \times 50 = 0.012$. To obtain the percentage effect we multiply this by 100: 1.2

- Test the null hypothesis that `ros` has no effect on salary against the alternative that `ros` has a positive effect. Carry out the test at the 10% significance level.

Solution: The 10% critical value for a one-tailed test using $df = \infty$ is obtained from T-test table as 1.282. The t statistic on `ros` is $0.00024/0.00054 \approx 0.44$ which is well below the critical value. Therefore, we fail to reject H_0 at the 10

- Would you include `ros` in a final model explaining CEO compensation in terms of firm performance? Explain.

  Solution: Based on this sample, the estimated `ros` coefficient appears to be different from zero only because of sampling variation. On the other hand, including `ros` may not be causing any harm; it depends on how correlated it is with the other independent variables (although these are very significant even with `ros` in the equation).

2.

The variable `rdintens` is expenditures on research and development (RD) as a percentage of sales. Sales are measured in millions of dollars. The variable `profmarg` is profits as a percentage of sales.

Using the data in `RDCHEM.RAW` for 32 firms in the chemical industry the following equation is estimated:

$$\text{rdintens} = 0.472 + 0.321\log(\text{sales}) + 0.050\text{profmarg}$$
$$(1.369) \quad (0.216) \quad (0.046)$$
$$n = 32 \quad R^2 = 0.099$$

- Interpret the coefficient on log(sales). In particular, if sales increases by 10% what is the estimated percentage point change in rdintens? Is this an economically large effect?

  Solution: Holding `profmarg` fixed:

  $$\Delta\text{rdintens} = 0.321\Delta\log(\text{sales}) = \frac{0.321}{100}[100\cdot\Delta\log(\text{sales})] \approx 0.00321(\%\Delta\text{sales}).$$

  Therefore, if $\Delta\log(\text{sales}) = 10$, $\Delta\text{rdintens} \approx 0.032$ or only about 3/100 of a percentage point. For such a large percentage increase in sales, this seems like a practically small effect.

- Test the hypothesis that R&D intensity does not change with sales against the alternative that it does increase with sales. Do the test at the 5% and 10% levels.

Solution:
$$H_0 : \beta_1 = 0 \quad \text{vs} \quad H_1 : \beta_1 > 0.$$
Where β_1 is the population slope on log(sales). The t statistic is $0.321/0.216 \approx 1.486$. The 5% critical value for a one-tailed test with $df = 32 - 3 = 29$ is obtained from the Table as 1.699; so we cannot reject H_0 at the 5% level. But the 10% critical value is 1.311; since the t statistic is above this value, we reject H_0 in favor of H_1 at the 10% level.

- Does `profmarg` have a statistically significant effect on `rdintens`?

  Solution: Not really. Its t statistic is only 1.087 which is well below even the 10% critical value for a one-tailed test.

3.

Are rent rates influenced by the student population in a college town? Let `rent` be the average monthly rent paid on rental units in a college town in the United States. Let `pop` denote the total city population, `avginc` the average city income, and `pctstu` the student population as a percentage of the total population. One model to test for a relationship is

$$\log(\text{rent}) = \beta_0 + \beta_1 \log(\text{pop}) + \beta_2 \log(\text{avginc}) + \beta_3 \text{pctstu} + u$$

- State the null hypothesis that size of the student body relative to the population has no ceteris paribus effect on monthly rents. State the alternative that there is an effect.

  Solution:
  $$H_0 : \beta_3 = 0 \quad \text{vs} \quad H_1 : \beta_3 \neq 0.$$

- What signs do you expect for β_1 and β_2?

  Solution: Other things equal, a larger population increases the demand for rental housing, which should increase rents. The demand for overall housing is higher when average income is higher, pushing up the cost of housing including rental rates.

- The equation estimated using 1990 data from `RENTAL.RAW` for 64 college towns is:

  $$\log(\text{rent}) = 0.043 + 0.066 \log(\text{pop}) + 0.507 \log(\text{avginc}) + 0.0056\text{pctstu}$$
  $$(0.844) \quad (0.039) \quad (0.081) \quad (0.0017)$$
  $$n = 64 \quad R^2 = 0.458$$

  What is wrong with the statement: "A 10% increase in population is associated with about a 6.6% increase in rent"?

Solution: The coefficient on log(pop) is an elasticity. A correct statement is that "a 10% increase in population increases rent by $0.066 \times 10 = 0.66\%$."

- Test the hypothesis stated in part (a.) at the 1% level.

Solution: With $df = 64 - 4 = 60$, the 1% critical value for a two-tailed test is 2.660. The t statistic is about 3.29, which is well above the critical value. So β_3 is statistically different from zero at the 1% level.