Financial Markets

Dr. Andrea Rigamonti

andrea.rigamonti@econ.muni.cz

Lecture 10

NOT REQUIRED FOR THE EXAM

Content:

- Relaxing the CAPM assumptions
- Arbitrage pricing theory
- Multifactor models

Relaxing the CAPM assumptions

The standard CAPM has strict and unrealistic assumptions:

- All investors have homogeneous expectations (they expect the same probability distribution of returns).
- All investors want to invest in an optimal portfolio based on Markowitz's mean-variance framework (lowest volatility for a given expected return).
- All investors can lend and borrow any amount of money at the risk-free rate.
- All investors have the same one-period horizon.
- All assets are infinitely divisible.
- There are no taxes and transaction costs.

Relaxing the CAPM assumptions

- There is no inflation or any change in interest rates, or inflation is fully anticipated.
- Capital markets are efficient (they are in equilibrium).

Alternative versions of the CAPM have been developed by relaxing some assumptions.

No alternative model is required if we disallow short selling, as it can be shown that the model does not change.

Relaxing the CAPM assumptions

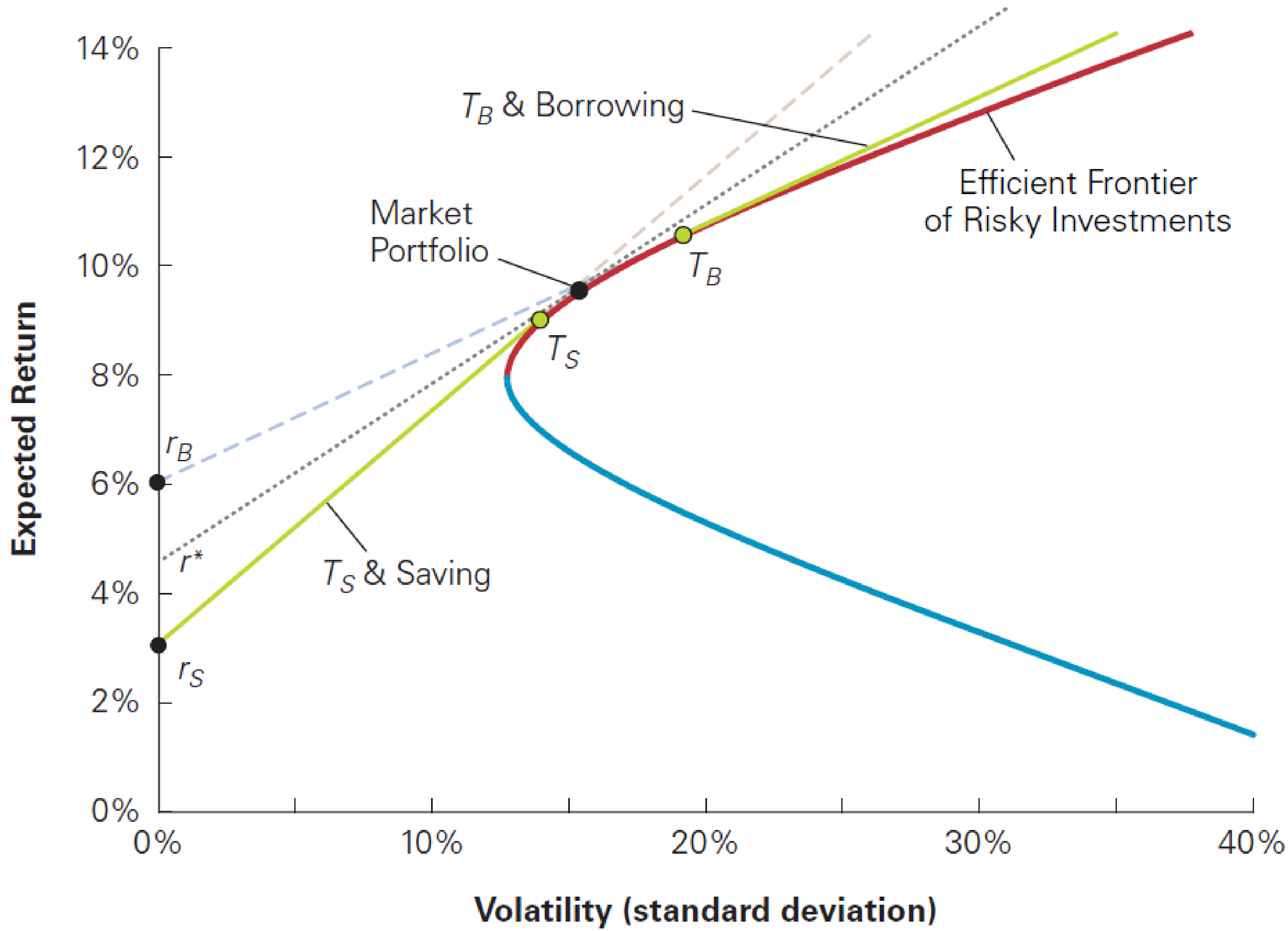
Different borrowing and lending rates

- Assuming an investor can lend at the risk-free rate is realistic (can buy bonds that pay a saving rate R_s)
- But borrowing at the risk-free rate is generally not possible: investors typically pay a higher rate R_B
- Different rates imply different tangent portfolios: T_S for the saving rate and T_B for the borrowing rate
- The market portfolio is no longer the unique efficient portfolio of risky securities
- The SML still holds for some rate R^* between R_s and R_B:

$$E[R_i] = R^* + \beta_i(E[R_m] - R^*)$$

Relaxing the CAPM assumptions

Different investors choose different portfolios on the line between T_S and T_B.



Relaxing the CAPM assumptions

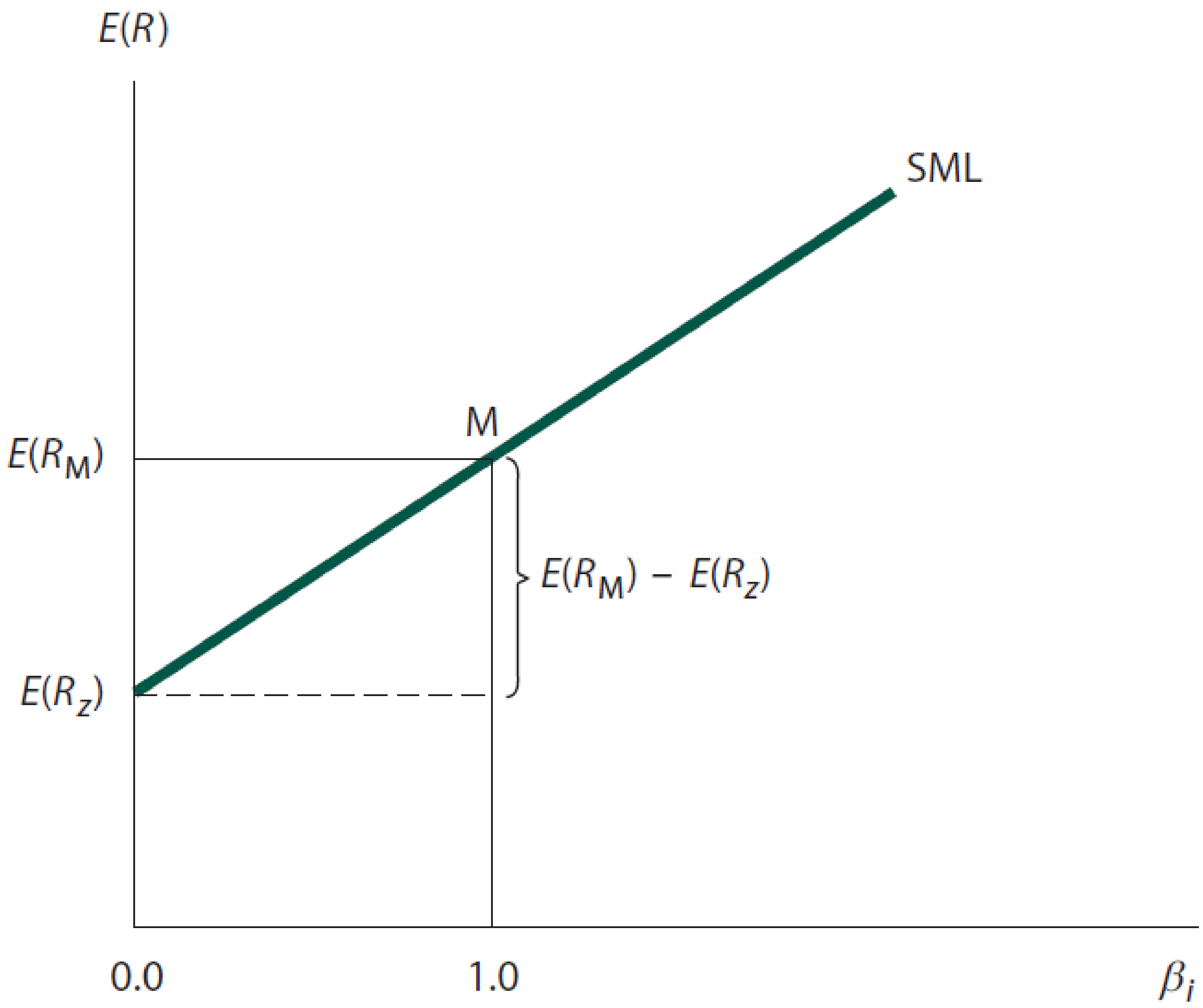
Zero-Beta CAPM

- In the standard CAPM the return of zero-beta assets is the risk-free rate, but this was rejected empirically
- Black (1972) drops the assumption of unlimited lending and borrowing at the risk-free rate, and considers portfolios uncorrelated with the markets (i.e., with $\beta = 0$)
- The return of these R_Z portfolios replaces R_f in the SML:

$$E[R_i] = E[R_Z] + \beta_i(E[R_m] - E[R_Z])$$

- Empirical support for this model is mixed

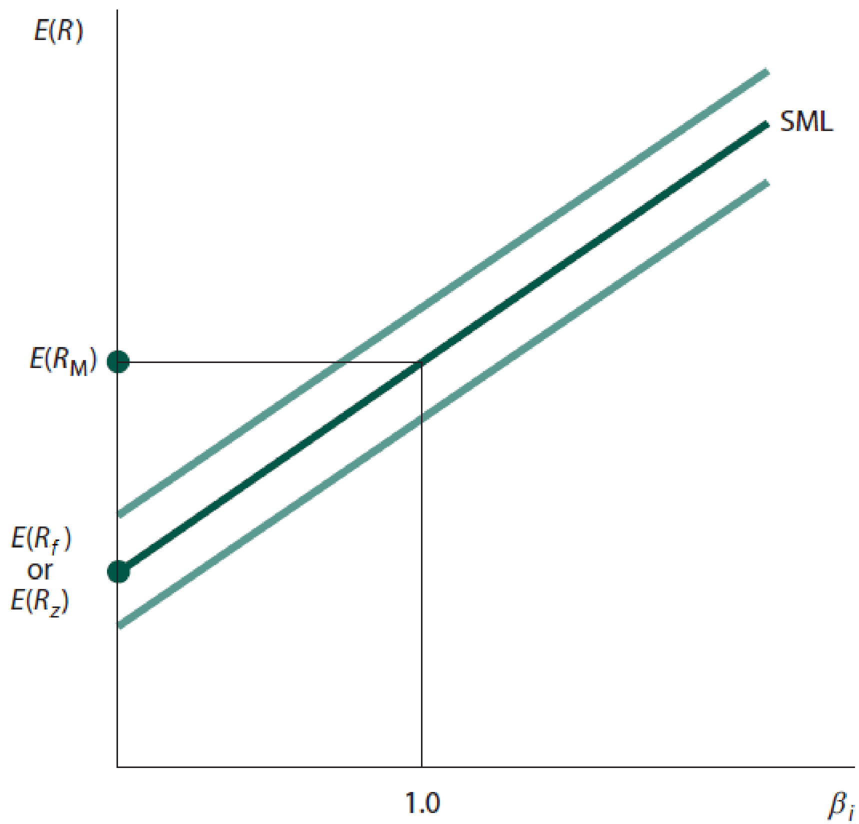
Relaxing the CAPM assumptions



Relaxing the CAPM assumptions

CAPM with transaction costs

- Investors will not correct small mispricings when the costs of buying and selling nullify the potential gains.
- The SML is a band rather than a line, and the greater the transaction costs, the wider the band becomes



Relaxing the CAPM assumptions

CAPM with heterogeneous expectations, investment horizons, and taxes

- Different expectations about risk and return, and different investment horizons imply different CML and/or SML.
- The composite graph would be a band instead of a line.
- Differences in taxation also have similar effects
- Taxation likely impacts on CML and SML but the exact effect is difficult to determine

Relaxing the CAPM assumptions

Final considerations:

- Dropping single assumptions does not significantly change the CAPM model
- The empirical support for CAPM is however very weak
- CAPM is still studied for its theoretical importance
- For practical purposes, more elaborate asset pricing models need to be used

Arbitrage pricing theory

To overcome the shortcomings of CAPM, Ross (1976,1977) developed the **Arbitrage Pricing Theory (APT)**.

APT rests on three assumptions:

- Capital markets are perfectly competitive.
- Investors always prefer more wealth for a given risk.
- The stochastic process generating asset returns can be expressed as a linear function of a set of K risk factors, and all unsystematic risk is diversified away.

Arbitrage pricing theory

The model relies on the **law of one price**: equivalent securities must have the same price.

Arbitrage opportunities are quickly exploited, bringing the markets into equilibrium.

As in CAPM, homogeneous expectations are assumed. But, contrary to CAPM, it is NOT assumed that:

- Investors have quadratic (i.e., mean-variance) utility functions
- Returns of securities are normally distributed
- There is a market portfolio that contains all the assets and is mean-variance efficient

Arbitrage pricing theory

Mathematically, APT requires that the return of stock i is linearly related to a set of k **factors**:

$$R_i = \alpha_i + b_{i1} f_1 + b_{i2} f_2 + \cdots + b_{ik} f_k + \varepsilon_i$$

- α_i is the expected return of i if all factors have value 0
- $f_1, \dots, f_k$ are a set of k factors with zero mean
- $b_1, \dots, b_k$ (called "loadings") specify the sensitivity of the stock return to changes in the corresponding factor
- ε_i is a random error term with mean 0 which indicates idiosyncratic shocks affecting the return of i. This risk can be diversified away.

Arbitrage pricing theory

The factors $f_1, \dots, f_k$ are assumed to impact the return of all the assets.

Factors are “variables that proxy for a common source of risk” on the market (DeMiguel et al., 2018)

The loadings $b_1, \dots, b_k$, on the contrary, are specific to each asset i: each asset has different sensitivity to each factor, which is why they have different returns.

APT does not say what these factors are. APT is therefore difficult to be tested and used in its general form.

Multifactor models

In practice some specific multifactor model is used.

The most important is the **Fama-French three-factor model** (Fama & French, 1993):

$$R_i = R_f + b_{i1}\left(R_m - R_f\right) + b_{i2}SMB + b_{i3}HML$$

- $\boldsymbol{SMB}$ **(Small Minus Big)** is the return difference between a portfolio of small cap and one of large cap stocks
- $\boldsymbol{HML}$ **(High Minus Low)** is the return difference between a portfolio of high Book-to-market (known as "growth" stocks) and one of low Book-to-market stocks (known as "value" stocks)

Multifactor models

Notice that the first factor is also in the CAPM.

CAPM is also a factor model: one with only one factor.

The Fama-French three-factor model is however empirically based. It does not require all the restrictive theoretical assumptions required by CAPM.

A variation is the **Carhart four-factor model** (Carhart, 1997):

$$R_i = R_f + b_{i1}\left(R_m - R_f\right) + b_{i2}SMB + b_{i3}HML + b_{i4}MOM$$

- $\boldsymbol{MOM}$ **(Momentum)** is the return difference between a portfolio of best performing and one of worst performing stocks.

Multifactor models

In 2015 Fama and French modified their model and proposed the **Fama-French five-factor model**:

$$R_i = R_f + b_{i1}(R_m - R_f) + b_{i2}SMB + b_{i3}HML + b_{i4}RMW + b_{i5}CMA$$

- $\boldsymbol{RMW}$ **(Robust Minus Weak)** is the return difference between a portfolio of stocks from high operating profitability ("robust") companies and one of stocks from low operating profitability ("weak") companies.

- **CMA (Conservative Minus Aggressive)** is the return difference between a portfolio of stocks from high companies that invest conservatively and one of stocks from companies that invest aggressively.