Financial Management

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Lecture 7

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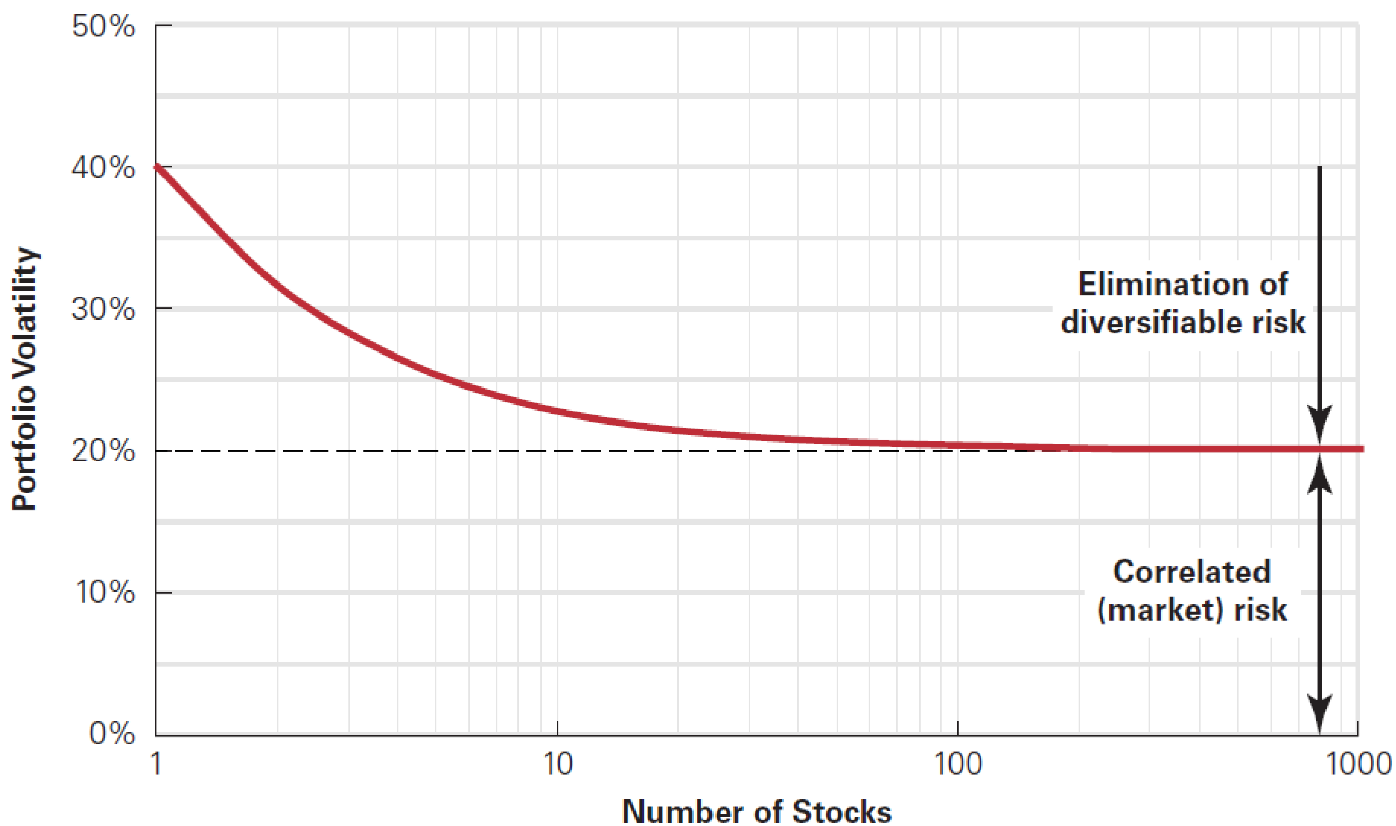
The CAPM model

When investing we face two types of risk:

- **common (or systematic) risk** related to the entire market;
- **independent (or specific or idiosyncratic) risk** related to single assets. This can be removed through **diversification**.



The CAPM model

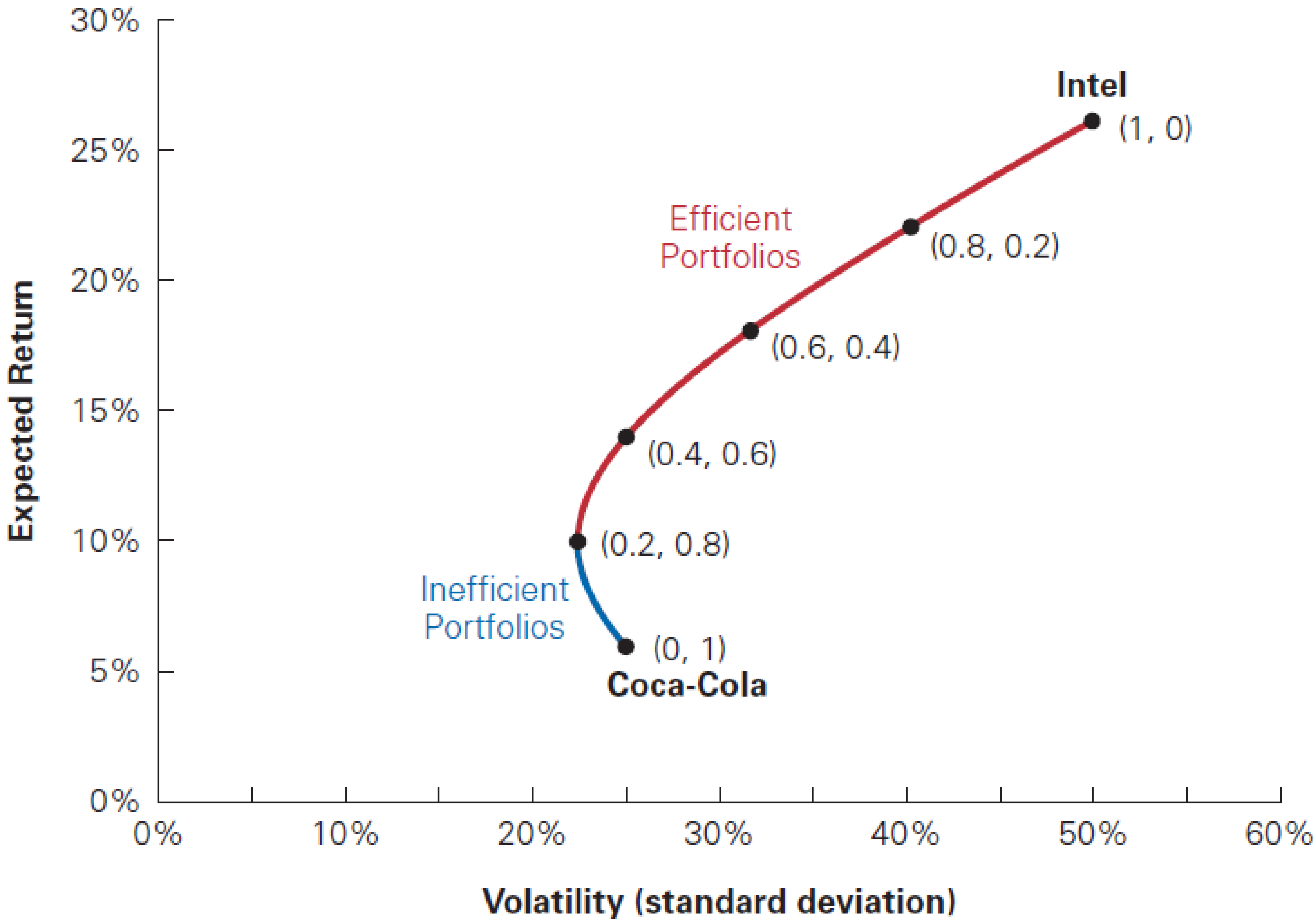
Idiosyncratic risk can be reduced with larger portfolios without reducing expected return because the deviations in the return of one asset can be compensated by the return of other assets.

We say that a portfolio is an **efficient portfolio** when it is NOT possible to find another portfolio that has either:

- a higher expected return without increasing the volatility;
- or a lower volatility without lowering the expected return.

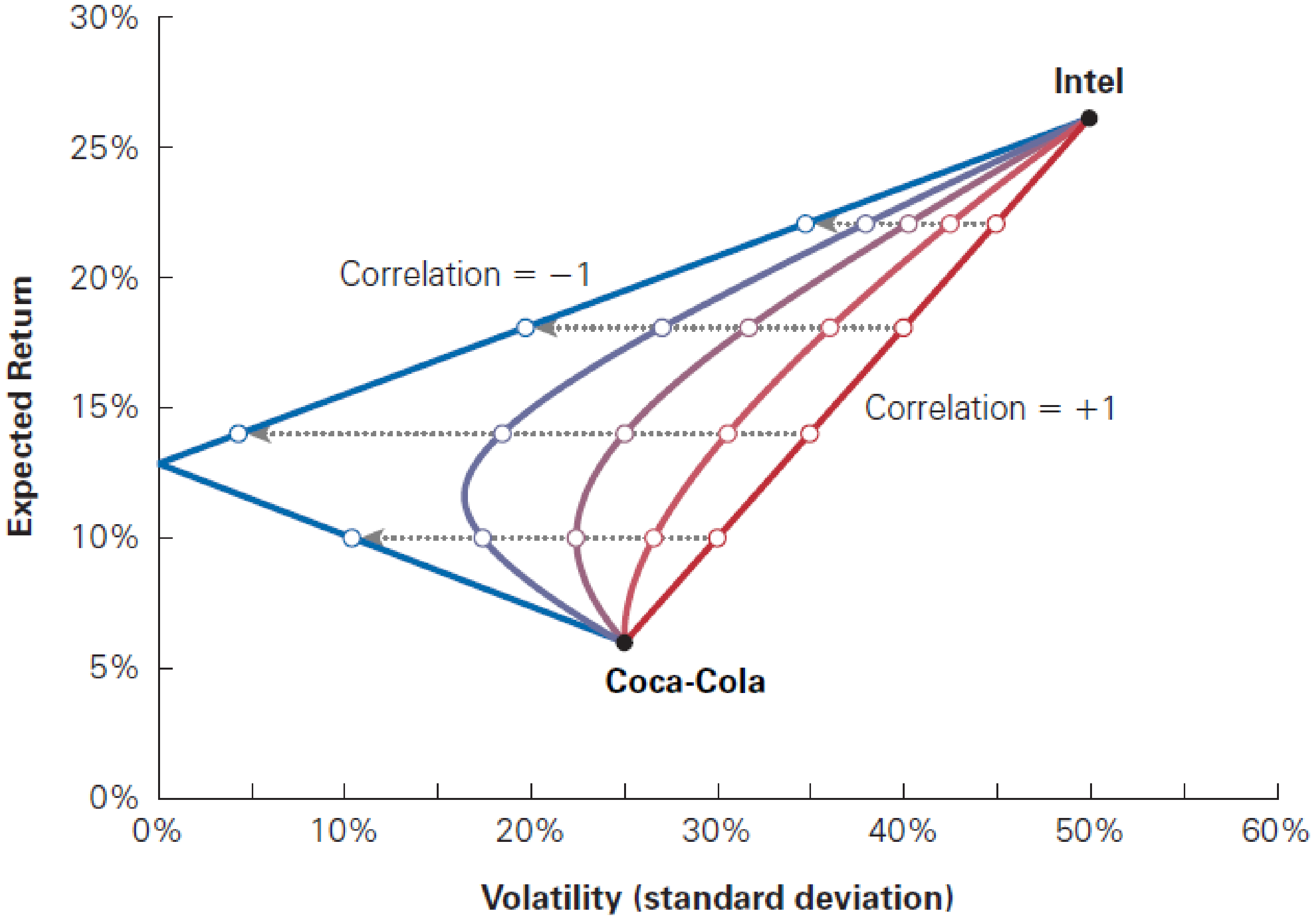
Let us consider a simple example with two stocks.

The CAPM model



The CAPM model

The lower the correlation between the assets, the lower the volatility that we can obtain for our portfolio given a certain return (or the higher the return given a certain volatility).



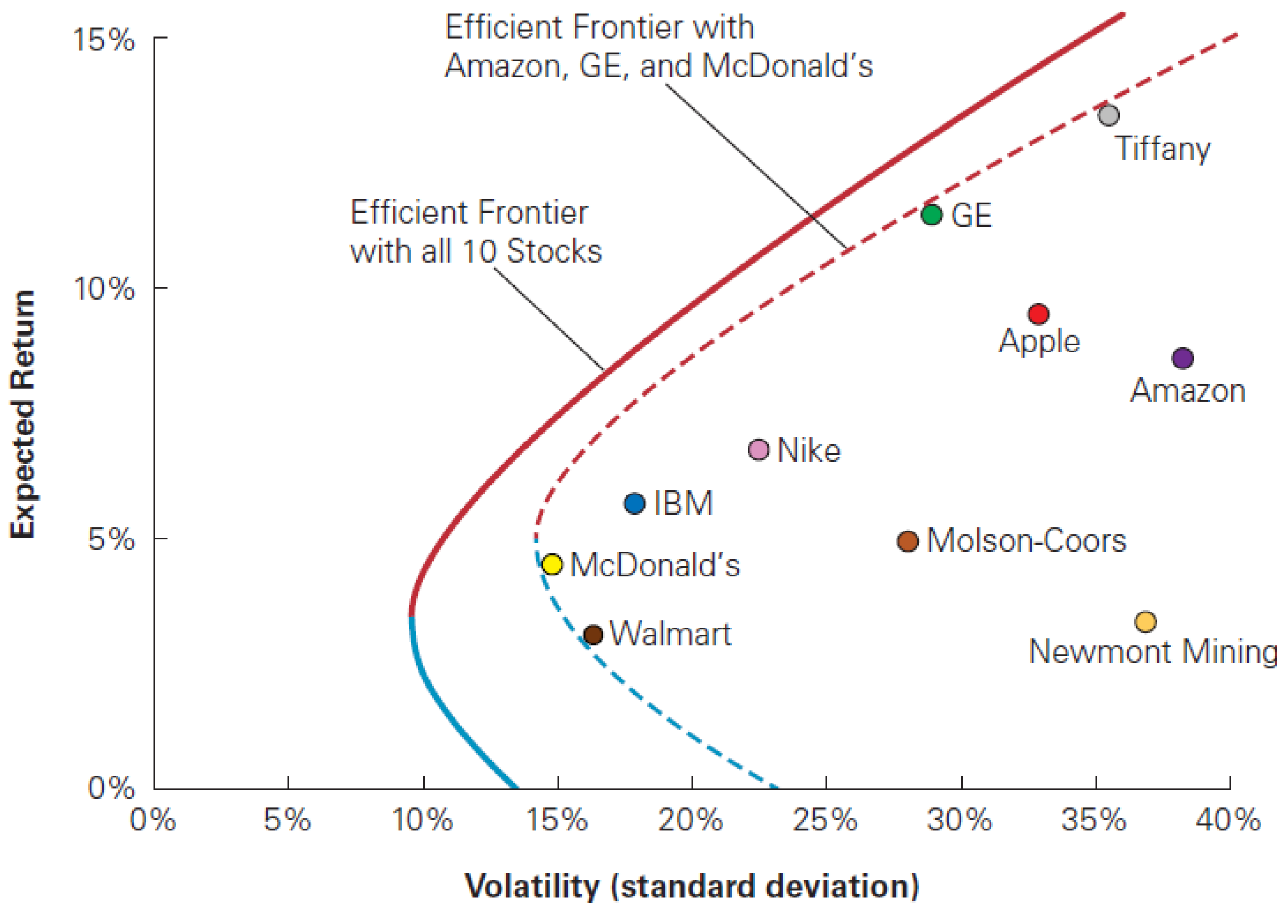
The CAPM model

- Perfectly positively correlated stocks (red line): the volatility of the portfolio is equal to the weighted average volatility of the two stocks: there is no diversification.
- Perfectly negatively correlated stocks (blue line): all risk is diversified and it is possible to hold a riskless portfolio
- In intermediate cases, the lower the correlation, the more risk can be reduced (and the more the curve bends to the left)

In practice, assets tend to be positively correlated, or at most slightly negatively correlated. However, even combining positively correlated assets allows for efficiency gains if the correlation is less than 1.

The CAPM model

Efficient frontier: the set of optimal portfolios that offer the highest expected return for a given risk (or the lowest risk for a given expected return). Efficiency tends to increase with the number of assets:



The CAPM model

Law of one price: equivalent securities must have the same price. This requires efficient markets:

- no arbitrage opportunities (or at least not large enough to compensate for transaction costs)
- operations by individual investors cannot affect the market prices

Under the above conditions we have that:

- idiosyncratic risk can be diversified away
- systematic risk can be eliminated only by sacrificing expected returns.

The CAPM model

Therefore:

- <u>the risk premium for this diversifiable risk is zero</u>: no compensation for holding firm-specific risk.
- <u>the risk premium of a security is determined only by its systematic risk</u>, i.e. by its correlation with the market.

To measure the systematic risk of a stock, we must determine how much of the variability of its return is due to systematic shocks that affect the economy as a whole.

The price changes of a portfolio with <u>only</u> systematic risk can reflect systematic shocks to the economy. A natural one is the market portfolio, but it is difficult to measure it. A good approximation is a large stock index, like the S&P 500.

The CAPM model

We can then measure the systematic risk of a security from the sensitivity of the security’s return to the return of the market portfolio, known as the **beta (β)** of the security.

- The beta of a security is the expected % change in its return given a 1% change in the return of the market portfolio.
- By definition the market has beta equal to 1.
- A stock with beta greater than 1 variates more than the market in response to systemic risk shocks; a stock with beta lower than 1 moves less than the market in response to systemic risk.

The CAPM model

The **Market Risk Premium** is the risk premium investors earn by holding market risk, and is defined as the difference between the market portfolio's expected return and the **risk-free interest rate**:

$$Market\ Risk\ Premium = E[R_{Mkt}] - R_f$$

The risk-free interest rate is the rate paid by a hypothetical asset that bears no risk. In practice, the interest rate of short-term government bonds from financially solid countries is typically used.

The market has β=1, so an investment with, e.g., β=2 should require a risk premium twice the market risk premium.

The CAPM model

From this reasoning we obtain the **Capital Asset Pricing Model (CAPM)**, which states that the expect return of a security is equal to the risk-free rate plus its beta times the market risk premium:

$$E[R_i] = R_f + \beta_i(E[R_m] - R_f)$$

The CAPM model

Instead of diversifying risky investments, we can also reduce risk by keeping some money in a **risk-free investment**.

Or we can increase expected return by borrowing money to invest in the stock market.

Let's put a fraction *x* of our money in an arbitrary risky portfolio with returns R_p, while leaving the remaining fraction (1 - *x*) in risk-free Treasury bills with a yield of R_f .

The expected return of this portfolio, whose returns we indicate with R_{xp}, is:

$$E[R_{xp}] = (1-x)R_f \ + \ xE[\,R_p] = R_f + x(E[R_p] - R_f)$$

The CAPM model

The volatility of the portfolio is:

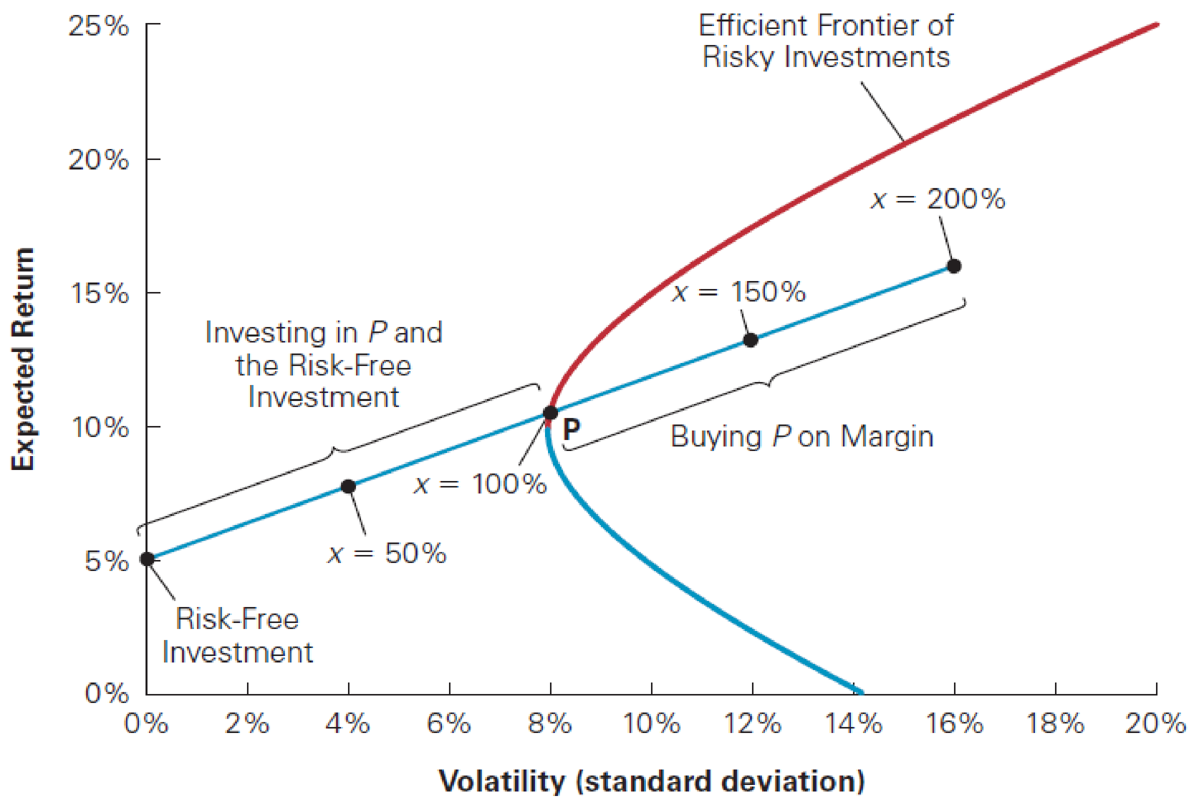
$$SD\left(R_{xp}\right) =$$

$$\sqrt{(1-x)^2 Var\left(r_f\right) + x^2 Var\left(R_p\right) + 2(1-x)xCov\left(r_f\,, R_p\right)} =$$

$$\sqrt{0 + x^2 Var\left(R_p\right) + 0} = \sqrt{x^2 Var\left(R_p\right)} = xSD\left(R_p\right)$$

The blue line in the next figure illustrates combinations of volatility and expected return for different choices of *x*.

The CAPM model



As the fraction *x* invested in *P* increases, both risk and risk premium increase proportionally. Hence the line is straight from the risk-free investment through *P*. If we increase *x* beyond 100 we are short selling the risk-free investment, so we must pay the risk-free return (we borrow money at R_f).

The CAPM model

Borrowing money to invest in stocks is referred to as **buying stocks on margin** or using leverage. The portfolio obtained in such way is known as a levered portfolio.

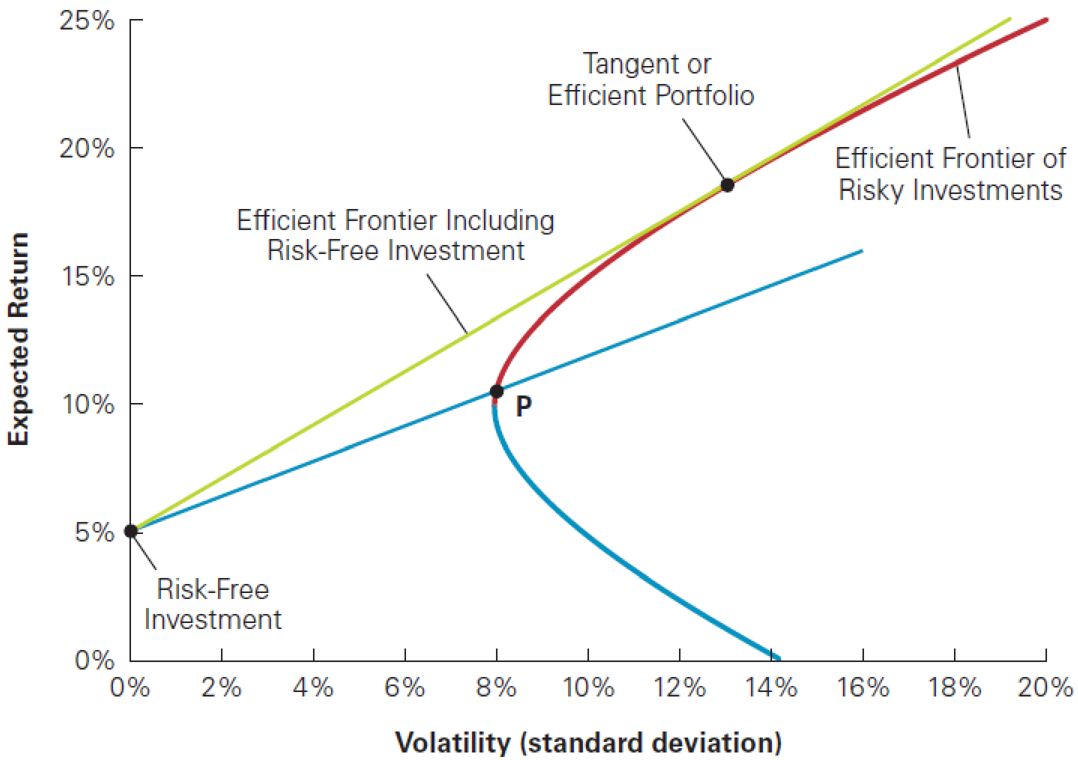
By combining the risk-free asset with a portfolio higher on the efficient frontier than *P*, we get a line that is steeper than the line through *P*. If the line is steeper, then for any level of volatility we will earn a higher expected return.

The slope of the line through a given portfolio *P* is often referred to as the **Sharpe ratio** of the portfolio:

$$SR = \frac{Portfolio\ excess\ return}{Portfolio\ excess\ return\ volatility} = \frac{E[R_p - R_f]}{SD(R_p - R_f)}$$

The CAPM model

The optimal portfolio to combine with the risk-free asset is the one with the highest Sharpe ratio, where the line with the risk-free investment is tangent to the efficient frontier of risky investments. The portfolio that generates this tangent line is known as the **tangent portfolio**, and it provides the biggest reward per unit of volatility of any portfolio available.



The CAPM model

- The tangent portfolio is efficient. Once we include the risk-free investment, all efficient portfolios are combinations of the risk-free investment and the tangent portfolio.
- Every investor should invest in the tangent portfolio independently of his preferences, which will determine only how much to invest in the tangent portfolio versus the risk-free investment.
- This allows the investor to earn the highest possible expected return for any level of volatility he is willing to bear.

The CAPM model

The CAPM model rests on three assumptions:

1. Investors can buy and sell all securities at competitive market prices (without incurring taxes or transactions costs) and can borrow and lend at the risk-free rate.
2. Investors hold only efficient portfolios of traded securities, that is, portfolios that yield the maximum expected return for a given level of volatility.
3. Investors have homogeneous expectations regarding the volatilities, correlations, and expected returns of securities.

The CAPM model

If investors have **homogeneous expectations**, they will all identify the same portfolio as having the highest Sharpe ratio in the economy, and will all demand the same efficient portfolio of risky securities (that is, the tangent portfolio) adjusting only their investment in risk-free securities.

If every investor is holding the tangent portfolio, then the combined portfolio of risky securities of all investors must also equal the tangent portfolio. And because every security is owned by someone, the sum of all investors' portfolios must equal the portfolio of all risky securities available in the market, that is, the market portfolio.

The CAPM model

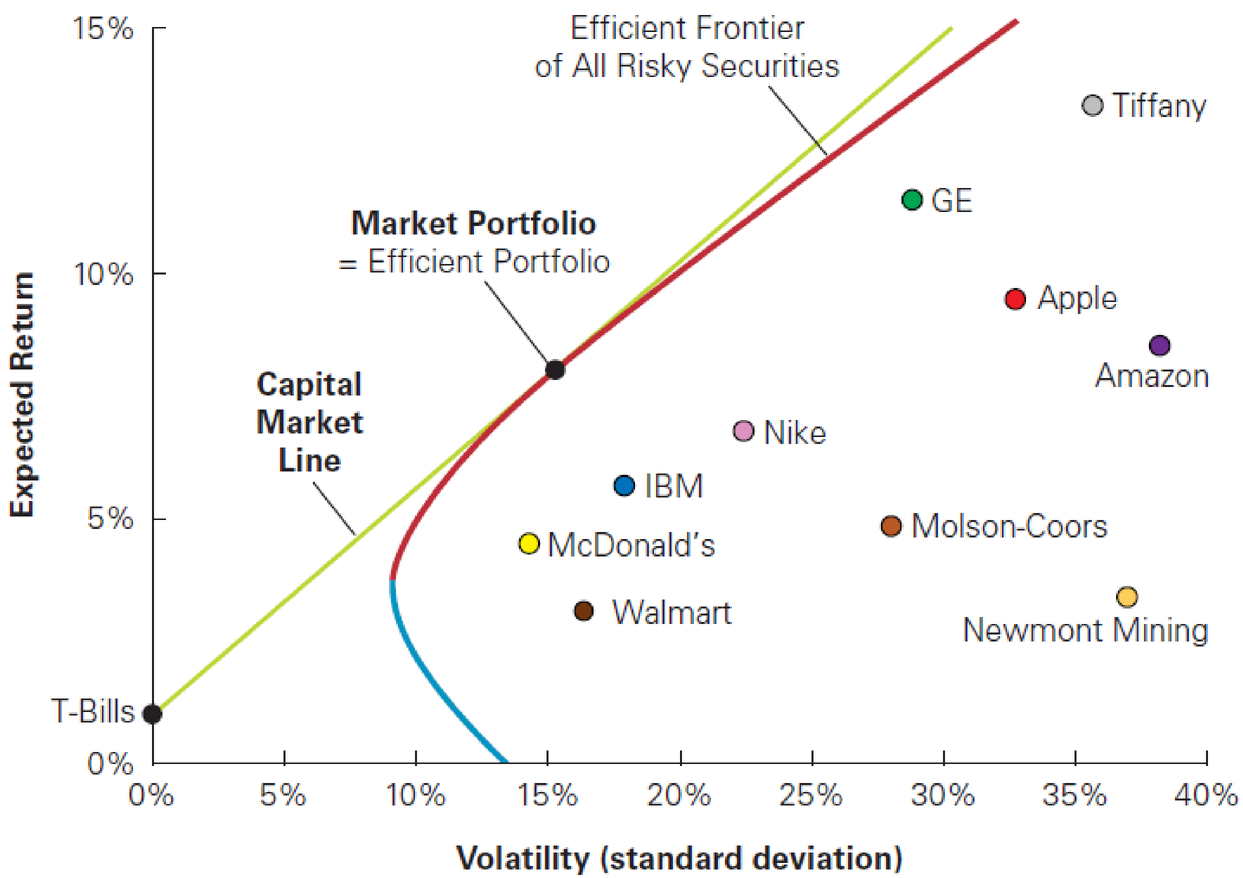
Therefore, the **efficient, tangent portfolio** of risky securities (the portfolio that all investors hold) **must equal the market portfolio**.

If a security is not part of the efficient portfolio, no investor wants to own it, and its demand does not equal its supply.

Its price falls, causing its expected return to rise until it becomes an attractive investment: prices adjust so that the efficient portfolio and the market portfolio coincide.

The CAPM model

When the tangent line goes through the market portfolio, it is called **capital market line (CML)**. According to CAPM, all investors should choose a portfolio on the CML, by holding a combination of the risk-free asset and the market portfolio.



The CAPM model

As the beta of a security i expresses the fraction of its volatility that is common with the market, we can write:

$$\beta_i = \frac{SD(R_i) * Corr(R_i, R_{Mkt})}{SD(R_{Mkt})}$$

and replacing $Corr(R_i, R_{Mkt})$ with $\frac{Cov(R_i, R_{Mkt})}{SD(R_i)SD(R_{Mkt})}$ we get:

$$\beta_i = \frac{Cov(R_i, R_{Mkt})}{Var(R_{Mkt})}$$

To determine the appropriate risk premium for any investment, we must rescale the market risk premium by the amount of market risk present in the security's returns, measured by its beta with the market.

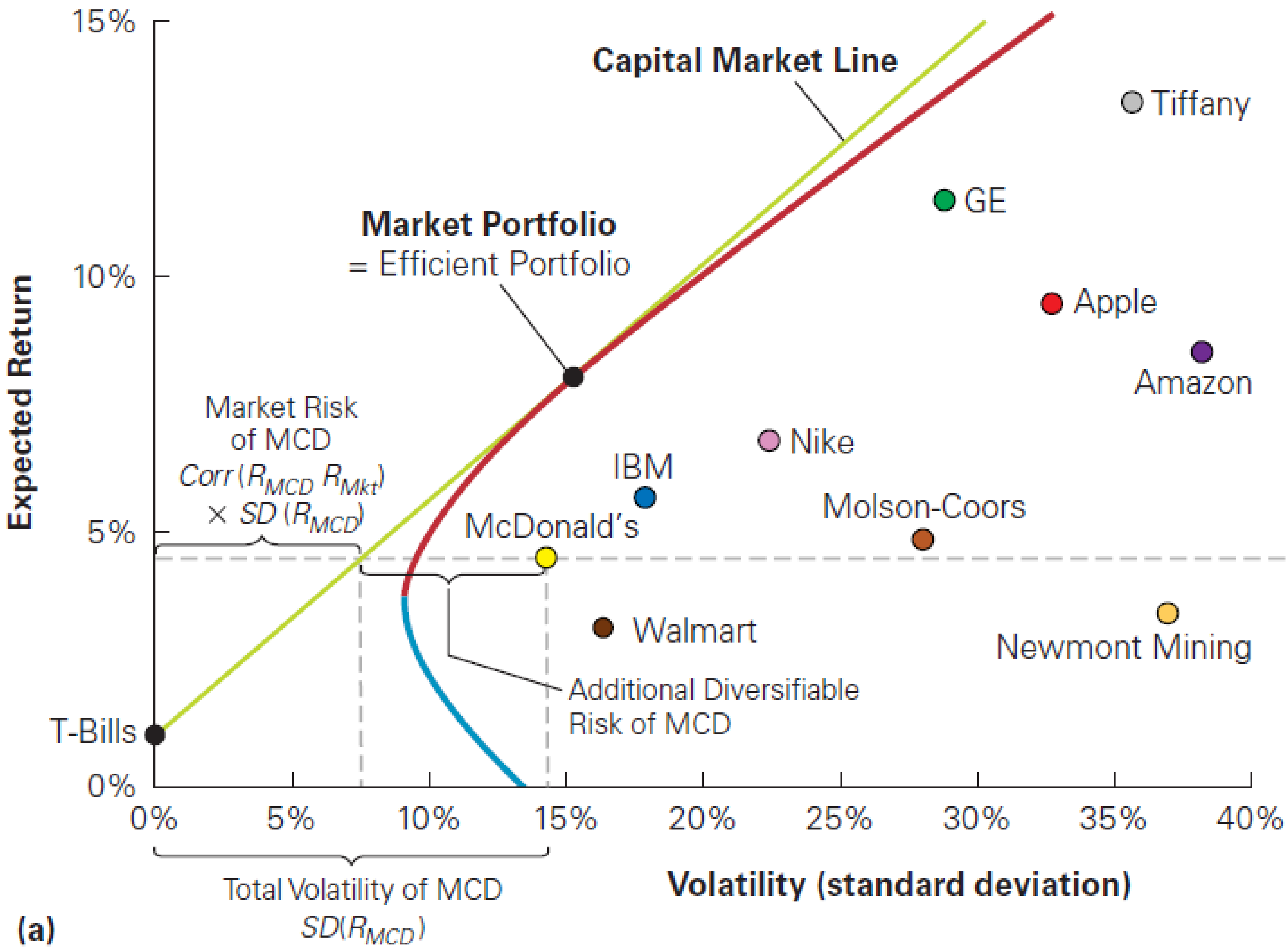
The CAPM model

CAPM implies that there is a linear relationship between a stock's beta and its expected return.

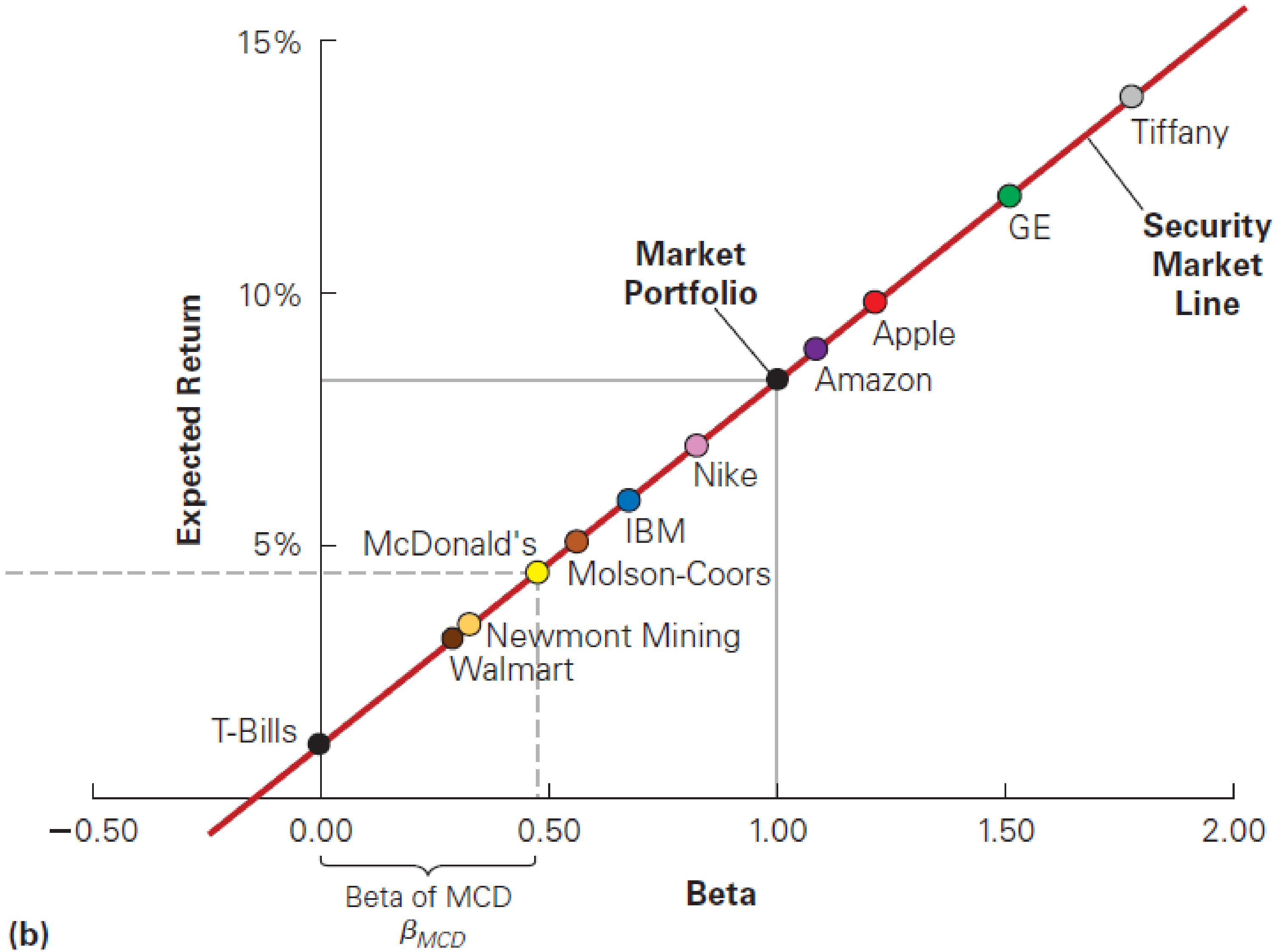
The **security market line (SML)** is the line along which all individual securities should lie when plotted according to their expected return and beta.

- The capital market line shows no clear relationship between an individual stock's volatility and its expected return because total volatility also includes diversifiable risk that receives no compensation.
- The relationship between risk and return for individual securities becomes evident only when we measure market risk rather than total risk.

The CAPM model



The CAPM model



The CAPM model

Because the security market line applies to all tradable investment opportunities, we can also apply it to portfolios.

The beta of a portfolio is simply the weighted average beta of the securities in the portfolio:

$$\beta_p = \sum_i w_i \beta_i$$

Estimation of CAPM parameters

To apply the CAPM we need an estimate of the risk-free rate, of the market risk premium, and of betas.

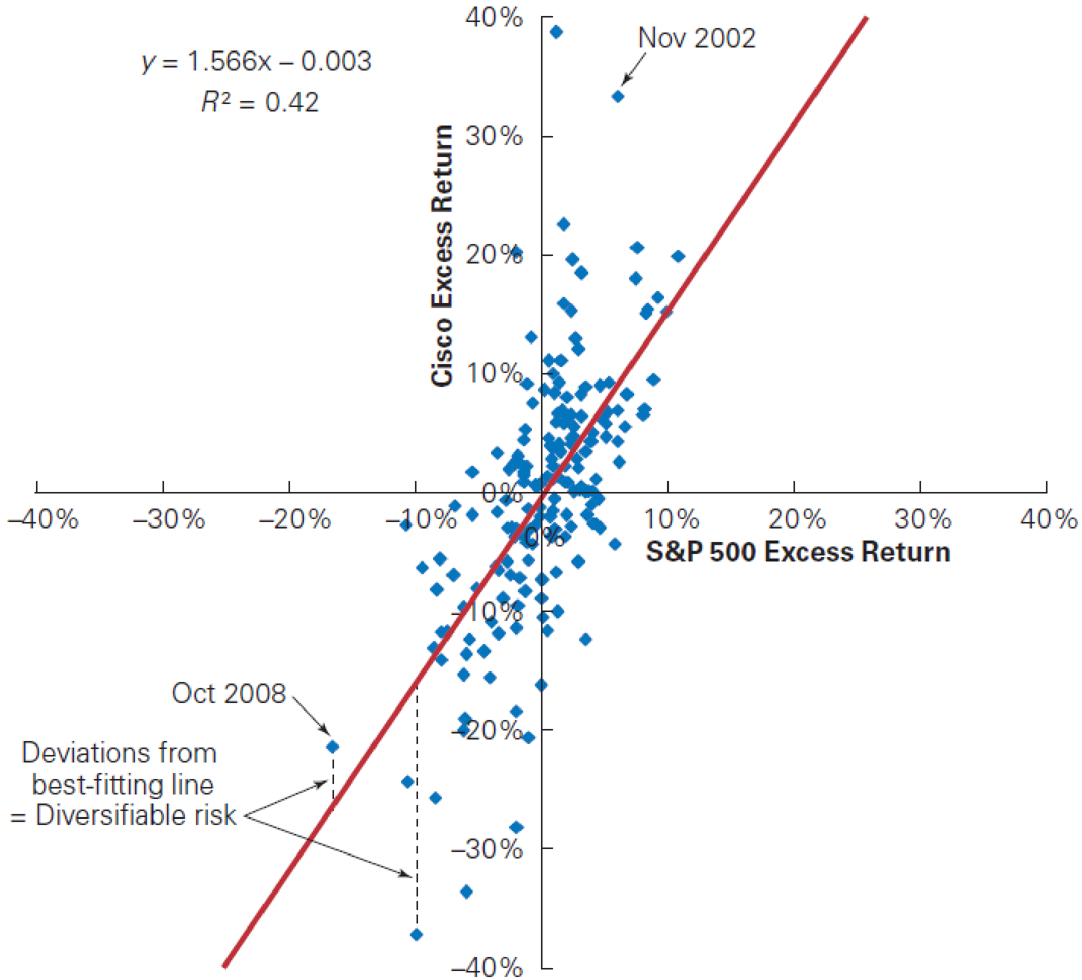
The yields on U.S. Treasury securities are generally used as proxy for the risk-free rate.

The risk premium can be estimated from past data or using some fundamental analysis approach (e.g. assuming a certain expected growth model for the market).

A stock's beta is generally estimated from its past sensitivity to market risk.

Estimation of CAPM parameters

One can exploit the fact that the beta corresponds to the slope of the best-fitting line in the plot of the security's excess returns versus the market excess return.



Estimation of CAPM parameters

To identify the best-fitting line through the set of points we can use the linear regression:

$$\left(R_i - R_f\right) = \alpha_i + \beta_i\left(R_{Mkt} - R_f\right) + \varepsilon_i$$

where ε_i is the error (or residual) term, which on average is zero.

The intercept α_i is called **Jensen's alpha**, and it measures the historical performance of the security relative to the expected return predicted by the security market line.

According to the CAPM, α_i should not be significantly different from zero.

Estimation of CAPM parameters

The linear regression we are considering is a simple linear regression model with a single regressor:

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

The equation of the fitted line is therefore:

$$\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i$$

The OLS estimators are given by:

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$