Test 2

(Solution)

Financial Investments
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December 2024.

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1. A mutual fund reports the following yearly returns (investment in the first calendar year of the year and results in the last calendar day of the corresponding year).

   - Year 1: 4%
   - Year 2: 5%
   - Year 3: -1%
   - Year 4: 0%

   (a) What is the total rate of return from the beginning of year 1 to end of year 4.

$$\begin{aligned} 1 + R_{0-4} &= (1+0.04)(1+0.05)(1-0.01)(1+0.00) \\ &= (1.04)(1.05)(0.99)(1) \\ &= 1.08108 \end{aligned}$$

$$\Longrightarrow R_{0-4} = 1.08108 - 1 = 8.108\%$$

   (b) What is the realized yield of the mutual fund over this 4-year period?

   - Geometric mean:

$$(1+R_Y)^4 = 1.08108 \Leftrightarrow R_Y = \sqrt[4]{1.08} - 1 \approx 1.97\%$$

2. Consider a risk-free rate of 1% and an economy where the feasible future state of the price are $S_T = \{90, 100, 110, 120\}$, each escenario with the following probabilities $\mathrm{P}^{\mathbb{Q}} = \{1/3, 1/3, 1/6, 1/6\}$. What is the fair value of a Put option with maturity T and strike 100?

   - In each feasible future scenario (time T), the payoff of the Put option (P) is:

$$P_T = \begin{cases} \max(100-90, 0) = 10 & \text{with probability } 1/3 \\ \max(100-100, 0) = 0 & \text{with probability } 1/3 \\ \max(100-110, 0) = 0 & \text{with probability } 1/6 \\ \max(100-120, 0) = 0 & \text{with probability } 1/6 \end{cases}$$

   - The expected value of the Put option at time T is given by

$$\langle P_T \rangle = \mathbb{E}(P_T) = 10 \cdot \frac{1}{3} + 0 + 0 + 0 = 3.33$$

- Considering an interest rate of 1% from the inception time up to maturity, the present value (premium) is equal to the discounted expected payoff; i.e.,

$$P_0 = \frac{3.33}{1.01} = 3.30$$

3. Compute the payoff (at maturity) and the total profit for each strategy, according to the value of the underlying asset at maturity, and plot them. In addition, point out the purpose of each strategy.

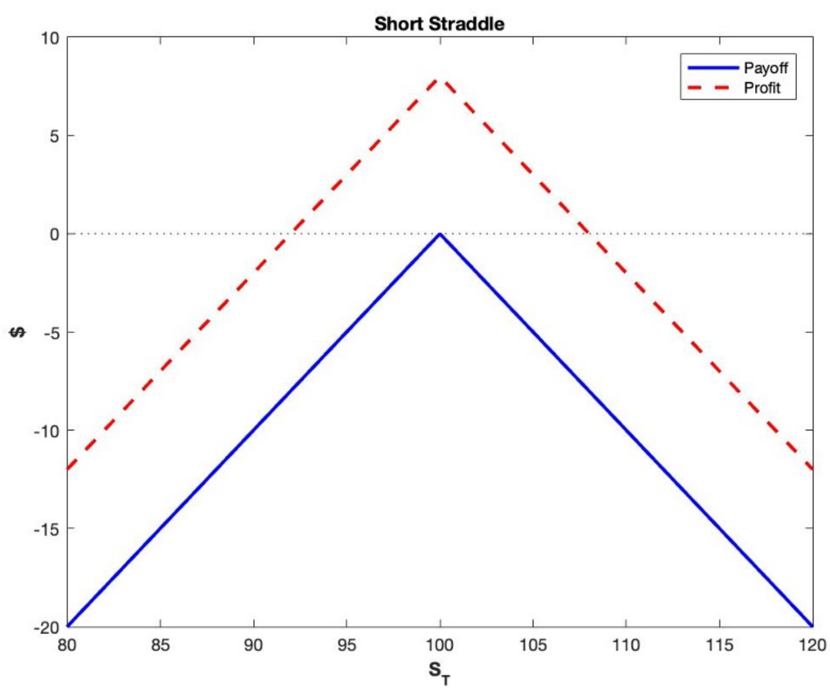
(a) Short Straddle: Selling of 1 European Call (premium equal to 3) and selling of 1 European Put (premium equal to 5). Both options are based on the same underlying, with identical strikes and maturities.

- Payoff of the strategy according the value of the underlying at maturity:

	$S_T < X$	$S_T \geq X$
$-P_T^1(X_1)$	$S_T - X$	0
$-C_T^2(X_2)$	0	$X - S_T$
Total	$S_T - X$	$X - S_T$

- Since the investor takes a short position in both option, he received at inception time the sum of the two premiums: 3+5=8.

- Payoff and profit diagram (using $X = 100$):



- This strategy is designed to take advantage of low volatility scenarios, when the future price is around the strike price.

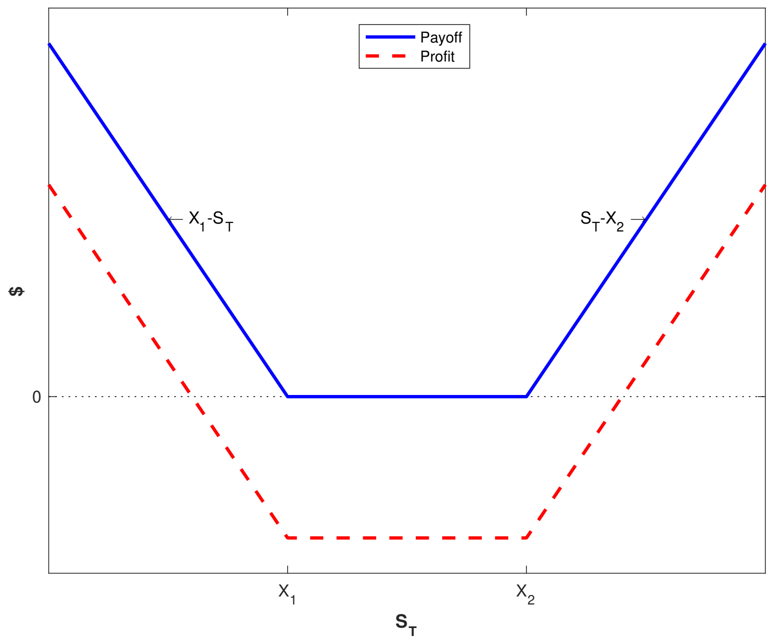
(b) Vertical Combination: Purchasing of a Call with exercise price X_2 and a Put with exercise price X_1, with X_2 greater than X_1.

- Payoff of the strategy according the value of the underlying at maturity:

	$S_T < X_1$	$X_1 \leq S_T \leq X_2$	$S_T \geq X_2$
$P_T^1(X_1)$	$X_1 - S_T$	0	0
$C_T^2(X_2)$	0	0	$S_T - X_2$
Total	$X_1 - S_T$	0	$S_T - X_2$

- Since the investor takes a long position in the two options, he paid at inception time the sum of the two premiums: $C_0 + P_0$.

- Payoff and profit diagram:



- In this strategy, the maximum profit is achieved when expected prices are outside of the interval $[X_1, X_2]$
- This strategy is designed to take advantage of high volatility scenarios.

4. Consider assets A and B. Asset A is featured by a return of 5% per year and risk (measured by annualized standard deviation) of 10%. On the other hand, Asset B has 3% of return and 9% of volatility. The correlation among them is $\rho_{AB} = -0.5$. The risk-free rate of return is 2%.

   (a) Compute the Sharpe ratio for the portolio composed of 60% (of the initial wealth) in asset A and 40% in asset B.

   - $R_A = 5\%$, $R_B = 3\%$, $\sigma_A = 10\%$, $\sigma_B = 9\%$, $\rho_{AB} = -0.5$, $w_A = 0.6$, and $w_B = 0.4$.
   - In order to compute the Sharpe ratio, we need to find both, risk and return of the portfolio.
   - Return:

$$R_P = w_A \cdot R_A + w_B * R_B = 4.2\%$$

   - Risk:

$$\begin{aligned}
\sigma_P^2 &= \text{var}\left(R_p\right) \\
&= \text{var}\left(w_A \cdot R_A + w_B * R_B\right) \\
&= \text{var}\left(w_A R_A\right) + \text{var}\left(w_B R_B\right) + 2\rho_{AB}\sqrt{\text{var}\left(w_A R_A\right) \cdot \text{var}\left(w_B R_B\right)} \\
&= w_A^2 \text{var}\left(R_A\right) + w_B^2 \text{var}\left(R_B\right) + 2\rho_{AB}\sqrt{w_A^2 \text{var}\left(R_A\right) \cdot w_B^2 \text{var}\left(w_B R_B\right)} \\
&= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2\rho_{AB} w_1 w_2 \sigma_A \sigma_B \\
&= 0.002736
\end{aligned}$$

$$\longrightarrow \sigma_P = \sqrt{\sigma_P^2} = 5.23\%$$

- Sharpe ratio:

$$SR_P = \frac{R_P - R_f}{\sigma_P} = 0.42$$

(b) How the results changes if $\rho_{AB} = 0$

- $\rho_{AB} = 0 \rightarrow \sigma_P^2 = 0.004896 \rightarrow \sigma_P \approx 7\% \rightarrow SR_P = 0.31$

(c) How the results changes if $\rho_{AB} = 0.5$

- $\rho_{AB} = 0. \rightarrow \sigma_P^2 = 0.007056 \rightarrow \sigma_P = 8.4\% \rightarrow SR_P = 0.26$