

## Week 1 – Algebraic Expressions and Basic Equations

### Weekly Goals

- Practice simplifying algebraic expressions involving powers, roots, and fractions.
- Understand and apply the rules of exponents and radicals.
- Manipulate and simplify rational expressions.
- Solve linear and basic quadratic equations.

### Solved Examples – With Detailed Steps

**Example 1:** Simplify the expression:

$$(a^3b^4c^5) \div (-a^3c^2)$$

**Step-by-step:**

- Use the rule  $\frac{x^m}{x^n} = x^{m-n}$ .
- Split:  $\frac{a^3}{-a^3} = -1$ ,  $\frac{b^4}{1} = b^4$ ,  $\frac{c^5}{c^2} = c^3$
- Result:  $-b^4c^3$

$$\boxed{-b^4c^3}$$

**Example 2:** Simplify:

$$\frac{1 + \frac{1}{x-1}}{1 - \frac{1}{x+1}}$$

- Numerator:  $\frac{x-1}{x-1} + \frac{1}{x-1} = \frac{x}{x-1}$ , Denominator:  $\frac{x+1}{x+1} - \frac{1}{x+1} = \frac{x}{x+1}$
- Convert division to multiplication and cancel:  $\frac{x}{x-1} \div \frac{x}{x+1} = \frac{x}{x-1} \cdot \frac{x+1}{x} = \frac{x+1}{x-1}$

$$\boxed{\frac{x+1}{x-1}}$$

**Example 3:** Solve:

$$2x^2 - 12x + 36 = x^2 + 2x - 13$$

- Rearranged:  $x^2 - 14x + 49 = 0$
- Factor:  $(x - 7)^2 = 0 \Rightarrow x = 7$

$$\boxed{x = 7}$$

**Example 4:** Simplify:

$$\frac{x^2 - 9}{x^2 + 5x + 6} \cdot \frac{x + 2}{x - 3}$$

- Factor:  $\frac{(x-3)(x+3)}{(x+2)(x+3)} \cdot \frac{x+2}{x-3}$
- Cancel all terms

$$\boxed{1}$$

**Example 5:** Simplify:

$$\frac{1}{\sqrt{x}+1} - \frac{\sqrt{x}}{x-1}$$

- Rationalize first term:  $\frac{1}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} = \frac{\sqrt{x}-1}{x-1}$
- Subtract:  $\frac{\sqrt{x}-1-\sqrt{x}}{x-1} = \frac{-1}{x-1}$

$$\boxed{-\frac{1}{x-1}}$$

**Example 6:** Simplify:

$$\frac{\sqrt[3]{a \cdot \sqrt{b}}}{\sqrt[3]{\sqrt{a} \cdot b^2}}$$

- Convert radicals to powers:

$$\frac{a^{1/3}(b^{1/2})^{1/3}}{(a^{1/2})^{1/3}(b^2)^{1/3}} = \frac{a^{1/3}b^{1/6}}{a^{1/6}b^{2/3}}$$

- Subtract the powers:

$$a^{1/3-1/6}b^{1/6-2/3} = a^{1/6}b^{-1/2}$$

- Back to radicals:

$$\boxed{\frac{\sqrt[6]{a}}{\sqrt{b}}}$$

## Practice Problems (for seminar)

Simplify the expressions

1.  $\left(\frac{3}{1+a} - 1\right) \cdot \left(\frac{3}{2-a} - 1\right)$
2.  $\left(\frac{1}{(a-b)^2} - \frac{1}{(a-b)^2+4ab}\right) \div \left(\frac{ab}{a^2-b^2}\right)$
3.  $\frac{(x^2-9)}{x^2+5x+6} \cdot \frac{x+2}{x-3}$
4.  $(x^2y^{-3}) \cdot (x^{-1}y^4)$
5.  $\left(\frac{x^2-x(x-2)-4}{x^2-4}\right)$
6. Simplify:

$$\frac{\sqrt{x}-2}{x-4}$$

(Hint: factor the denominator and express as a conjugate if needed)

7. Rationalize the denominator:

$$\frac{5}{2-\sqrt{3}}$$

**Solve the equations**

8.  $x^2 + 2x + 1 = 0$

9.  $2x^2 - 2x + 1 = x^2 - 2x$

10.  $(x + 1)^2 = 2x + 5$