Week 2 – Absolute Value and Inequalities

Weekly Goals

- Understand the definition and graph of the absolute value function.
- Solve equations and inequalities involving absolute value.
- Practice solving linear and quadratic inequalities.

Solved Examples – With Detailed Steps

Example 1: Solve the equation:

$$2|x-3|-4=0$$

Steps:

- Isolate the absolute value: $|x-3|=2$
- Solve two cases:

$$x-3=2 \Rightarrow x=5, \quad x-3=-2 \Rightarrow x=1$$

$$\boxed{x=1 \text{ or } x=5}$$

Example 2: Solve the inequality:

$$|2x+1|<5$$

Steps:

- Rewrite as compound inequality: $-5<2x+1<5$
- Subtract 1: $-6<2x<4$
- Divide by 2: $-3<x<2$

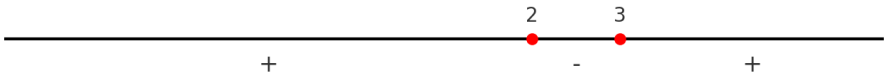
$$\boxed{x \in (-3,\ 2)}$$

Example 3: Solve the quadratic inequality:

$$x^2-5x+6 \leq 0$$

Steps:

- Factor: $(x-2)(x-3) \leq 0$
- Determine sign changes on intervals:
  - both terms are negative so the product is positive for $x<2$
  - $x-2>0$ and $x-3<0$ for $x \in [2,\ 3]$
  - both terms and their product are positive for $x>3$



$$\boxed{x \in [2,\ 3]}$$

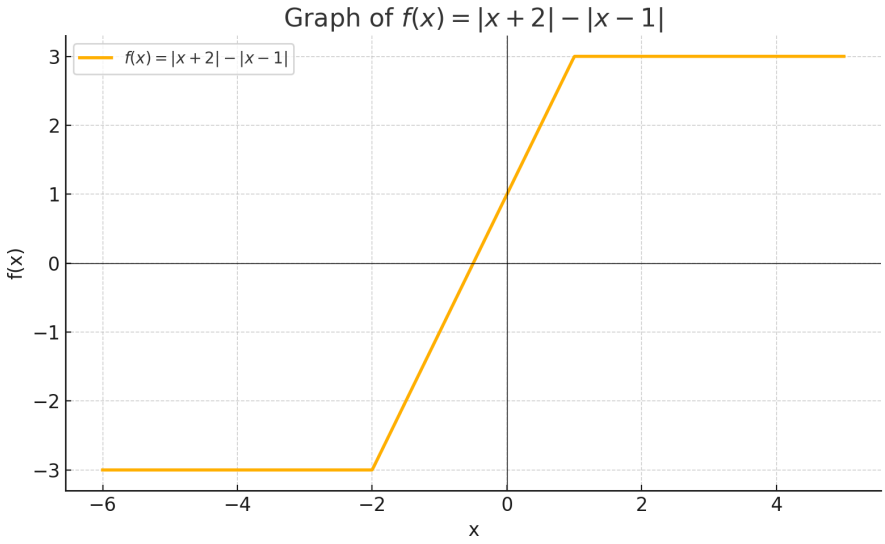
Example 4: Graph the function:

$$f(x) = |x + 2| - |x - 1|$$

Steps:

- Identify critical points: $x = -2,\ x = 1$
- Split into intervals and analyze:
  - $x < -2$: $f(x) = -(x + 2) - (1 - x) = -3$
  - $-2 \leq x < 1$: $f(x) = x + 2 - (1 - x) = 2x + 1$
  - $x \geq 1$: $f(x) = x + 2 - (x - 1) = 3$

We obtain piecewise function: $f(x) = \begin{cases} -3 & x < -2 \\ 2x + 1 & -2 \leq x < 1 \\ 3 & x \geq 1 \end{cases}$



Example 5: Solve the equation:

$$|2x - 3| = |x + 1|$$

Steps:

- Find critical points where expressions inside absolute values are zero:

$$2x - 3 = 0 \Rightarrow x = \frac{3}{2}, \quad x + 1 = 0 \Rightarrow x = -1$$

- Divide the real line into intervals:

  1. **Case 1:** $x < -1$
     $|2x-3| = -(2x-3),\ |x+1| = -(x+1)$
     $-2x+3 = -x-1 \Rightarrow -x = -4 \Rightarrow x = 4 \notin (-\infty, -1)$ discard
  2. **Case 2:** $-1 \leq x < \frac{3}{2}$
     $|2x-3| = -(2x-3),\ |x+1| = x+1$
     $-2x+3 = x+1 \Rightarrow -3x = -2 \Rightarrow x = \frac{2}{3} \in [-1,\ \frac{3}{2})$ accept
  3. **Case 3:** $x \geq \frac{3}{2}$
     $|2x-3| = 2x-3,\ |x+1| = x+1$
     $2x-3 = x+1 \Rightarrow x = 4 \in [\frac{3}{2}, \infty)$ accept

$$\boxed{x = \frac{2}{3},\ x = 4}$$

Practice Problems for Seminar

Absolute Value

1. Solve: $|3x-6| = 9$
2. Solve: $2|x-1|+3 = 9$
3. Solve: $|x-2| = |3x+1|$
4. Graph: $f(x) = |x| - |x-3|$
5. Graph: $f(x) = 2x + |x-2|$

Inequalities

6. Solve: $x^2 - 4x > 5$
7. Solve: $(x+3)(x+1) \leq -1$
8. Solve: $|x+2| \geq 4$
9. Solve: $|x-4| < x$