

Week 9 – Piecewise Functions and Graphs

Weekly Goals

- Analyze and graph piecewise-defined functions.
- Interpret continuity and jump behavior in graphs.
- Review key concepts and typical problem types from previous weeks.

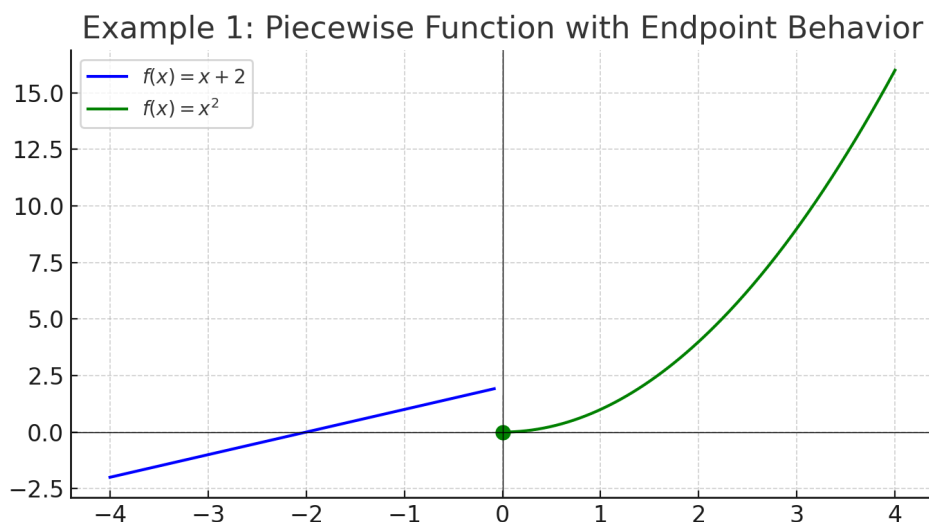
Solved Examples – With Detailed Steps

Example 1: Define and sketch:

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

Steps:

- For $x < 0$: linear segment
- For $x \geq 0$: parabola starting at origin
- Evaluate continuity at $x = 0$: $\lim_{x \rightarrow 0^-} = 2$, $\lim_{x \rightarrow 0^+} = 0 \Rightarrow$ discontinuous

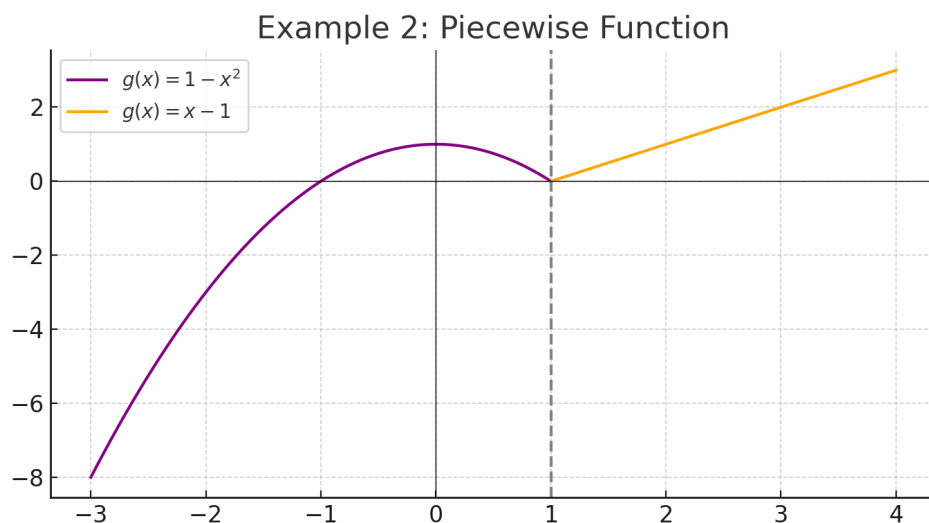


Example 2: Analyze:

$$g(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 1 \\ x - 1 & \text{if } x > 1 \end{cases}$$

Steps:

- Left part: downward parabola
- Right part: linear growth from $x = 1$
- Evaluate limits and value at $x = 1$: continuous

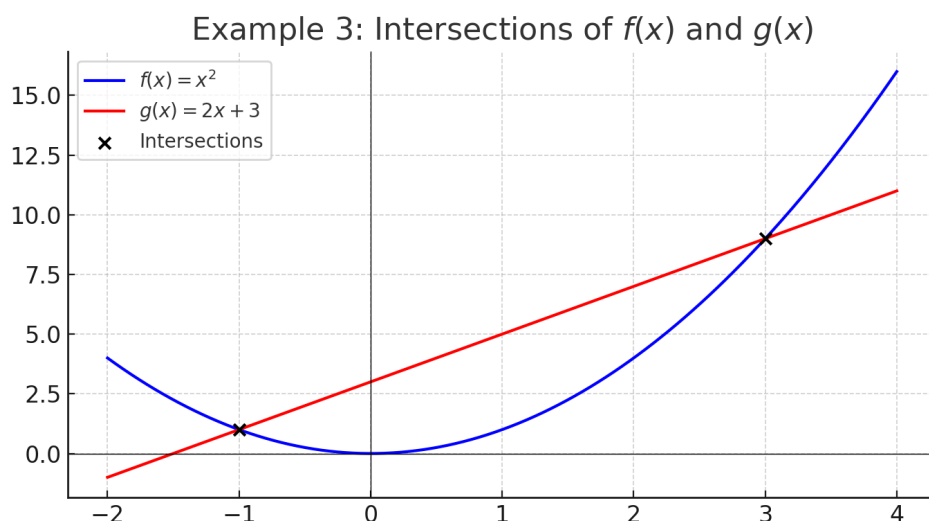


Example 3: Sketch the graphs of $f(x) = x^2$ and $g(x) = 2x + 3$ in one coordinate system and find their points of intersection.

Steps:

- Set $x^2 = 2x + 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0$
- Roots: $x = -1, x = 3$
- Plug in to find points: $(-1, 1), (3, 9)$

Intersections at $(-1, 1)$ and $(3, 9)$



Practice Problems for Seminar

Piecewise Functions

1. Define and sketch:

$$f(x) = \begin{cases} -x & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

2. Is the following function continuous at $x = 2$? Sketch and decide.

$$g(x) = \begin{cases} 3x - 1 & \text{if } x < 2 \\ x^2 - 1 & \text{if } x \geq 2 \end{cases}$$

3. Sketch:

$$f(x) = \begin{cases} 2 & \text{if } x < 0 \\ -x + 2 & \text{if } x \geq 0 \end{cases}$$

Graph – Practice Problems

4. Sketch and find intersections of $f(x) = x^2 - 4$, $g(x) = -x^2 + 2$
5. Sketch and find intersections of $f(x) = x^2 - 2x$, $g(x) = x + 2$

Review Problems

6. Solve: $\log_2(x - 3) = 4$
7. Rewrite $f(x) = 2(x - 1)^2 + 3$ as a transformation of x^2
8. Find the inverse of $f(x) = \frac{x+4}{3}$