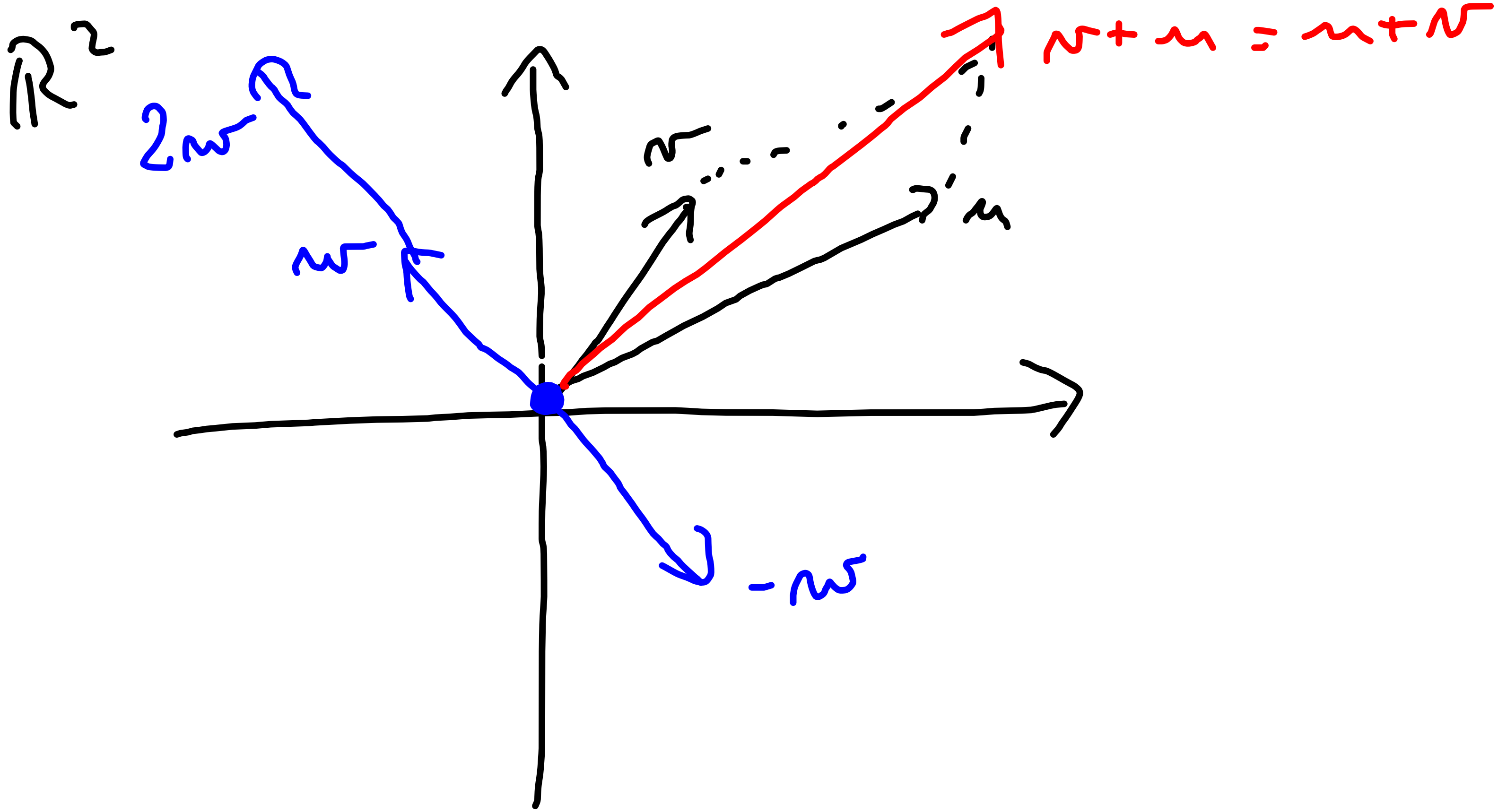

$\mathbb{R}^2$
$2w$
w
$-w$
v
u
$v+u = u+v$

$\mathrm{Mat}_2(\mathbb{R})$

Operace:

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\in\, \mathrm{Mat}_2\mathbb{R}} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \underbrace{\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}}_{\in\, \mathrm{Mat}_2\mathbb{R}}$$

$V \times V \to V$

$$\underbrace{r}_{\in\,\mathbb{R}} \cdot \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\in\, \mathrm{Mat}_2\mathbb{R}} = \underbrace{\begin{pmatrix} ra & rb \\ rc & rd \end{pmatrix}}_{\in\, \mathrm{Mat}_2\mathbb{R}} \qquad \mathbb{K} \times V \to V$$

Ověříme V vlastnosti v.p.:

(1) $$\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix}\right) + \begin{pmatrix} i & j \\ k & l \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix} = \boxed{\begin{pmatrix} (a+e)+i & (b+f)+j \\ (c+g)+k & (d+h)+l \end{pmatrix}}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \left(\begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix}\right) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e+i & f+j \\ g+k & h+l \end{pmatrix} =$$

$$= \boxed{\begin{pmatrix} a+(e+i) & b+(f+j) \\ c+(g+k) & d+(h+l) \end{pmatrix}}$$

$\mathbb{R}$ je asoc., po složkách

(2) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix} =$

$= \begin{pmatrix} e+a & f+b \\ g+c & h+d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(3) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} ? \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

hledaná nula je $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(4) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} ? \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$ je hledaný opačný prvek k $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(5) $r \in \mathbb{R}$

$$r \cdot \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} \right) = r \cdot \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix} =$$

$$= \boxed{\begin{pmatrix} r(a+e) & r(b+f) \\ r(c+g) & r(d+h) \end{pmatrix}}$$

$$r \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} + r \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ra & rb \\ rc & rd \end{pmatrix} + \begin{pmatrix} re & rf \\ rg & rh \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} ra+re & rb+rf \\ rc+rg & rd+rh \end{pmatrix}}$$

(6) $$(r+s)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} (r+s)a & (r+s)b \\ (r+s)c & (r+s)d \end{pmatrix}$$

$$r \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} + s\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad //$$

(7) $$r \cdot \left(s \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = r \begin{pmatrix} sa & sb \\ sc & sd \end{pmatrix} = \begin{pmatrix} rsa & rsb \\ rsc & rsd \end{pmatrix}$$

$$= rs \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

(8) $$1 \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1a & 1b \\ 1c & 1d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

(1) $\mathbb{R}^3$ nad $\mathbb{R}$

$$(a+b)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (a+b)x \\ y \\ z \end{pmatrix}$$

$$a\begin{pmatrix} x \\ y \\ z \end{pmatrix} + b\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax \\ y \\ z \end{pmatrix} + \begin{pmatrix} bx \\ y \\ z \end{pmatrix} = \begin{pmatrix} (a+b)x \\ 2y \\ 2z \end{pmatrix}$$

NENÍ V.P.!

OPERACE:

(2) $u \oplus v = \underset{\mathbb{R}_+}{u} \cdot \underset{\mathbb{R}_+}{v} \in \mathbb{R}_+$

$\underset{\mathbb{R}}{r} \circ \underset{\mathbb{R}^+}{u} = u^r \in \mathbb{R}_+$

(1) AXIOMY:

$(u \oplus v) \oplus w = u \cdot v \oplus w = u \cdot v \cdot w = u \oplus v \cdot w$
$= u \oplus (v \oplus w)$

(2) $u \oplus v = u \cdot v = v \cdot u = v \oplus u$

(3) $u \oplus o = u \quad u \cdot o = u \leadsto o = 1$ ← je hledaná nula

hledáme nulu

(4) $u \oplus (-u) = 1$
$u \cdot (-u) = 1$

$\frac{1}{u}$ je opačný prvek k u

(5)
$a \circ (u \oplus v) = a \circ uv = (uv)^a = u^a v^a =$
$= (a \circ u) \cdot (a \circ v) = (a \circ u) \oplus (a \circ v)$

(6) $(a+b) \circ u = u^{a+b} = u^a u^b =$
$= (a \circ u) \oplus (b \circ u)$

(7) $a \circ (b \circ u) = a \circ (u^b) = u^{ba} =$
$= u^{ab} = (ab) \circ u$

(8) $1 \circ u = u^1 = u$ je v.p.

---

(3) $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid x \geq 0 \right\}$ není v.p.

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $(-2) \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

$1+x,\ 1-x,\ 2+x-x^2$

$$a_1(1+x)+a_2(1-x)+a_3(2+x-x^2)=0$$

$$a_1+a_1x+a_2-a_2x+2a_3+a_3x-a_3x^2=0$$

$$(a_1+a_2+2a_3)+(a_1-a_2+a_3)x-$$
$$-a_3x^2=0+0x+0x^2$$

$$\begin{aligned} a_1+a_2+2a_3&=0\\ a_1-a_2+a_3&=0\\ -a_3&=0 \end{aligned} \qquad \begin{pmatrix} 1 & 1 & 2\\ 1 & -1 & 1\\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2\\ 0 & -2 & -1\\ 0 & 0 & -1 \end{pmatrix}$$

má pouze nulové řešení $\Rightarrow$ LN

$$a_1\begin{pmatrix}1&1\\2&2\end{pmatrix}+a_2\begin{pmatrix}2&2\\1&1\end{pmatrix}+a_3\begin{pmatrix}1&1\\1&1\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$\begin{pmatrix}a_1+2a_2+a_3 & a_1+2a_2+a_3\\ 2a_1+a_2+a_3 & 2a_1+a_2+a_3\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$\begin{aligned}a_1+2a_2+a_3&=0\\ a_1+2a_2+a_3&=0\\ 2a_1+a_2+a_3&=0\\ 2a_1+a_2+a_3&=0\end{aligned}$$

$$\begin{pmatrix}1&2&1\\1&2&1\\2&1&1\\2&1&1\end{pmatrix}\sim\begin{pmatrix}1&2&1\\0&3&2\\0&0&0\\0&0&0\end{pmatrix}$$

p

$\Rightarrow$ LZ

$$\begin{pmatrix}1&1\\1&1\end{pmatrix}=\frac{1}{3}\begin{pmatrix}1&1\\2&2\end{pmatrix}+\frac{1}{3}\begin{pmatrix}2&2\\1&1\end{pmatrix}$$

(1) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \,\middle|\, x_1 + x_2 = 0 \right\} =: U$

$$\left. \begin{array}{ll} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & x_1 + x_2 = 0 \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} & y_1 + y_2 = 0 \end{array} \right\} \text{prvky z } U$$

je součet v U?

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}$$

$$x_1 + y_1 + x_2 + y_2 = (x_1 + x_2) + (y_1 + y_2)$$
$$= 0 + 0 = 0$$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $x_1 + x_2 = 0$

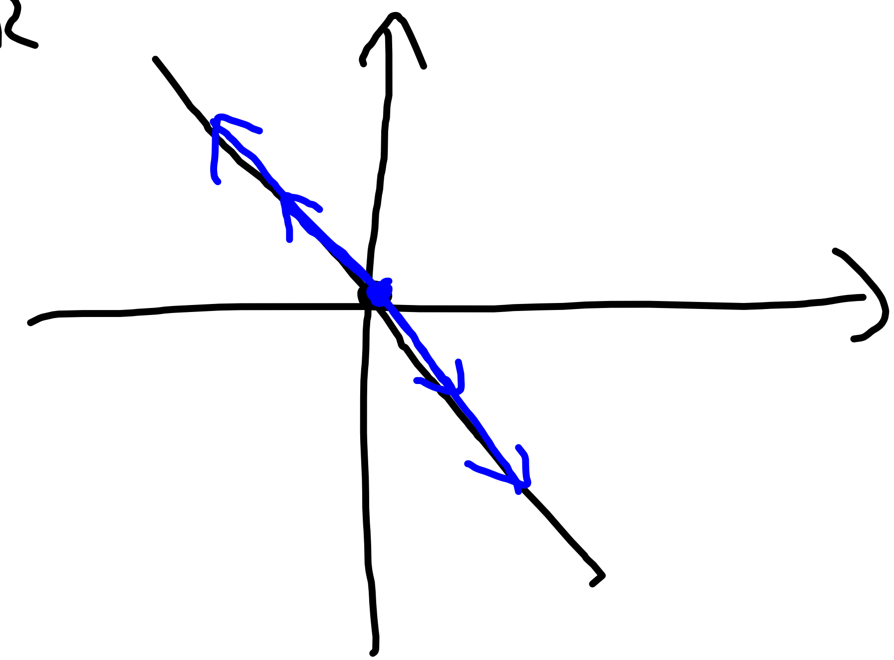
je $r \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ v U?

$$r \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} r x_1 \\ r x_2 \end{pmatrix}$$

$$r x_1 + r x_2 =$$
$$= r(x_1 + x_2) =$$
$$= r \cdot 0 = 0$$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $0 + 0 = 0$

$\mathbb{R}^2$

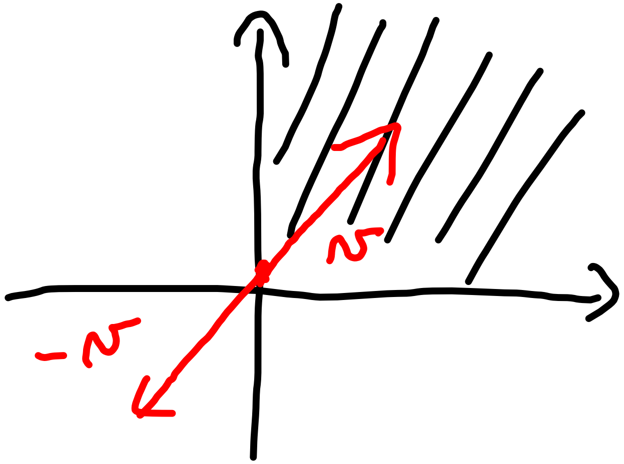


$$x_1 + x_2 = 0 \Rightarrow \begin{pmatrix}1\\-1\end{pmatrix}$$
$$x_1 = -x_2$$

$$\left\{ \begin{pmatrix}x_1\\x_2\end{pmatrix} \mid x_1 \geq 0,\ x_2 \geq 0 \right\}$$

$$(-2)\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$$

NE



$\{f \mid \exists g : f = g(x^2+1)\} =: U$

$f_1, f_2 \in U \quad f_1 = g_1(x^2+1)$

$\qquad\qquad\quad f_2 = g_2(x^2+1)$

$f_1 + f_2 = g_1(x^2+1) + g_2(x^2+1) =$

$= (g_1+g_2)(x^2+1)$

$f \in U, \; f = g(x^2+1)$

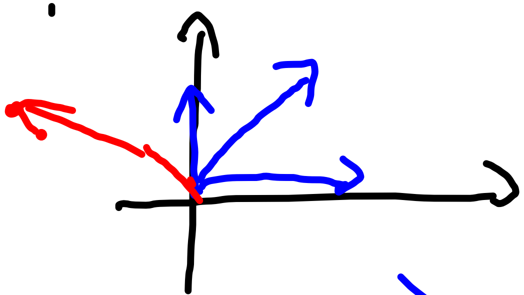
$rf = r(g(x^2+1)) = (rg)(x^2+1)$

$0 = 0 \cdot (x^2+1) \in U \qquad$ ANO

V v.p. $M \subset V$

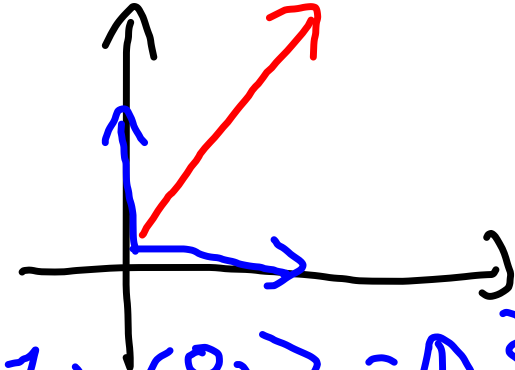
$\langle M \rangle$ nebo $[M]$

Př:

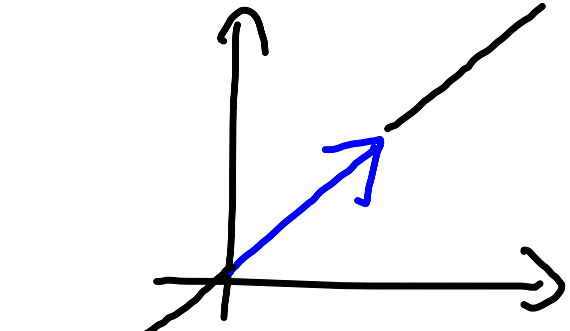


$$\left\langle \binom{1}{0}, \binom{0}{1}, \binom{1}{1} \right\rangle = \mathbb{R}^2$$

Př:



$$\left\langle \binom{1}{0}, \binom{0}{1} \right\rangle = \mathbb{R}^2$$



$$\left\langle \binom{1}{1} \right\rangle = \left\{ \binom{r}{r} \,\middle|\, r \in \mathbb{R} \right\}$$

$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ lib.

$$a\begin{pmatrix}1\\0\\1\end{pmatrix} + b\begin{pmatrix}1\\1\\1\end{pmatrix} + c\begin{pmatrix}1\\1\\0\end{pmatrix} + d\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$\uparrow$ cokoliv

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & v_1 \\ 0 & 1 & 1 & 1 & v_2 \\ 0 & 0 & -1 & 0 & v_3 \end{array}\right)$$

r

$d = t$

$c = -v_3$

b ..

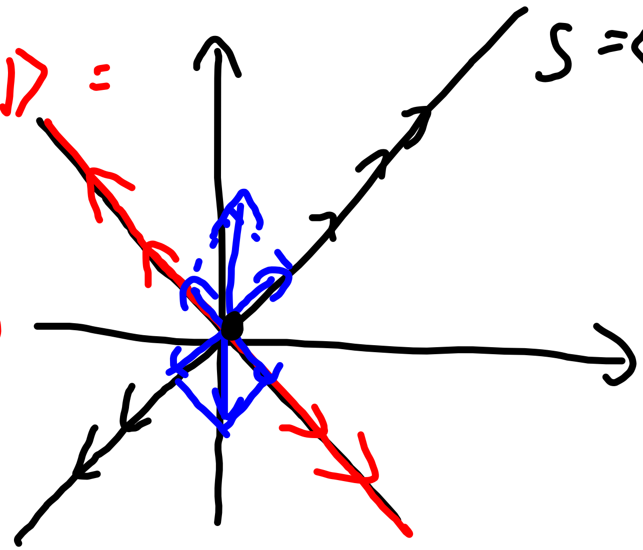
ano

$$\begin{pmatrix}1\\0\\7\end{pmatrix} = a\begin{pmatrix}1\\1\\1\end{pmatrix} + b\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\left(\begin{array}{cc|c}1&0&1\\1&1&0\\1&0&7\end{array}\right) \sim \left(\begin{array}{cc|c}1&0&1\\0&1&-1\\0&0&6\end{array}\right)$$ nemá řešení!

$\mathbb{R}^2$

$T = \left\langle \begin{pmatrix}1\\-1\end{pmatrix} \right\rangle = \left\{ \begin{pmatrix}t\\-t\end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$



$S = \left\langle \begin{pmatrix}1\\1\end{pmatrix} \right\rangle = \left\{ \begin{pmatrix}t\\t\end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$

$$S + T = \mathbb{R}^2$$

1a) $S \cap T = \{0\}$

$v \in S \cap T \iff v \in S \wedge v \in T$

řešení $x = y$ řešení $x = -y$

$S \cap T$ je řešení $\begin{array}{l} x = y \\ x = -y \end{array}$ Řešení je $\begin{array}{l} x = 0 \\ y = 0 \end{array}$

je přímý

$S + T = \mathbb{R}^2$

(2) $v \in S \cap T \iff v \in S \wedge v \in T$

řešení $x - 2y - 3z = 0$ řešení $x = z$

Řešíme

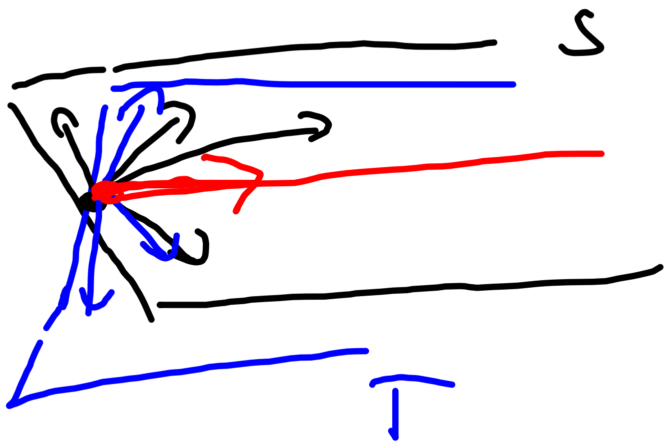
$x - 2y - 3z = 0$

$x = z$

$$\begin{pmatrix} 1 & -2 & -3 \\ 1 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -3 \\ 0 & 2 & 2 \end{pmatrix}$$

z

není přímý



$S + T = \mathbb{R}^3$

$S \cap T: \quad x = y = 0 \qquad \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

$\qquad\qquad y = z = 0$

ji mij

$S + T = \mathbb{R}^2$