$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = u \qquad \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = v$$

$$c \cdot (u+v) = \begin{pmatrix} c \cdot (a_1 + b_1) \\ c \cdot (a_2 + b_2) \\ \vdots \\ c \cdot (a_n + b_n) \end{pmatrix} = \begin{pmatrix} c \cdot a_1 \\ \vdots \\ c \cdot a_n \end{pmatrix} + \begin{pmatrix} c \cdot b_1 \\ \vdots \\ c \cdot b_n \end{pmatrix}$$

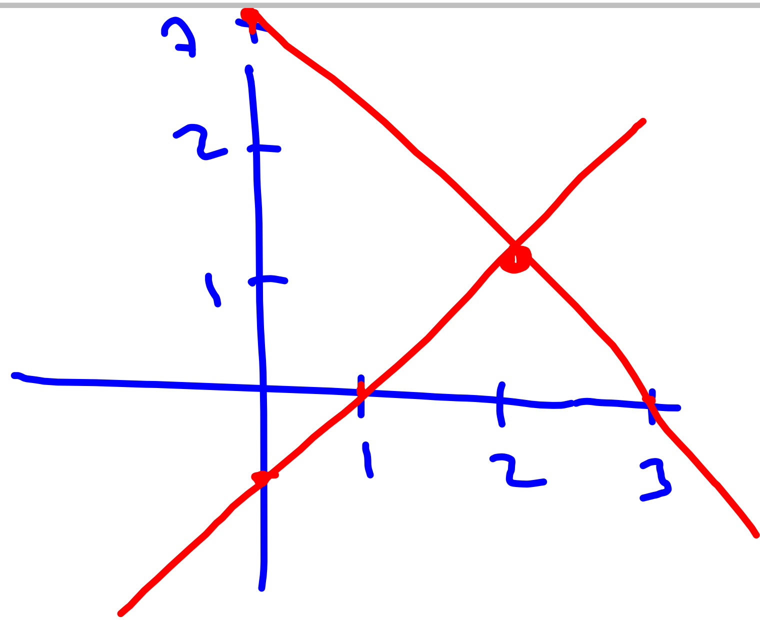
$$= c \cdot u + c \cdot v$$

$$x_1 + x_2 = 3$$

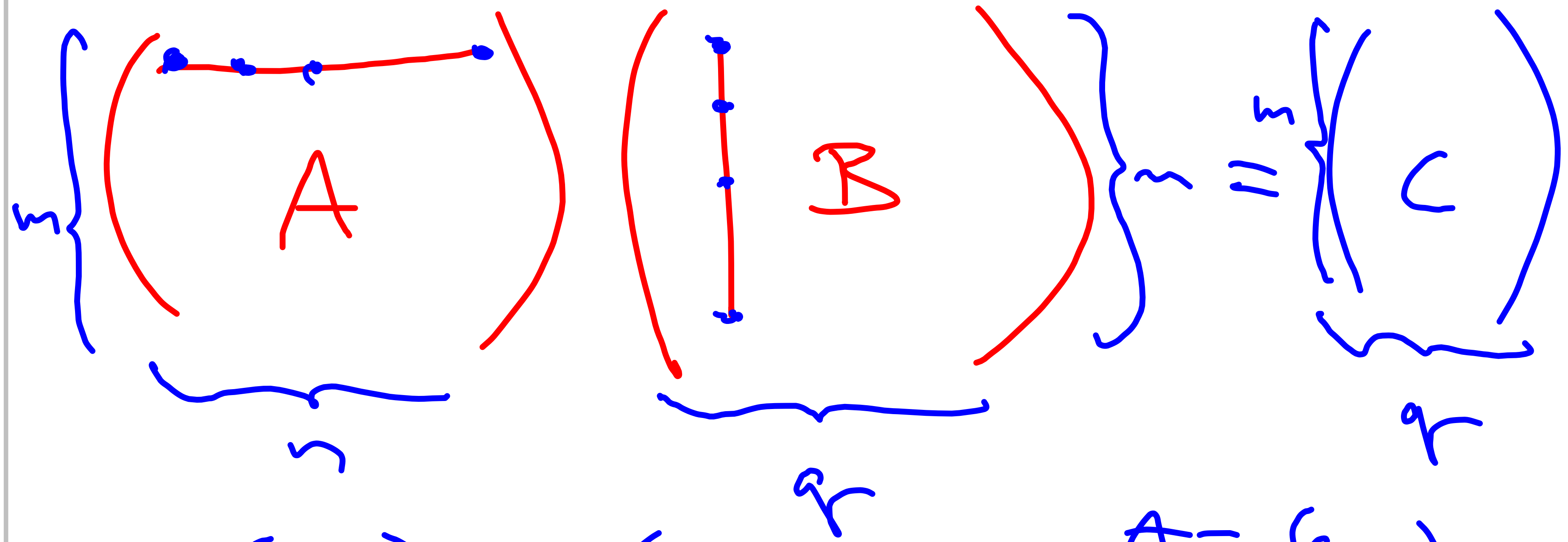
$$x_1 - x_2 = 1$$

$$2x_1 = 4 \Rightarrow x_1 = 2$$

$$2x_2 = 2$$



$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$



$$A\cdot(B\cdot C) = (A\cdot B)\cdot C$$

$$A = (a_{ij})$$
$$B = (b_{jk})$$
$$C = (c_{kl})$$

$$B\cdot C = \left(\sum_k b_{jk}\cdot c_{kl}\right)$$

$$A\cdot(B\cdot C) = \left(\sum_j \left(a_{ij}\cdot \sum_k b_{jk}\cdot c_{kl}\right)\right) = \left(\sum_{j,k} a_{ij}\cdot(b_{jk}\cdot c_{kl})\right)$$

$$(A\cdot B)\cdot C = \left(\sum_{j,k} (a_{ij} b_{jk}) c_{kl}\right) = \left(\sum_k \left(\sum_j a_{ij} b_{jk}\right)\cdot c_{kl}\right)$$

$$A\cdot(B+C) = \left(\sum_j a_{ij}\cdot(b_{jk}+c_{jk})\right)$$

$$= \left(\sum_j a_{ij}\, b_{jk} + \sum_j a_{ij}\, c_{jk}\right)$$

$$= A\cdot B + A\cdot C$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A \cdot x = b \quad / \cdot A^{-1}$$

$$A^{-1} \cdot (A \cdot x) = A^{-1} \cdot b$$

$$\shortparallel$$

$$(A^{-1} A) \cdot x = A^{-1} \cdot b$$

$$\shortparallel$$

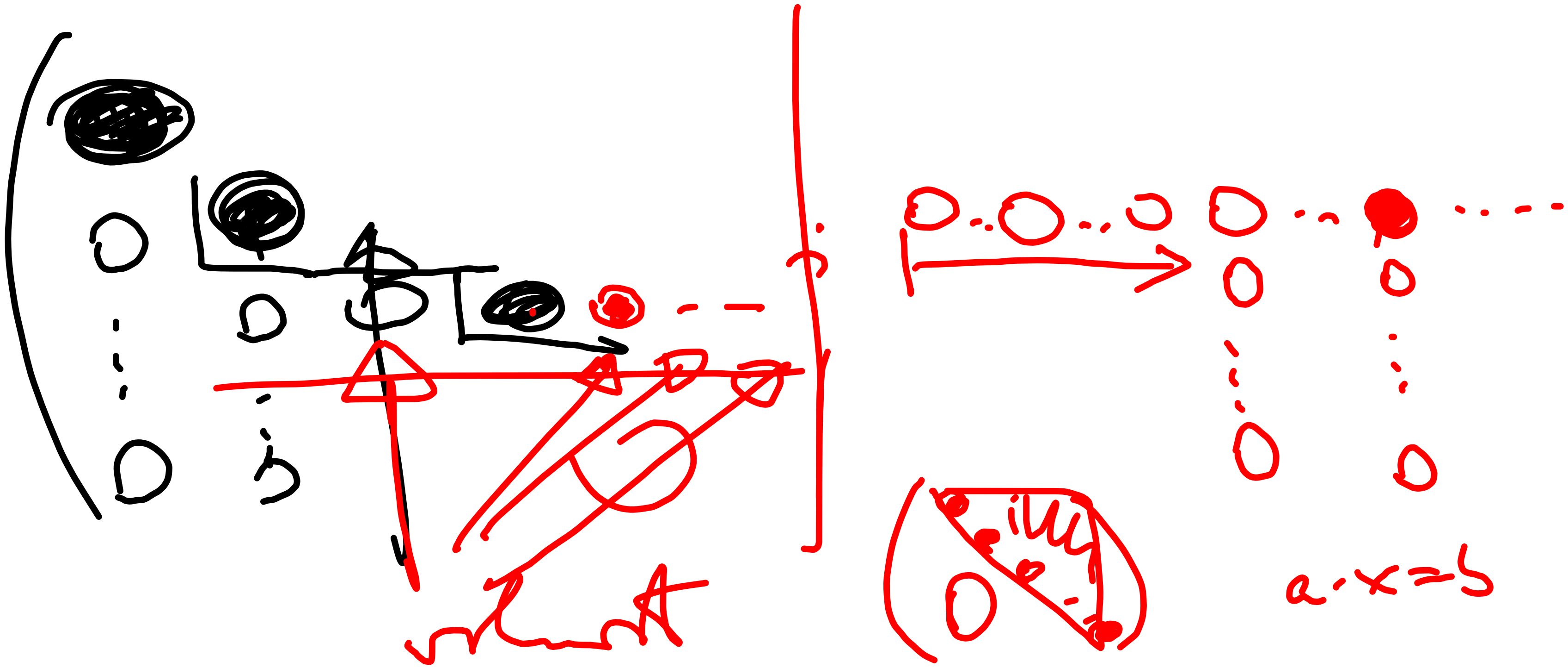
$$\boxed{x = A^{-1} \cdot b}$$

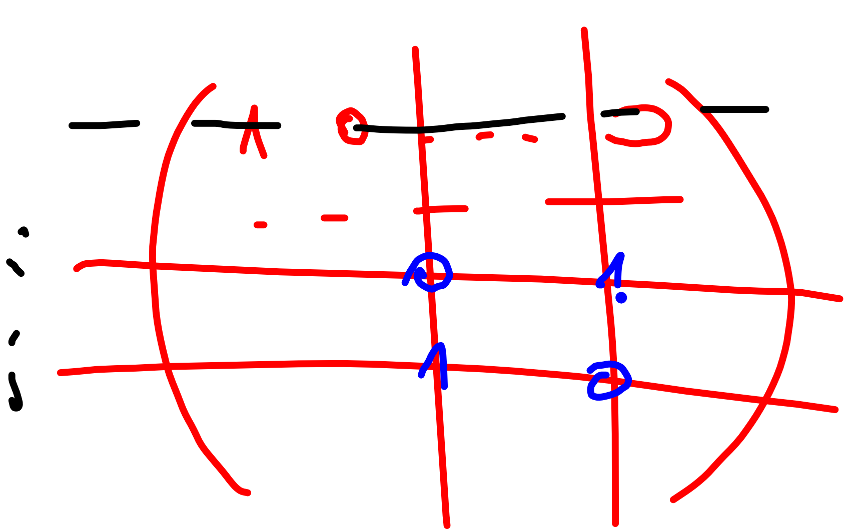
$$\underbrace{(B^{-1} \cdot \underbrace{(A^{-1}) \cdot (A)}_{E} \cdot B)} = E$$

$A \cdot x = b$

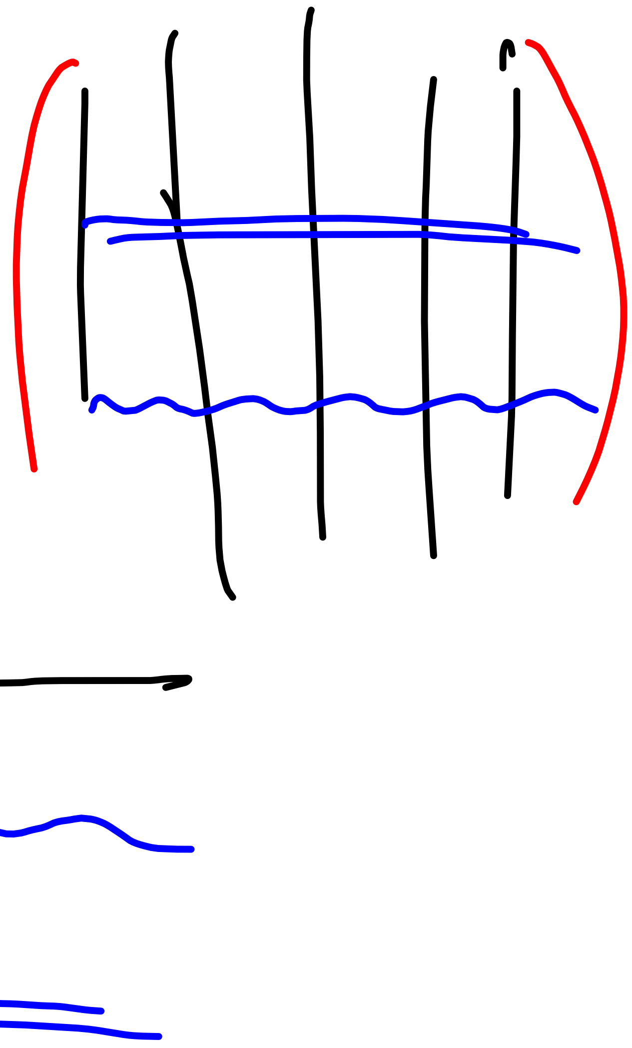
$E \cdot x = x = \tilde{b}$

$$(A \mid b) \rightsquigarrow (E \mid \tilde{b})$$





0
1
1
2



$$(P_k \cdots (P_2(P_1 A))\cdots)$$

Gaus. el. $= \quad A \mapsto \underbrace{P\cdot A}_{\text{ve schod. tvaru (řádk.)}} = A'$

$$B \mapsto \underbrace{B'}_{\text{sloupcový schod. tvar}} = B(Q_1 \cdots Q_\ell)$$

$$A \rightsquigarrow P\cdot A\cdot Q = m\left\{\underbrace{\begin{pmatrix} \overbrace{E_h}^{h} & 0 \\ 0 & 0 \end{pmatrix}}_{n}\right.$$

Inv. matice:

$A \mapsto \underline{P'} \cdot A = A'$ (řádk. odst. tvar)

ex. $A^{-1} \Rightarrow$ ex. $(A')^{-1} = A^{-1} \cdot (P')^{-1}$

$$\begin{pmatrix} * \\ \hline 0 \; \cdots \; 0 \end{pmatrix} \begin{pmatrix} * \end{pmatrix} = \begin{pmatrix} ? \\ \hline 0 \; \cdots \; 0 \end{pmatrix}$$

$\Rightarrow A'$ nemá nulový řádek

$\Rightarrow$ ex. P'' tak, že $P = P'' \cdot P'$, $\underline{\underline{P \cdot A = E}}$

$$A \rightsquigarrow A'' = \underbrace{P}_{\substack{\| \\ A^{-1}}} \cdot A = E$$

$$\underset{\sim}{(A|E)} \rightsquigarrow P \cdot (A \;\; E)$$

$$= (E \;\; P)$$

$$\begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 3 & 5 & | & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \| & -2 & 1 \\ 0 & 1 & \| & 3/2 & -1/2 \end{pmatrix}$$

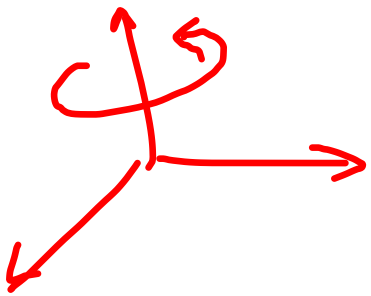
$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

✓

$$1\cdot\begin{pmatrix}1\\1\\0\end{pmatrix} + 1\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix} + 1\cdot\begin{pmatrix}0\\-2\\-1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$u_1 \qquad u_2 \qquad u_3$ jsou lineárně závislé

$A \cdot x =$ rotace x kolem osy o úhel



$$\begin{pmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$