$\overline{TIK}$

$\mathbb{R}_2[x], \mathbb{R}$ $+$ ... abelovská grupa

$a, b \in \mathbb{R}$ $\quad (a \cdot b)\, u$ $\quad u \in \mathbb{R}_2[x]$

$= a(b u)$ $\quad v \in$

$(a+b)\, u = au + bu$

$a(u+v) = au + av$

$1 u = u$

PŘ: $\mathbb{R}$ nad $\mathbb{C}$

$U \subseteq \mathbb{R}_2[x]$

$f \in U \iff f(-x) = -f(x)$

$ax^2 + bx + c =: f$

$a(-x)^2 + b(-x) + c = -(ax^2 + bx + c)$

$ax^2 - bx + c = -ax^2 - bx - c$

$2ax^2 + 2c = 0 \implies a = c = 0$

$U = \{ bx \mid b \in \mathbb{R} \} \subseteq \mathbb{R}_2[x]$

$b_1 x + b_2 x = (b_1 + b_2) x \in U$

$r(bx) = (rb) x \in U$

$b = 0 \qquad 0 \cdot x = 0 \in U$

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix}$$

$a_1+b_1+c_1+d_1=0$ $\qquad a_2+b_2+c_2+d_2=0$

$\Downarrow$?

$$a_1+a_2+b_1+b_2+c_1+c_2+d_1 d_2=0$$

$$v\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} va & vb \\ vc & vd \end{pmatrix}$$

$$va+vb+vc+vd=0$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad 0+0+0+0=0$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = a \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} + b \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$$

$$1 = 3a + 3b$$

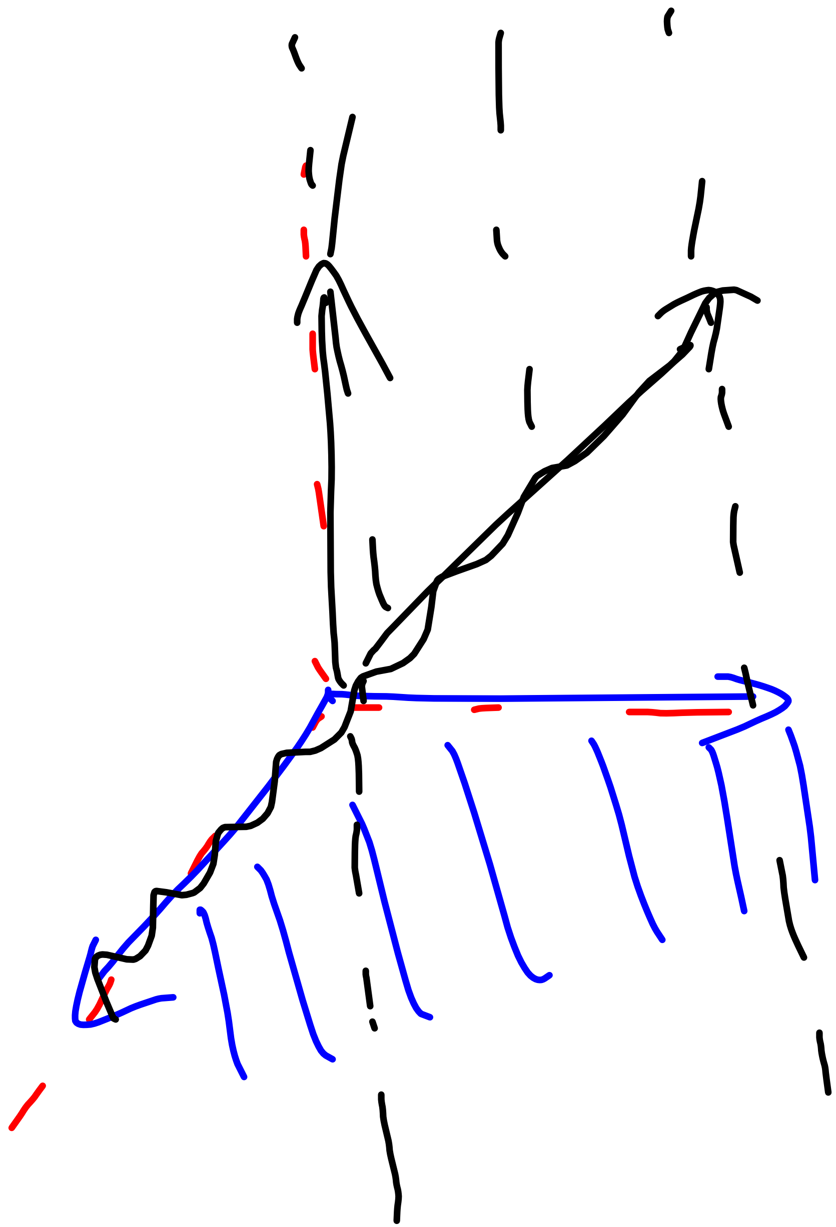
$$2 = 3a + b$$

$$3 = 2a - b$$

$$4 = 2a - 2b$$

$a = 1$

$b = -1$



$x + y = 0$ $\quad$ $y = -x$

$x + z = 0$ $\quad$ $z = -x$

$(x, -x, -x)$

$(1, -1, -1) \in S \cap T$

M_1, M_2, M_3

$\underline{IV}$ =: Π

LN

$a \in A$

$\longrightarrow \varphi \longleftarrow$

$\mathbb{R}_2[x]$ $\quad ax^2+bx+c$

$$= d\cdot 1 + e\,x + f\,x^2$$

$$\Rightarrow \langle 1, x, x^2\rangle = \mathbb{R}_2[x]$$

$$ax^2+bx+c\cdot 1 = 0 \Rightarrow a=b=c=0$$

$1+x,\ x+x^2,\ x^2$

$$ax^2+bx+c = d(1+x)+e(x+x^2)+f(x^2) =$$

$$= (e+f)x^2 + (d+e)x + d(1)$$

$d = c$
$e = b-c$
$f = a-b+c$

$2x^2+x+1$

$d = 1$
$e = b = 0$
$f = a = 2$

$$1(1+x)+2x^2 = 2x^2+x+1$$

$$a(1+x)+b(x+x^2)+c(x^2)=0$$

$a=0$
$a+b=0$
$b+c=0$

$\Rightarrow$ $a=0$, $b=0$, $c=0$

je to báze

$1+x,\ x,\ 1$

$$a(1+x)+b(x)+c(1)=0$$

$$a+c=0$$
$$a+b=0$$

$$a=t \quad b=-t \quad c=-t$$

$$1+x=(1)+(x)$$

$$1+x,\ x,\ x+x^2,\ x^2$$

$$a(1+x)+b(x)+c(x+x^2)+d(x^2)=0$$

$$a=0$$

$$a+b+c=0$$

$$c+d=0$$

$$a=0 \quad c=t \quad b=-t \quad d=-t$$

$$x+x^2=(x)+(x^2)$$

$$v = 1 + x + x^2$$

$$\alpha_1 = (1, x, x^2)$$

$$1 + x + x^2 = (1) \cdot 1 + (1) \cdot x + (1) \cdot x^2$$

$$(v)_{\alpha_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$w = 2x + 4x^2 \qquad (w)_\alpha = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$$

$v = x^2 + x + 2$ $\qquad \alpha_1 = (x^2, x, 1)$

$(v)_{\alpha_1} = (1, 1, 2)^T$

$\alpha_2 = (1+x, x+x^2, x^2)$

$1 + x + x^2 = a(1+x) + b(x+x^2) + c\,x^2$

$$\begin{aligned} 1 &= a & a &= 1 \\ 1 &= a + b & b &= 0 \\ 1 &= b + c & c &= 1 \end{aligned}$$

$$(1 + x + x^2)_{\alpha_2} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\mathbb{R}^3$ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\dim \mathbb{R}^3 = 3$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 2 & 4 & 6 & 0 \\ 1 & 2 & 3 & 6 & 9 & 0 \\ 0 & 3 & 3 & 6 & 9 & 0 \end{array}\right) \sim \begin{pmatrix} 1 & 1 & 2 & 4 & 6 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 3 & 3 & 6 & 9 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 4 & 6 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\langle \Pi \rangle = \langle (1,1,0), (1,2,3) \rangle$$

$\mathbb{R}_2[x]$

$$\langle 1, 1+x \rangle = a\cdot 1 + b\cdot(1+x) =$$

$$= (a+b) + bx = \underline{\underline{c + bx}}$$

$$a\,1 + b(1+x) = 0$$

$$a+b = 0 \Rightarrow a = 0$$
$$b = 0$$

$$\alpha = \boxed{\langle 1, 1+x, x^2 \rangle} = \mathbb{R}_2[x]$$

$$a\,1 + b(1+x) + c(x^2) = 0$$

$$a+b = 0 \Rightarrow a = 0$$
$$b = 0$$
$$c = 0$$

$$2 + 3x + x^2 = a(1) + b(1+x) + c(x^2)$$

$$2 = a+b \qquad a = -1$$
$$3 = b$$
$$1 = c$$

$$(2+3x+x^2)_\alpha = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$$

$f: \mathbb{R}_3[x] \longrightarrow \mathbb{R}_2[x]$

$f(ax^3 + bx^2 + cx + d) =$

$= 3ax^2 + 2bx + c$

$f(u+v) = f(a_1x^3 + b_1x^2 + c_1x + d_1 +$

$+ a_2x^3 + b_2x^2 + c_2x + d_2) =$

$f((a_1+a_2)x^3 + (b_1+b_2)x^2 + (c_1+c_2)x + (d_1+d_2))$

$= 3(a_1+a_2)x^2 + 2(b_1+b_2)x + (c_1+c_2) =$

$= (3a_1x^2 + 2b_1x + c_1) + (3a_2x^2 + 2b_2x + c_2)$

$f(u) + f(v) = (3a_1x^2 + 2b_1x + c_1) +$

$+ (3a_2x^2 + 2b_2x + c_2)$

$f(k(ax^3 + bx^2 + cx + d)) =$

$f((ka)x^3 + (kb)x^2 + (k \cdot c)x + kd) =$

$= 3 \cdot (k \cdot a)x^2 + 2k \cdot bx + k \cdot c$

$k \cdot f(w) = k(3ax^2 + 2bx + c)$

$\alpha_1 = (1, x, x^2)$

$\alpha_2 = (1+x, x, x^2)$

$v = 1 + x + x^2 \in \mathbb{R}_2[x]$

$(v)_{\alpha_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad (v)_{\alpha_2} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$$(v)_{\alpha_2} = T_{\alpha_2\alpha_1}(v)_{\alpha_1}$$

$1 = a(1+x) + b(x) + cx^2$

$(1)_{\alpha_2} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $\quad (x)_{\alpha_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad (x^2)_{\alpha_2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$T_{\alpha_2\alpha_1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$(1+x+x^2)_{\alpha_1} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$T_{\alpha_2\alpha_1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$f: \mathbb{R}_3[x] \longrightarrow \mathbb{R}_2[x]$

$f(ax^3 + bx^2 + cx + d) = 3ax^2 + 2bx + c$

$\alpha = (1, x, x^2, x^3)$ $\beta = (1, x, x^2)$

$$\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3a \\ 2b \\ c \end{pmatrix}$$

$$(f)_{\beta\alpha} (v)_{\alpha} = (f v)_{\beta}$$

$f: \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$

$f(ax^2+bx+c) = 2ax+b$

$\alpha = (1+x,\ x+x^2,\ x^2)$ $\quad \underline{\varepsilon = (x^2, x, 1)}$

$$f_{\varepsilon\varepsilon} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad T_{\varepsilon\alpha} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_{\alpha\varepsilon} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & +1 \end{pmatrix}$$

$f_{\alpha\alpha} = \underline{T_{\alpha\varepsilon} f_{\varepsilon\varepsilon} T_{\varepsilon\alpha}}$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & -1 & -2 \end{pmatrix}$$