1) $\vec{f}_1 = \frac{1}{x}$ $\qquad$ $f_2 = \frac{1}{x^2} - \frac{3}{5x}$

$$\vec{f}_L = \frac{1}{x^2} + \ell \cdot \frac{1}{x} \quad , \ell \in \mathbb{R}$$

$$0 = \vec{f}_1 \cdot \vec{f}_L = \vec{f}_1 \cdot \frac{1}{x^2} + \ell \cdot \|\vec{f}_1\|^2 \Rightarrow$$

$$\Rightarrow \ell = - \frac{\vec{f}_1 \cdot \frac{1}{x^2}}{\|\vec{f}_1\|^2} = - \frac{\frac{3}{8}}{\frac{1}{2}} = - \frac{3}{4}$$

$$\|\vec{f}_1\|^2 = \int_1^2 \frac{1}{x} \cdot \frac{1}{x} \, dx = \left[- \frac{1}{x} \right]_1^2 = \frac{1}{2}$$

$$\vec{f}_1 \cdot \frac{1}{x^2} = \int_1^2 \frac{1}{x^3} \, dx = \left[- \frac{1}{2x^2} \right]_1^2 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

$$\vec{f}_3 = \frac{1}{x^3} + a \cdot \vec{f}_2 + b \cdot \vec{f}_1$$

$$0 = \vec{f}_2 \cdot \vec{f}_3 = \vec{f}_2 \cdot \frac{1}{x^3} + a \cdot \|\vec{f}_2\|^2 \Rightarrow$$

$$\Rightarrow a = -\frac{\vec{f}_2 \cdot \frac{1}{x^3}}{\|\vec{f}_2\|^2}$$

$$\|f_2\|^2 = \int_1^2 \left(\frac{1}{x^2} - \frac{3}{4x}\right)\left(\frac{1}{x^2} - \frac{3}{4x}\right) dx =$$

$$= \int_1^2 \left(\frac{1}{x^4} - \frac{3}{2x^3} + \frac{9}{16x^2}\right) dx$$

$$=$$

$$p = \frac{\boxed{\frac{1}{x^5} \cdot \vec{f_1}}}{\|\vec{f_1}\|^2} \cdot \vec{f_1} + \frac{\boxed{\frac{1}{x^5} \cdot f_2}}{\|\vec{f_2}\|^2} \cdot \vec{f_2} + \frac{\boxed{\frac{1}{x^5} \cdot f_3}}{\|\vec{f_3}\|^2} \cdot \vec{f_3}$$

vzdálenost: $\left\| \frac{1}{x^5} - p \right\|^2 = \int_{-1}^{2} \left(\frac{1}{x^4} - \frac{15}{32} f_1 - \frac{69}{50} f_2 - \frac{9}{4} f_3 \right)^2 dx$

$$\langle 1, \sin(x), \cos x, \sin(2x), \cos(2x), \dots, \sin(nx), \cos(nx), \dots \rangle$$

na int $\langle -\pi, \pi \rangle$

Vyjádření funkce na $\langle -\pi, \pi \rangle$ ve formě

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$$

se říká Fourierova řada dané fce.

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin nx \, dx \qquad n = 0, \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos x \, dx \qquad n = 1, \dots$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot 1 \, dx = 0$$

$n \geq 1$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin(nx)\, dx = \frac{1}{\pi}\left[-\frac{1}{n} x \cos(nx) + \underline{\frac{1}{n^2}\sin(nx)}\right]_{-\pi}^{\pi}$$

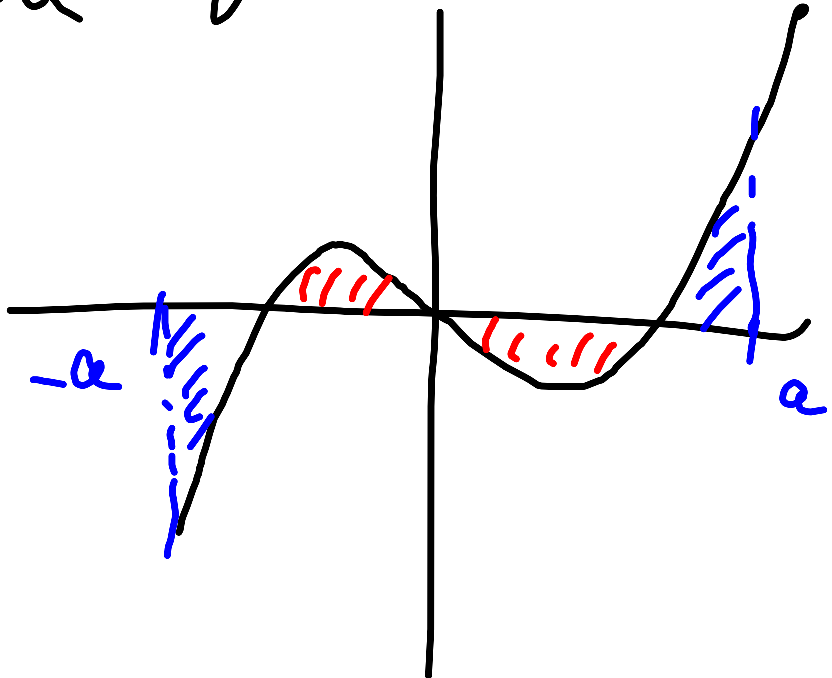
$$\int x \cdot \sin(nx) = -\frac{1}{n} x \cos(nx) + \frac{1}{n}\int \cos(nx) =$$

$$\left[\begin{array}{ll} u = x & u' = 1 \\ v' = \sin(nx) & v = -\frac{1}{n}\cos(nx) \end{array}\right]$$

$$= -\frac{1}{n} x \cos(nx) + \frac{1}{n^2}\sin(nx)$$

$$= \frac{1}{\pi}\left[-\frac{1}{n}\pi(-1)^n + \frac{1}{n}(-\pi)(-1)^n\right] = \frac{2}{n}(-1)^{n+1}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cdot \cos(nx)\, dx = 0$$



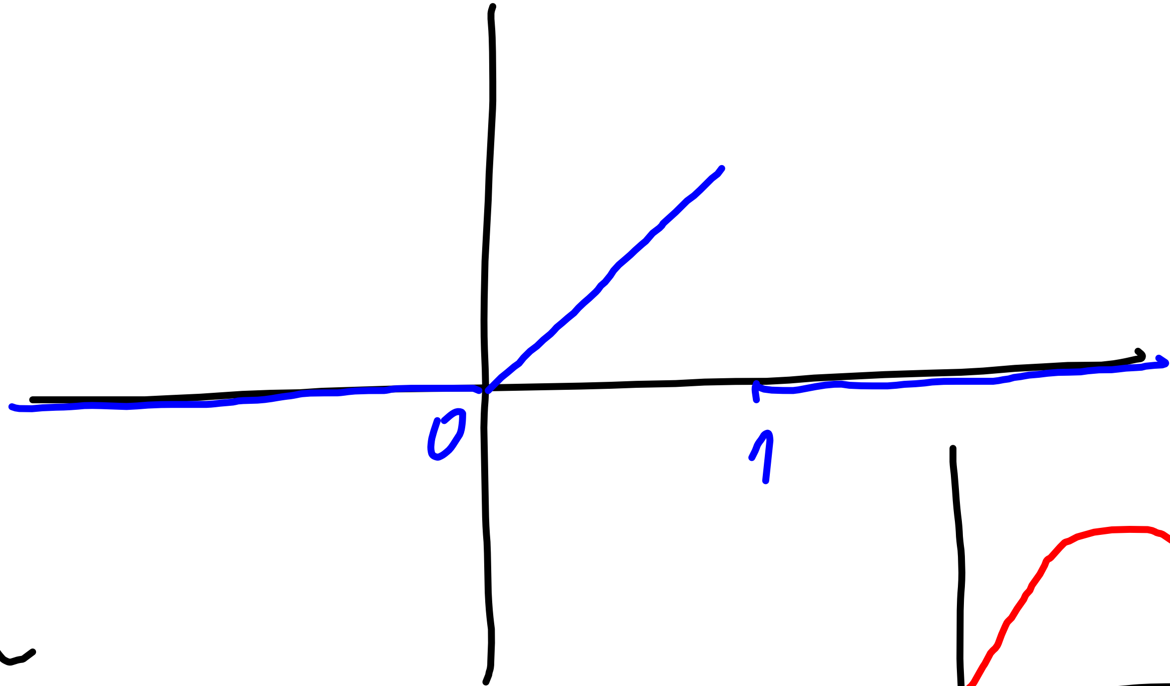
$$x = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$

$$x + 1 = 1 + 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$

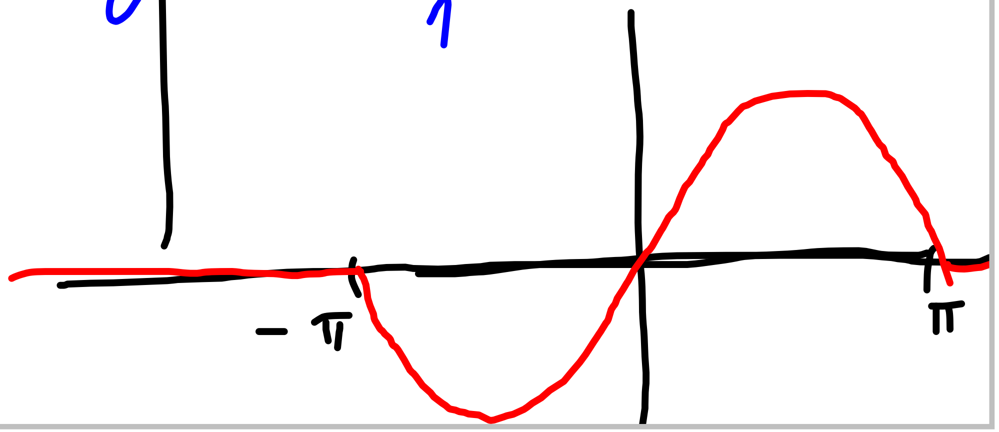
$$\frac{1}{\pi} \int_{-\pi}^{\pi} (x+1) = 2$$

$$f_1 * f_2(t) = \int_{-\infty}^{\infty} f_1(t-x)\, f_2(x)\, dx$$

$f_1 \sim$



$f_2 \sim$



$$f_1 * f_2(t) = \int_{-\infty}^{\infty} f_1(t-x) \cdot f_2(x)\, dx =$$

pro která t je $f_1(t-x) \cdot f_2(x) = 0$ pro lib. x ?:

$$t \in (-\infty, -\pi) \cup (\pi + 1, \infty)$$

– pro která t budou x takové, že $t - x \in \langle 0, 1\rangle$ ležet v $\langle -\pi, \pi\rangle$?

Pro je ~~ccc~~ $\langle t-1, t\rangle \subset \langle -\pi, \pi\rangle$
$\Leftrightarrow$ $t \in \langle -\pi + 1, \pi\rangle$

Pro dané t je

$$\int_{-\infty}^{\infty} \Gamma(t-x)\sin(x)\,dx = \int_{t-1}^{t} (t-x)\sin(x)\,dx =$$

$$= \Big[-t\cos(x)\Big]_{t-1}^{t} - \int_{t-1}^{t} x\sin(x)\,dx =$$

$$= -t\cos(t) + t\cos(t-1) - \Big[-x\cos x + \sin x\Big]_{t-1}^{t} =$$

$$= \cancel{-t\cos(t)} + \cancel{t\cdot\cos(t-1)} + \cancel{t\cos t} - \sin t - (\cancel{t}-1)\cos(t-1)$$

$$+ \sin(t-1) = \cos(t-1) + \sin(t-1) - \sin(t)$$

Integrujeme vlastně přes interval

$\langle s-1, s\rangle \cap \langle -\pi, \pi\rangle$.

a) Pro $s \in \langle -\pi, -\pi+1)$ je to interval

$\langle -\pi, s\rangle$

b) Pro $s \in \langle -\pi+1, \pi\rangle$ je to interval

$\langle s-1, s\rangle$

c) Pro $s \in \langle \pi, \pi+1\rangle$ je to interval

$\langle s-1, \pi\rangle$

a) $t \in \langle -\pi, -\pi+1)$:

$$f_1 * f_2(t) = \int_{-\infty}^{\infty} f_1(t-x) \cdot f_2(x)\,dx =$$

$$= \int_{-\pi}^{t} (t-x)\sin(x) =$$

$$= -\sin(t) - t - \pi$$

c) $t \in (\pi, \pi+1)$

$$f_1 * f_2(t) = \int_{-\infty}^{\infty} f_1(t-x) \cdot f(x)\,dx =$$

$$= \int_{t-1}^{\pi} (t-x)\sin(x) = t - \pi + \sin(t-1) + \cos(t-1)$$

$$f_1 * f_2(t) = \begin{cases} \cos(t-1) - \sin(t) + \sin(t-1), & \text{pro } t \in \langle -\pi+1, \pi \rangle \\ -\sin(t) - t - \pi, & \text{pro } t \in \langle -\pi, -\pi+1 \rangle \\ t - \pi + \sin(t-1) + \cos(t-1), & \text{pro } t \in \langle \pi, \pi+1 \rangle \\ 0 & \text{jinak} \end{cases}$$