$$f_1 * f_1(t) = \int_{-\infty}^{\infty} f_1(x) \cdot f_1(t-x)\, dx$$

Zajímá nás interval, kde je $f_1(x) \cdot f_1(t-x)$ nenulová.
To závisí od t:

$f_1(x)$ je nenulová na $\langle 0,1 \rangle$

$f_1(t-x)$ je nenulová na $t-x \in \langle 0,1 \rangle \Leftrightarrow x \in \langle t-1, t \rangle$

$$f_1(x) \cdot f_1(t-x) \neq 0 \Leftrightarrow x \in \langle 0,1 \rangle \,\&\, x \in \langle t-1, t \rangle \Leftrightarrow$$
$$\Leftrightarrow x \in \langle 0,1 \rangle \cap \langle t-1, t \rangle$$

pro $t \in \langle 0,1 \rangle$ je $\langle 0,1 \rangle \cap \langle t-1, t \rangle = \langle 0, t \rangle$
$t \in \langle 1,2 \rangle$ je $\langle 0,1 \rangle \cap \langle t-1, t \rangle = \langle t-1, 1 \rangle$

$s \in \mathbb{R} - \langle 0, 2 \rangle$ je $\langle 0, 1 \rangle \cap \langle s-1, s \rangle = \emptyset$

$$\int_{-\infty}^{\infty} f_1(x) f_1(s-x) =$$

$$f_1 * f_2(s) = \begin{cases} s & ; \text{ pro } s \in \langle 0, 1 \rangle \\ 2-s & ; \text{ pro } s \in \langle 1, 2 \rangle \\ 0 & , \text{ jinak} \end{cases}$$

a) $s \in \langle 0, 1 \rangle$

$$= \int_0^s dx = [x]_0^s = s$$

b) $s \in \langle 1, 2 \rangle$

$$= \int_{s-1}^{1} dx = [x]_{s-1}^{1} = 2-s$$

c) $= 0$

2, opět hledáme interval, kde je $f_2(x)$ a zároveň $f_1(s-x)$ nenulové.

$$\left(\int_{-\infty}^{\infty} f_2(x) \cdot f_1(s-x)\,dx \right)$$

$$f_2(x) \neq 0 \Rightarrow x \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$$

$$f_1(s-x) \neq 0 \Leftrightarrow x \in \langle s-1, s \rangle$$

$$f_2(x) \cdot f_1(s-x) \neq 0 \Leftrightarrow x \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle \cap \langle s-1, s \rangle$$

a) $s \in \left\langle -\frac{\pi}{2}, -\frac{\pi}{2}+1 \right\rangle$:

$$\left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle \cap \langle s-1, s \rangle = \left\langle -\frac{\pi}{2}, s \right\rangle$$

b) $s \in \left\langle -\frac{\pi}{2}+1, \frac{\pi}{2} \right\rangle$: $\left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle \cap \langle s-1, s \rangle = \langle s-1, s \rangle$

$t \in \langle \frac{\pi}{2}, \frac{\pi}{2}+1 \rangle$:

$$\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle \cap \langle t-1, t \rangle = \langle t-1, \frac{\pi}{2} \rangle$$

$$\int_{-\infty}^{\infty} f_2(x) \cdot f_1(t-x)\, dx =$$

a) $$\int_{-\frac{\pi}{2}}^{t} \cos(x)\, dx = [\sin(x)]_{-\frac{\pi}{2}}^{t} = \sin(t)+1$$

b) $$\int_{t-1}^{t} \cos(x)\, dx = [\sin x]_{t-1}^{t} = \sin(t) - \sin(t-1)$$

c) $$\int_{t-1}^{\frac{\pi}{2}} \cos(x)\, dx = [\sin x]_{t-1}^{\frac{\pi}{2}} = 1 - \sin(t-1)$$

$$f_1 * f_2(t) = \begin{cases} \sin(t)+1, & \text{pro } t \in \left(-\frac{\pi}{2}, -\frac{\pi}{2}+1\right) \\ \sin(t)-\sin(t-1), & \text{pro } t \in \left\langle -\frac{\pi}{2}+1, \frac{\pi}{2}\right) \\ 1-\sin(t-1), & \text{pro } t \in \left\langle \frac{\pi}{2}, \frac{\pi}{2}+1\right) \\ 0, & \text{jinak} \end{cases}$$

3) $\langle 1, \sin x, \cos(x), \sin(2x), \cos(2x), \dots \rangle$

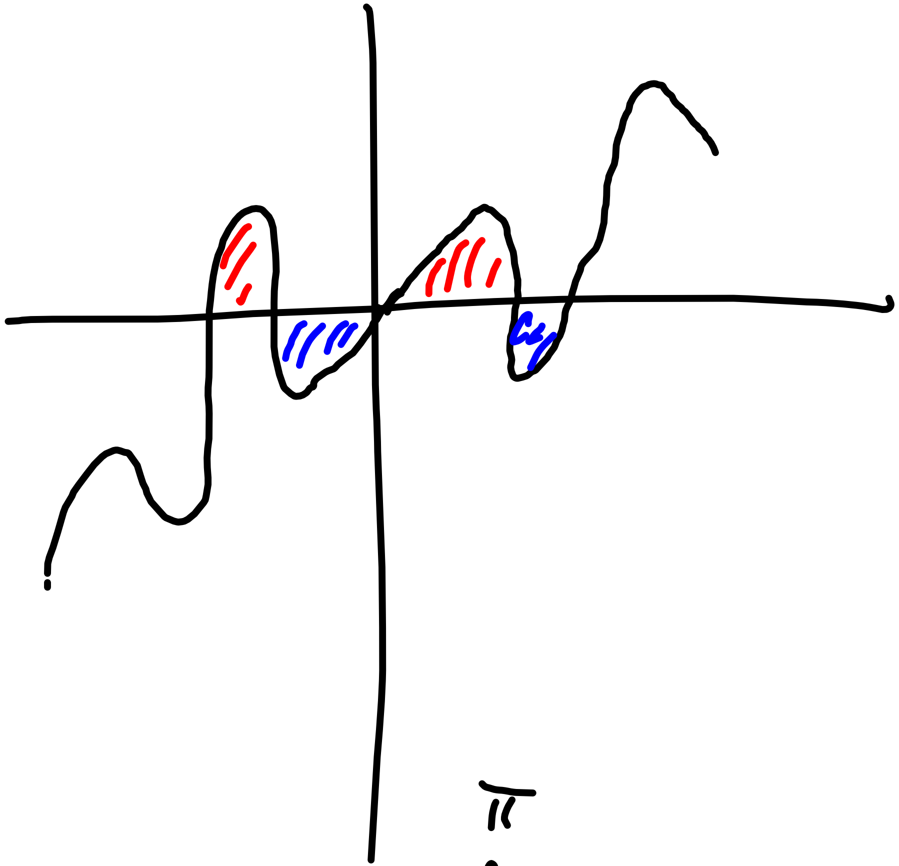
$$\int_{-\pi}^{\pi} x^2 \cdot 1 = \left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{2\pi^3}{3} \qquad \|1\| = 2\pi$$

$$x^2 \mp \frac{\pi^2}{3} + \sum_{i=1}^{n} \left(a_n \sin(nx) + b_n \cos(nx)\right)$$

$$\pi a_n = \int_{-\pi}^{\pi} x^2 \cdot \sin(nx)\, dx =$$

$$\begin{bmatrix} u = x^2 & u' = 2x \\ v' = \sin(nx) & v = -\frac{1}{n}\cos(nx) \end{bmatrix}$$

$$= \left[-\frac{x^2}{n}\cos(nx)\right]_{-\pi}^{\pi} + \frac{2}{n}\int_{-\pi}^{\pi} x \cos nx = 0$$



$$\pi b_n = \int_{-\pi}^{\pi} x^2 \cdot \cos(nx)\,dx =$$

$$\left[\begin{array}{ll} u = x^2 & u' = 2x \\ v' = \cos nx & v = \frac{1}{n}\sin(nx) \end{array}\right]$$

$$= \left[\frac{x^2}{n}\sin(nx)\right]_{-\pi}^{\pi} - \frac{2}{n}\int x \cdot \sin(nx)\,dx =$$

$$= -\frac{2}{\pi n}\left[\left[x\cdot\left(-\frac{1}{n}\right)\cos(nx)\right]_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi}\cos(nx)\,dx\right] =$$

$$= \frac{2}{n^2}\left[x\cos(nx)\right]_{-\pi}^{\pi} - \frac{2}{n^3}\left[\sin(nx)\right]_{-\pi}^{\pi} =$$

$$= 4\cdot\frac{(-1)^n}{n^2} \qquad b_n = \frac{(-4)^n}{n^2}$$

$$x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{(-4)^n}{n^2}\cos(nx)$$

$$\int \frac{1}{(x-1)(2x^2+x+1)}\, dx$$

$$\frac{1}{(x-1)(2x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{2x^2+x+1}$$

$$1 = A(2x^2+x+1) + (Bx+C)(x-1)$$

$$x=1: \quad 1 = 4A \Rightarrow A = \frac{1}{4}$$

$$x^0: \quad 1 = A - C \Rightarrow C = -\frac{3}{4}$$

$$x^2: \quad 0 = 2A + B \Rightarrow B = -\frac{1}{2}$$

$$x^1: \quad 0 = A - B + C$$

$$\int \frac{1}{(x-1)(2x^2+x+1)}\,dx = \frac{1}{4}\int \frac{1}{x-1}\,dx - \int \frac{\frac{1}{2}x + \frac{3}{4}}{2x^2+x+1}\,dx$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{8}\int \frac{4x+6}{2x^2+x+1}\,dx =$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{8}\left[\int \frac{4x+1}{2x^2+x+1}\,dx + \int \frac{5}{2x^2+x+1}\,dx\right] =$$

$$[t = 2x^2+x+1,\ dt = (4x+1)\,dx]$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{8}\ln(2x^2+x+1) - \frac{5}{16}\int \frac{1}{x^2+\frac{1}{2}x+\frac{1}{2}}\,dx =$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{8}\ln(2x^2+x+1) - \frac{5}{16}\int \frac{1}{(x+\frac{1}{4})^2+\frac{7}{16}}\,dx$$

$$= -\frac{5}{7}\int \frac{1}{\frac{16}{7}(x+\frac{1}{4})^2+1}\,dx = -\frac{5}{7}\int \frac{1}{(\frac{4}{\sqrt{7}}x+\frac{1}{\sqrt{7}})^2+1}\,dx = \frac{5\sqrt{7}}{28}\int \frac{1}{t^2+1}\,dt$$

$$t = \frac{4}{\sqrt{7}}x + \frac{1}{\sqrt{7}},\ dt = \frac{4}{\sqrt{7}}\,dx$$

$$= \frac{5\sqrt{7}}{28} \operatorname{arctg}\left(\frac{4}{\sqrt{7}}x + \frac{1}{\sqrt{7}}\right)$$

$$= \frac{1}{4}\ln(x-1) - \frac{1}{8}\ln(2x^2+x+1) - \frac{5\sqrt{7}}{28}\operatorname{arctg}\left(\frac{4}{\sqrt{7}}x+\frac{1}{\sqrt{7}}\right)$$