$$\lim_{h\to 0} \frac{\underbrace{\sqrt[3]{x+h}}_{a} - \underbrace{\sqrt[3]{x}}_{b}}{h} = \frac{\left(\sqrt[3]{x+h} - \sqrt[3]{x}\right)\left(a^2+ab+b^2\right)}{h\left(a^2+ab+b^2\right)} =$$

$$\underline{x^3 - y^3 = (x-y)(x^2+xy+y^2)}$$

$$\underline{x^2 - y^2 = (x-y)(x+y)}$$

$$x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + \dots + xy^{n-2} + y^{n-1})$$

$$a^2+ab+b^2 = \left(\sqrt[3]{x+h}\right)^2 + \left(\sqrt[3]{x+h}\right)\sqrt[3]{x} + \left(\sqrt[3]{x}\right)^2$$

$$= \lim_{h\to 0} \frac{x+\cancel{h}-x}{\cancel{h}\left((x+h)^{\frac{2}{3}} + (x+h)^{\frac{1}{3}}x^{\frac{1}{3}} + x^{\frac{2}{3}}\right)} =$$

$$= \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3}x^{-\frac{2}{3}}$$

$y = x^2$

$$\sum_{n=1}^{\infty} \frac{1}{2^n \ln(n) \cdot n^2} y^n$$

$$r = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{2^n \ln(n)\cdot n^2}}} = \frac{1}{\underbrace{\frac{1}{2} \cdot \lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{\ln(n)\cdot n^2}}}_{\to 1}}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{\ln(n)\cdot n^2}} = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{\ln(n)\cdot n^2}} =$$

$$\lim_{n\to\infty} \sqrt[n]{\ln(n)\cdot n^2} = \lim_{n\to\infty} (\ln(n))^{\frac{1}{n}} \lim_{n\to\infty} (n^2)^{\frac{1}{n}} =$$

$$= \lim_{n\to\infty} e^{\underbrace{\frac{1}{n}\ln(\ln(n))}_{\to 0}} \cdot \lim_{n\to\infty} e^{\underbrace{\frac{2\ln(n)}{n}}_{\to 0}}$$

$$\lim_{n\to\infty} \frac{\ln(\ln(n))}{n} = \lim_{n\to\infty} \frac{\frac{1}{n\ln(n)}}{1} = 0$$

$$\lim_{n\to\infty} \frac{2\ln(n)}{n} = \lim_{n\to\infty} \frac{\frac{2}{n}}{1} = 0$$

Řada konverguje pro $\underset{\underset{x^2}{\parallel}}{y} \in (-2,2) \rightarrow x \in (-\sqrt{2},\sqrt{2})$

$$\lim_{n\to\infty} \left| \frac{\frac{1}{2^{n+1}\ln(n+1)(n+1)^2} \cdot x^{2n+2}}{\frac{1}{2^n \cdot \ln(n)\, n^2} \cdot x^2} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\ln(n)\cdot n^2}{2\ln(n+1)(n+1)^2} x^2 \right| < 1$$

$$x^2 < \frac{1}{\lim_{n\to\infty} |\ldots|}$$

Zkoumáme konvergenci pro $x = \sqrt{2}$:

$$\sum_{n=2}^{\infty} \frac{1}{2^n \ln(n)\, n^2}\, 2^n = \sum_{n=2}^{\infty} \frac{1}{n^2 \ln(n)} < \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$\Rightarrow$ konverguje

Mocninná řada konverguje pro

$x \in \langle -\sqrt{2}, \sqrt{2} \rangle$

$$\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$$

$\sqrt{n+1} \quad \sqrt{n}$

$$\lim_{n\to\infty} \frac{e^{n^2}}{n!} = \lim_{n\to\infty} \frac{e^n \cdot e^n \cdot e^n \cdots e^n}{n\cdot(n-1)\cdot(n-2)\cdots 1} \geq \lim_{n\to\infty} e^n = \infty$$

2) $$\lim_{x\to 0} \frac{\arcsin x}{x} \underset{x=\sin y}{=} \lim_{y\to 0} \frac{\arcsin(\sin y)}{\sin y} =$$

$$= \lim_{y\to 0} \frac{y}{\sin y} = 1$$

3) $$\lim_{x\to\infty} x - \ln(x)\sqrt{x} = \lim_{x\to\infty} \sqrt{x}\,\underbrace{\left(\sqrt{x} - \ln x\right)}_{>1} =$$

$$= \infty$$

Uvážíme mocninou řadu $\sum_{n=1}^{\infty} a_n x^n$.

Podle odmocninového kriteria tato konverguje pro

$$\lim_{n\to\infty} \sqrt[n]{|a_n x^n|} < 1 \iff |x| < \frac{1}{\lim_{n\to\infty} \sqrt[n]{|a_n|}}$$

Podle podílového kriteria řada konverguje pro

$$\lim_{n\to\infty} \left| \frac{a_{n+1} x^{n+1}}{a_n x^n} \right| < 1 \iff |x| < \frac{1}{\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|}$$

$$\Rightarrow \frac{1}{\lim_{n\to\infty} \sqrt[n]{|a_n|}} = \frac{1}{\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|} \Rightarrow$$

$$\Rightarrow \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots + \frac{f^n(a)}{n!}(x-a)^n + \boxed{\frac{f^{n+1}(\xi)}{(n+1)!}(x-a)^{n+1}}$$

$\xi \in (a, x)$

$\underline{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots}$

$$f(x) = \sum_{i=0}^{\infty} a_i x^i \Rightarrow f(0) = a_0$$

$$f'(0) = a_1$$

$$f''(0) = 2a_2$$

$$f^3(0) = 6a_3 \Rightarrow a_3 = \frac{f^3(0)}{3!}$$

$\vdots$

$T_0^3(\sin x)$

T_a^k ... Taylorův polynom k-tého řádu v bodě a

$(\sin x)' = \cos x$

$(\sin x)'' = -\sin x$

$(\sin x)''' = -\cos x$

$$T_0^3(\sin x) = 0 + x - \frac{x^3}{6} + \frac{f^4(\xi)}{4!}x^4$$

pro $x = \frac{1}{2}$ odhaduji daný polynom fce $\sin x$ s přesností $\frac{1}{2^4 \cdot 4!}$

$$\left(\frac{e^x}{x}\right)' = -\frac{e^x}{x^2} + \frac{e^x}{x} \qquad f(x) = \frac{e^x}{x}$$

$$\left(\frac{e^x}{x}\right)'' = +\frac{2e^x}{x^3} - \frac{e^x}{x^2} + \left(-\frac{e^x}{x^2} + \frac{e^x}{x}\right) =$$

$$= \frac{2e^x}{x^3} - \frac{2e^x}{x^2} + \frac{e^x}{x}$$

$$T_1^2\left(\frac{e^x}{x}\right) = \underbrace{\frac{e^1}{1}}_{f(1)} + \underbrace{0 \cdot \underbrace{(x-1)}}_{f'(1)\cdot x} + \underbrace{\frac{e}{2}\underbrace{(x-1)^2}}_{f^{(2)}(1)\cdot(x-1)^2} =$$

$$= \frac{e}{2}x^2 - ex + \frac{3}{2}e$$

$$(x-2)^{\frac{2}{3}} = \sqrt[3]{(x-2)^2}$$

$$f'(x) = \frac{2}{3}(x-2)^{-\frac{1}{3}} = \frac{2}{3}\frac{1}{\sqrt[3]{(x-2)}}$$

Derivace v bodě $x=2$ není definována

$$f''(x) = -\frac{2}{9}\frac{1}{(x-2)^{\frac{4}{3}}}$$

Druhá derivace není def.

Kritický bod $x=2$ ... nabývá minimum.

V krajních bodech intervalu (dosazení) zjistíme, že nabývá maximum fce na daném intervalu

$$f(x) = x^2 - \frac{2008}{x}$$

$$f'(x) = 2x + \frac{2008}{x^2} = 0$$

$$\Leftrightarrow 2x^3 + 2008 = 0$$

$$\Leftrightarrow x = -\sqrt[3]{1004}$$

jedná se o minimum