$$V = \pi r^2 \cdot v = 64 \Rightarrow v = \frac{64}{\pi r^2}$$

$$P = 2\pi r \cdot v + 2\pi r^2 =$$

$$= \frac{128}{r} + \pi r^2$$

$\mathcal{D} = \mathbb{R} - \{-1, \frac{1}{2}\}$

	$(-\infty, -1)$	$(-1, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
f	$+$	$-$	$+$

$$f'(x) = \frac{(2x-1)(2x^2+x-1) - (x^2-x+2)(4x+1)}{(2x^2+x-1)^2} =$$

$$= \frac{3x^2-10x-1}{(2x^2+x-1)^2} = 0$$

$$x_{1,2} = \frac{5}{3} \pm \frac{2\sqrt{7}}{3}$$

	$(-\infty, -1)$	$(-1, x_1)$	$(x_1, \frac{1}{2})$	$(\frac{1}{2}, x_2)$	(x_2, ∞)
f'	$+$	$+$	$-$	$-$	$+$
f	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

f má v x_1 maximum, v x_2 minimum

$$f''(x) = C \cdot \frac{x^3 - 5x^2 - x - 1}{(2x^2 + x - 1)^3} \overset{?}{=} 0$$

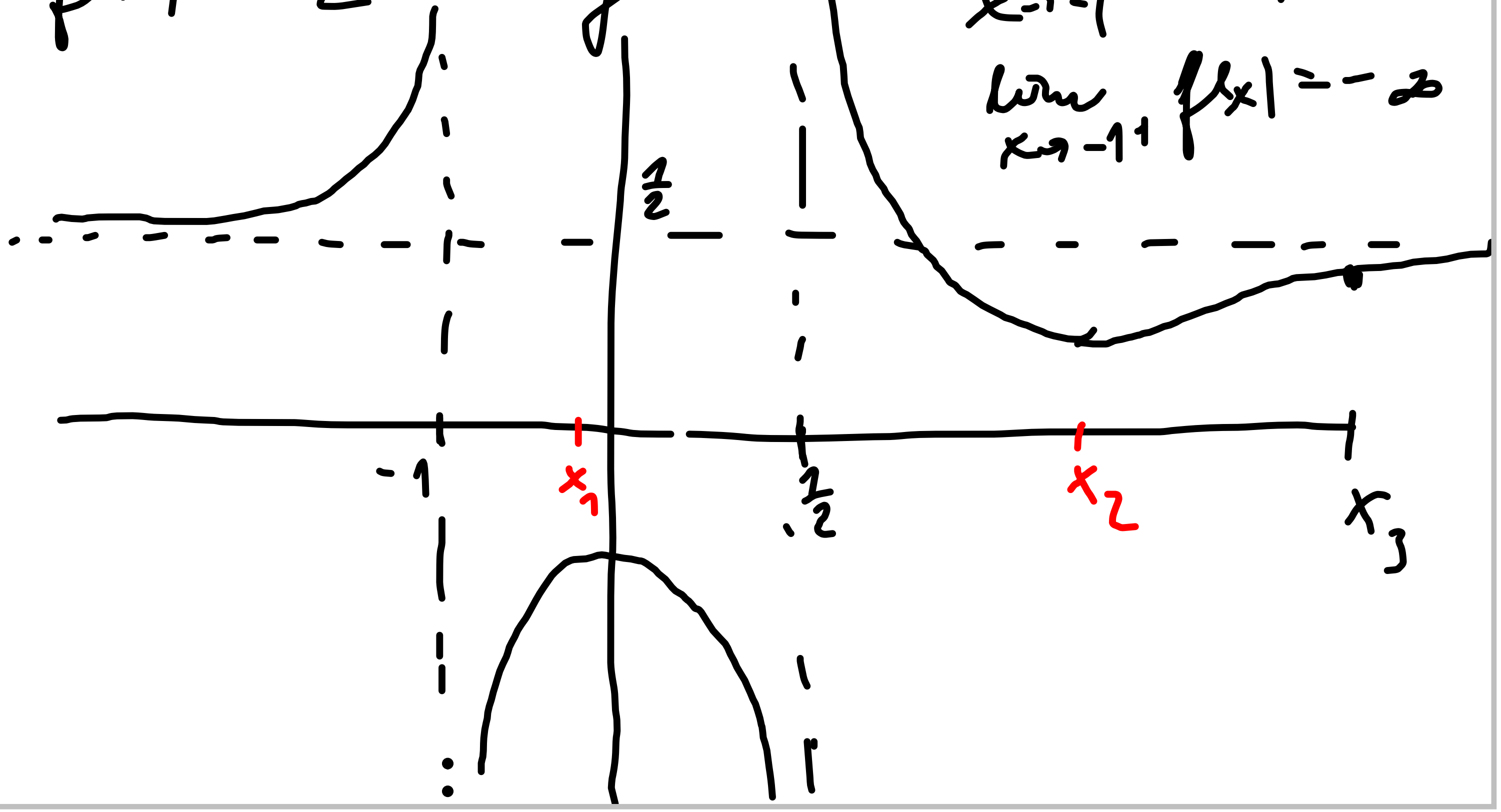
$x_3 = 5{,}\ldots$

asymptoty: $x = -1$, $x = \frac{1}{2}$ bez směrnice

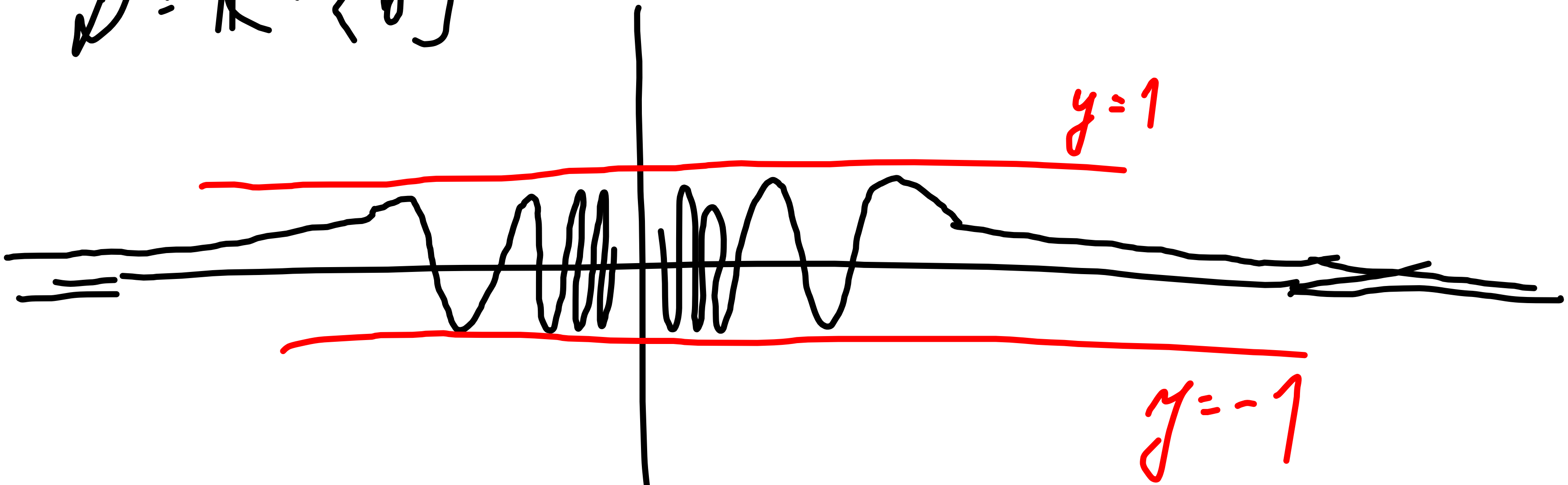
$\lim\limits_{x \to \infty} f(x) = \frac{1}{2} \Rightarrow y = \frac{1}{2}$

$\lim\limits_{x \to -1^-} f(x) = \infty$

$\lim\limits_{x \to -1^+} f(x) = -\infty$



$D = \mathbb{R} - \{0\}$



nulové body $\sin\left(\frac{1}{x^2}\right) = 0 \iff \frac{1}{x^2} = k\pi \quad k \in \mathbb{N}_0$

$$x = \pm \frac{1}{\sqrt{k\pi}}$$

$$f'(x) = -\frac{2}{x^3}\cos\left(\frac{1}{x^2}\right) = 0 \iff \cos\left(\frac{1}{x^2}\right) = 0$$

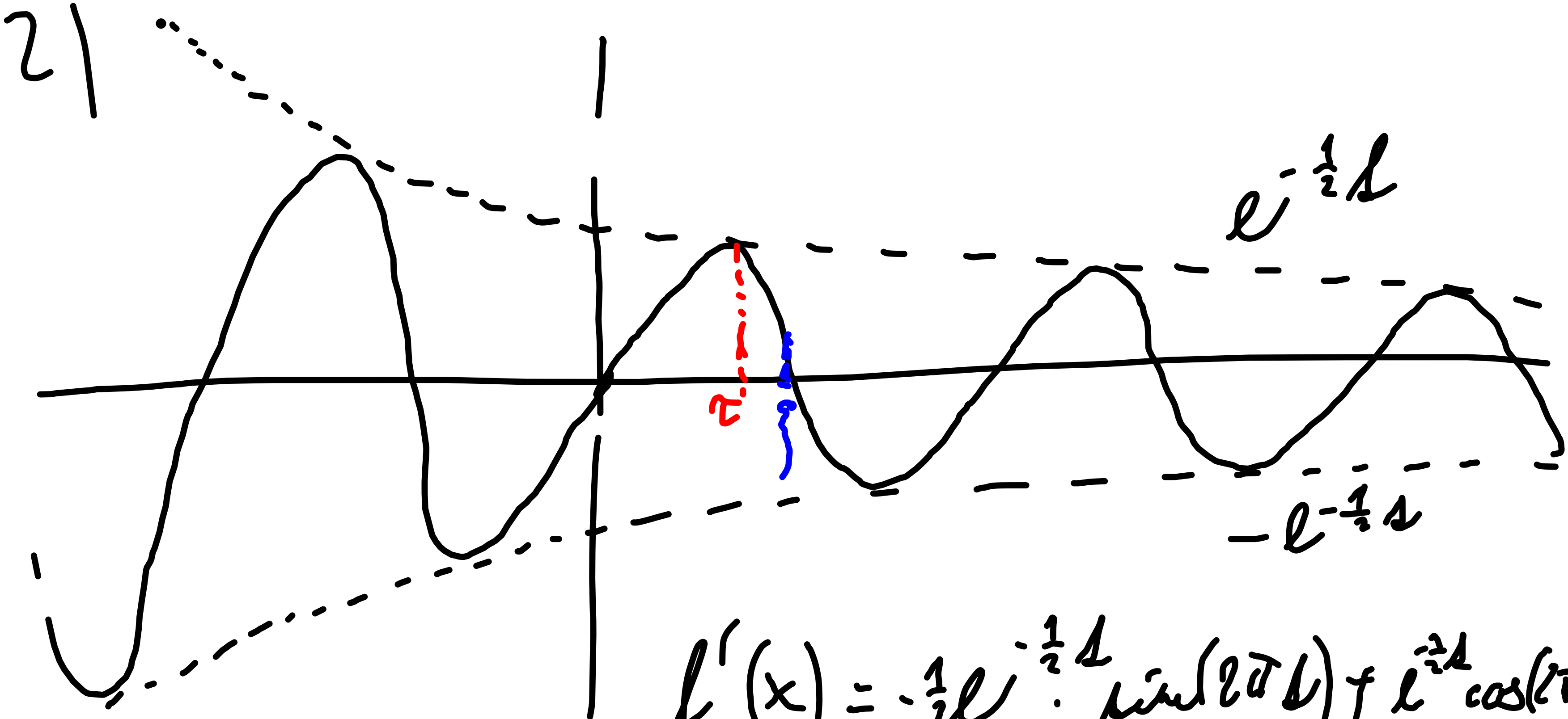
$$\iff \frac{1}{x^2} = \frac{\pi}{2} + k\pi \quad k \in \mathbb{N}_0$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{\frac{\pi}{2} + k\pi}} \quad k \in \mathbb{N}_0$$

asymptota bez směrnice není

se směrnicí: $y = 0$

2)



$$f'(x) = -\tfrac{1}{2}e^{-\frac{1}{2}t}\sin(2\pi t) + e^{-\frac{1}{2}t}\cos(2\pi t)\cdot 2\pi$$

$= 0 \iff \tan(2\pi t) = 4\pi \qquad \tan(2\pi\xi) = 4\pi$

$\implies t = \tau + \frac{k}{2},\ k \in \mathbb{Z}$

$$\int \frac{1}{x}\,dx = \ln x + C \qquad \text{pro } x > 0$$

$$\int \frac{1}{x}\,dx = -\int \frac{1}{|x|}\,dx = \qquad \text{pro } x < 0$$

$$-\ln|x| + C$$

$$\int g(f(x))\, f'(x)\cdot dx = \int g(u)\, du$$

$$\int \operatorname{tan} x\, dx = \int \frac{\sin x}{\cos x}\, dx = -\int \frac{1}{u}\, du =$$

$$\begin{bmatrix} u = \cos x \\ du = -\sin x\, dx \end{bmatrix}$$

$$= -\ln|u| + C = -\ln(\cos x) + C$$

$$\int \sin^2 x\,dx = \qquad \int u' v = uv - \int u v'$$

$$\left[\begin{matrix} \sin x = u' \\ \sin x = v \end{matrix}\right]$$

$$\int g(f(x))\cdot f'(x)\,dx = \left[\begin{matrix} u = f(x) \\ du = f'(x)\,dx \end{matrix}\right] = \int g(u)\,du$$

$$= -\cos x \sin x + \int \cos^2 x\,dx =$$

$$= -\cos x \sin x + \int (1-\sin^2 x)\,dx =$$

$$= -\cos x \sin x + x - \int \sin^2 x\,dx$$

$$\Rightarrow \int \sin^2 x = \frac{1}{2}\left(x - \cos x \sin x \right)$$

$$\int \arcsin x\,dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx =$$

$$\left[\begin{matrix} u' = 1 \\ v = \arcsin x \end{matrix}\right] \qquad \left[\begin{matrix} u = x^2 \\ du = 2x\,dx \end{matrix}\right]$$

$$= x \arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{1-u}}\,du =$$

$$= x \arcsin x + \sqrt{1-x^2}$$

Název: IV 15-19:01 (7 z 12)

$$\frac{x^3-1}{x^2+1} = x - \frac{x+1}{x^2+1}$$

$$x^3-1 : (x^2+1) = x$$

$$\frac{x^3+x}{\boxed{-x-1}}$$

$$\int \frac{2x}{x^2-4x+3}\,dx$$

rozložíme na parc. zlomky:

i) rozložíme jmenovatele:

$$x^2-4x+3 = (x-1)(x-3)$$

ii) $$\frac{2x}{x^2-4x+3} = \frac{A}{(x-1)} + \frac{B}{(x-1)} \qquad A, B \in \mathbb{R}$$

$$2x = A(x-3) + B(x-1)$$

$x=1 \quad 2 = -2A \Rightarrow A = -1$

$x=3 \quad 6 = 2B \Rightarrow B \Rightarrow 3$

$$\int \frac{2x}{x^2-4x+3}\,dx = -\int \frac{dx}{(x-1)} + 3\int \frac{dx}{(x-3)} =$$

$$= -\ln(x-1) + 3\ln(x-3)$$

$\underbrace{(x-\alpha)(x-\bar{\alpha})} \in \mathbb{R}(x)$ $\quad = (x^4-1) = (x-1)(x^3+x^2+x+1)$

$(x^2-1)(x^2+1)$

$$x^3+x^2+x+1 = (x+1)(x^2+1)$$

$$\frac{x^2+x+1}{x^3+x^2+x+1} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)}$$

$$x^2+x+1 = A(x^2+1)+(Bx+C)(x+1)$$

$$x=-1: \quad 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$x=0: \quad 1 = A + C \Rightarrow C = \frac{1}{2}$$

$$x=1: \quad 3 = A\cdot 2 + (B+C)(2) = 1 + 2B + 1 \Rightarrow B = \frac{1}{2}$$

$$\int \frac{x^2+x+1}{x^3+x^2+x+1}\,dx = \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{x+1}{x^2+1}\,dx =$$

$$= \frac{1}{2}\ln(x+1) + \frac{1}{2}\int \frac{x\,dx}{x^2+1}_{[u=x^2]} + \frac{1}{2}\int \frac{1}{x^2+1}\,dx =$$

$$= \frac{1}{2}\ln(x+1) + \frac{1}{4}\int \frac{1}{u+1}\,du + \frac{1}{2}\operatorname{arctg}(x) =$$

$$= \frac{1}{2}\ln(x+1) + \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\operatorname{arctg}(x)$$

$$\int \frac{1}{(x^2+1)^2}\,dx = K_2$$

$$\operatorname{arctg}(x) = \int \frac{1}{(x^2+1)}\,dx = \frac{x}{x^2+1} + \int \frac{2x^2}{(x^2+1)^2}\,dx$$

$$\left[\begin{array}{l} u' = 1 \\ v = \frac{1}{x^2+1} \end{array}\right] = \frac{x}{x^2+1} + 2\left(\int \frac{1\,dx}{x^2+1} - K_2\right)$$

$$\frac{x^2}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} = \frac{1}{x^2+1} - \frac{1}{(x^2+1)^2}$$

$$x^2 = (Ax+B)(x^2+1) + Cx + D$$

x^0 : $B + D = 0$

x : $A + C = 0$

x^2 : $B = 1 \Rightarrow D = -1$

x^3 : $A = 0 \Rightarrow C = 0$

$$K_2 = \frac{1}{2}\left(\operatorname{arctg}(x) + \frac{x}{x^2+1}\right)$$

$$\int \frac{dx}{x^2+x+3} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \frac{11}{4}} = \frac{4}{11}\int \frac{dx}{\frac{4}{11}\left(x+\frac{1}{2}\right)^2+1} =$$

$$= \frac{4}{11}\int \frac{dx}{\underbrace{\left(\frac{2}{\sqrt{11}}x + \frac{1}{\sqrt{11}}\right)}_{u}^2+1} = \frac{2}{\sqrt{11}}\int \frac{du}{u^2+1} =$$

$$du = \frac{2}{\sqrt{11}}\,dx$$

$$= \frac{2}{\sqrt{11}}\operatorname{arctg} u = \frac{2}{\sqrt{11}}\operatorname{arctg}\left(\frac{2}{\sqrt{11}}x + \frac{1}{\sqrt{11}}\right)$$