$$\lim_{n \to -\infty} \left[\left(\frac{2^{n} - 2^{-n}}{2^{n} + 2^{-n}} \right) \cdot \left(\frac{2^{n}}{2^{n}} \right) \right] = \lim_{n \to -\infty} \left(\frac{2^{2n} - 1}{2^{2n} + 1} \right) =$$

$$= \lim_{n \to \infty} \left(\frac{2^{-2n} - 1}{2^{-2n} + 1} \right) = \lim_{n \to \infty} \frac{\frac{1}{2^{2n}} - 1}{\frac{1}{2^{2n}} + 1} = \frac{0 - 1}{0 + 1} =$$

$$= -1$$

$$\lim_{x\to 2}\left(\frac{x-2}{\sqrt{x^2-4}}\cdot\frac{\sqrt{x^2-4}}{\sqrt{x^2-4}}\right)=\lim_{x\to 2}\frac{(x-2)\sqrt{x^2-4}}{x^2-4}=$$

$$=\lim_{x\to 2}\frac{\cancel{(x-2)}\sqrt{x^2-4}}{(x+2)\cancel{(x-2)}}=\lim_{x\to 2}\frac{\sqrt{x^2-4}}{x+2}=\frac{0}{4}=0$$

$$\boxed{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1}$$

$$\lim_{x\to 0}\left(\frac{\sin(\sin(x))}{x}\cdot\frac{\sin(x)}{\sin(x)}\right)=$$

$$=\lim_{x\to 0}\left(\frac{\sin\sin x}{\sin x}\cdot\frac{\sin x}{x}\right)=$$

$$=\lim_{x\to 0}\frac{\sin\sin x}{\sin x}\cdot\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\sin\sin x}{\sin x}=\{y=\sin x\}=\lim_{y\to 0}\frac{\sin y}{y}=1$$

$$\lim_{x\to 0} \frac{\sin^2(x)}{x} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right) \cdot \sin x =$$

$$\underbrace{\lim_{x\to 0} \frac{\sin x}{x}}_{1} \cdot \underbrace{\lim_{x\to 0} \sin x}_{0} = 0$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\boxed{\frac{1}{x^2}} = x^{-2} = f$$

$$\boxed{x^{-2} = -2x^{-2-1} = -\frac{2}{x^3}}$$

$$\left(\frac{1}{x^2}\right)' = \lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0}\frac{\frac{1}{x^2+2xh+h^2}-\frac{1}{x^2}}{h} =$$

$$= \lim_{h\to 0}\frac{x^2-(x^2+2xh+h^2)}{x^2(x^2+2xh+h^2)\cdot h} = \lim_{h\to 0}\frac{-2xh+h^2}{(x^4+2x^3h+h^2x^2)h} =$$

$$= \lim_{h\to 0}\frac{-2x+h}{x^4+2x^3h+h^2x^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

$\cos x$

$$\cos' x = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} =$$

$$\lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \leftarrow$$

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$e^{i(\varphi+\psi)} = e^{i\varphi} e^{i\psi}$$

$$\underline{\cos(\varphi+\psi)} + i \sin(\varphi+\psi) =$$

$$= (\cos\varphi + i\sin\varphi)\cdot(\cos\psi + i\sin\psi) =$$

$$= \underline{(\cos\varphi\cos\psi - \sin\varphi\sin\psi)} + i \,\_\_\_\_$$

$$= \lim_{h\to 0} \Bigg(\underbrace{\frac{\cos x(\cos h - 1)}{h}}_{0} - \underbrace{\frac{\sin x \sin h}{h}}_{-\sin x} \Bigg)$$

$$\lim_{h\to 0} \frac{\cos x(\cos h - 1)}{h} = \cos x \lim_{h\to 0} \frac{\cos h - 1}{h} = \leftarrow$$

$$\cos h \geq \cos^2 h$$

$$0 \leq 1 - \cos h \leq 1 - \cos^2 h = \sin^2 h$$

$h > 0$

$$0 \leq \frac{1 - \cos h}{h} \leq \frac{\sin^2 h}{h}$$

$$\lim_{h\to 0} \frac{\sin^2 h}{h} = 0 \Rightarrow \underline{\lim_{h\to 0} \frac{1 - \cos h}{h} = 0}$$

$$\boxed{(f\circ g)'(x) = f'(g(x))\cdot g'(x)}$$

1. $$\underbrace{\left(\frac{x}{1+x}\right)^5}$$

$$x \xrightarrow{g} \underbrace{\frac{x}{1+x}=y}\xrightarrow{f} y^5$$

$$g(x)=\frac{x}{1+x} \qquad f(y)=y^5$$

$$\left(\left(\frac{x}{1+x}\right)^5\right)' = 5y^4\cdot\frac{1}{(1+x)^2} =$$

$$= 5\cdot\left(\frac{x}{1+x}\right)^4\cdot\frac{1}{(1+x)^2} = \frac{5x^4}{(1+x)^6}$$

$$\left(\sqrt{\frac{x-1}{x+1}}\right)'$$

$$x \xrightarrow{g} \frac{x-1}{x+1}=y \xrightarrow{f} \sqrt{y}$$

$$g(x)=\frac{x-1}{x+1} \qquad f(y)=y^{\frac{1}{2}}$$

$$\| \quad f\circ g(x)$$

$$\| \quad f'(g(x))\cdot g'(x)$$

$$= \frac{1}{2}y^{-\frac{1}{2}}\cdot\frac{2}{(x+1)^2} = \frac{1\cdot\frac{2}{(x+1)^2}}{2\sqrt{\frac{x-1}{x+1}}} =$$

$$\left(\frac{x-1}{x+1}\right)' = \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$\Big| \; = \ldots$$

$f(x) = x^3 - x^2 + 3x - 5$

$f'(x) = 3x^2 - 2x + 3$

$3x^2 - 2x + 3 = 0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4 - 36}}{6} \notin \mathbb{R}$

$+$

x

$(-\infty, \infty)$ .... roste

$f'(0) = 3$

V: v okolí x

$f'(x) > 0$

$\Rightarrow f$ roste

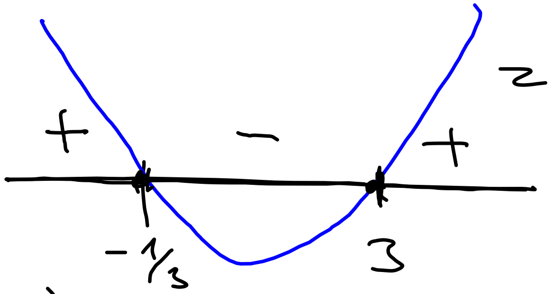
$f'(x) < 0$

$\Rightarrow f$ klesá

$f = x^3 - 4x^2 - 3x + 10$

$f'(x) = 3x^2 - 8x - 3$

$3x^2 - 8x - 3 = 0 \qquad x_{1,2} = \dfrac{8 \pm \sqrt{64+36}}{6} = \dfrac{8 \pm 10}{6} = \begin{matrix} 3 \\ -\frac{1}{3} \end{matrix}$



$f'(-10) = 300 + 80 - 3 > 0$

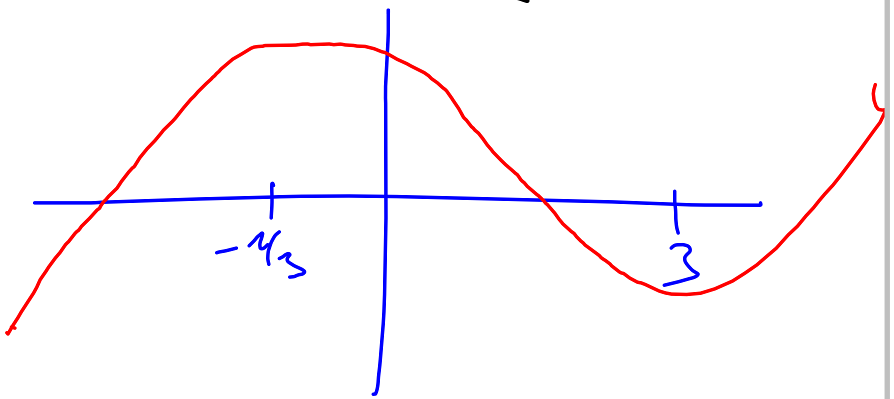
$f'(0) = -3 < 0$

$f'(10) = 300 - 80 - 3 > 0$

$(-\infty, -1/3)$ ... roste

$(-1/3, 3)$ ... klesá

$(3, \infty)$ ... roste



$$\frac{1}{x} = x^{-1} = f \qquad D_f: \mathbb{R} \setminus \{0\}$$

$$f'(x) = \frac{-1}{x^2} \qquad \frac{-1}{x^2} = 0 \quad \text{nikdy}$$

$$\begin{array}{ccc} - & & - \\ \hline & 0 & \end{array}$$

$$f'(-10) = \frac{-1}{100} < 0$$

$$f(10) = \frac{-1}{100} < 0$$

$(-\infty, \infty) \setminus \{0\}$ ... klesající

$(-\infty, 0)$ ... klesající

$(0, \infty)$ ... klesající

$$(a, b] = \{x \mid a \le x < b\}$$

$$(f^{-1} \circ f) = id$$

$$(f^{-1})'(f(x)) \cdot f'(x) = 1$$

$$\boxed{(f^{-1})'(f(x)) = \frac{1}{f'(x)}}$$

$\arccos' x$

$f = \cos x$

$f^{-1} = \arccos x$

$$(\arccos)'(\cos x) = \frac{1}{-\sin x} =$$

$$= \frac{-1}{\sqrt{1-\cos^2 x}}$$

$\{ y = \cos x \}$

$$\boxed{\arccos' y = \frac{-1}{\sqrt{1-y^2}}}$$

arctg x

$$\operatorname{tg}' x = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos x \cos x + \sin x \sin x}{\cos^2 x} =$$

$$= \frac{1}{\cos^2 x}$$

$f = \operatorname{tg} x$

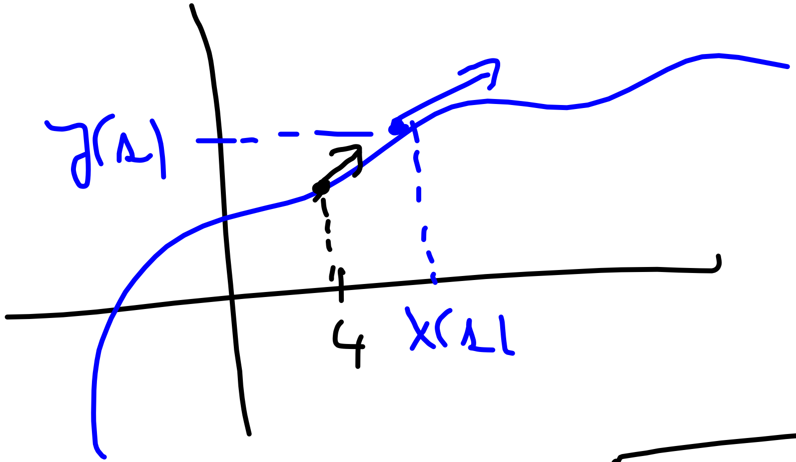
$f^{-1} = \operatorname{arctg} x$

$$\left(\operatorname{tg} x = \frac{\sin x}{\cos x} \qquad \cos^2 x = \operatorname{tg}^2 x \sin^2 x\right)$$

$$(\operatorname{arctg})'(\operatorname{tg} x) = \frac{1}{\operatorname{tg}' x} = \cos^2 x =$$

$$= \operatorname{tg}^2 x \cdot \sin^2 x =$$

$$= \ldots\ldots$$



$$\boxed{y = x^3 - 3x + 5} \qquad \boxed{x = \frac{1}{2}\sqrt{t} + 3}$$

$$x(1) = \frac{1}{2}\sqrt{1} + 3$$

$$x(4) = \frac{1}{2}\sqrt{4} + 3 = \underline{4}$$

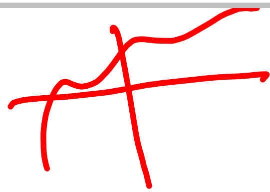
$$y(4) = 4^3 - 3 \cdot 4 + 5 = \ldots$$

$$y'(x) = 3x^2 - 3$$

$$y'(4) = 3 \cdot 16 - 3 = 48 - 3 = \underline{45}$$

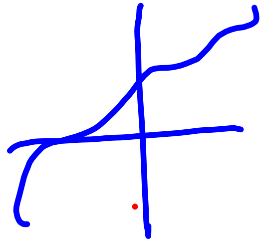
$$x = t^2 - t + 1$$

$$y = t^3 + 2t^2 - t + 1$$



$t = 2$

$$\begin{cases} x' = 2t - 1 \\ y' = 3t^2 + 4t - 1 \end{cases}$$



$$(x, y)(2) = (3, 15)$$

$$(x', y')(2) = (3, 19)$$

$$x = 3 + 3t \Rightarrow t = \frac{x-3}{3}$$

$$y = 15 + 19t$$

$$y = 15 + 19 \cdot \frac{x-3}{3}$$

$$3y = 45 + 19x - 57$$

$$3y - 19x = -12$$

$$y = \frac{-12 + 19x}{3}$$