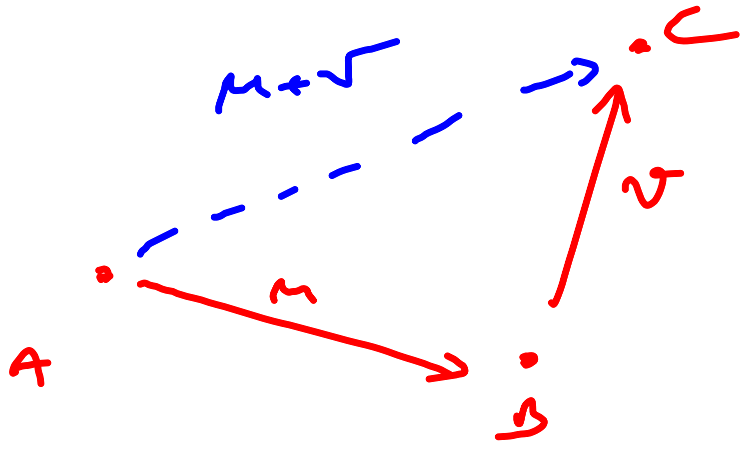
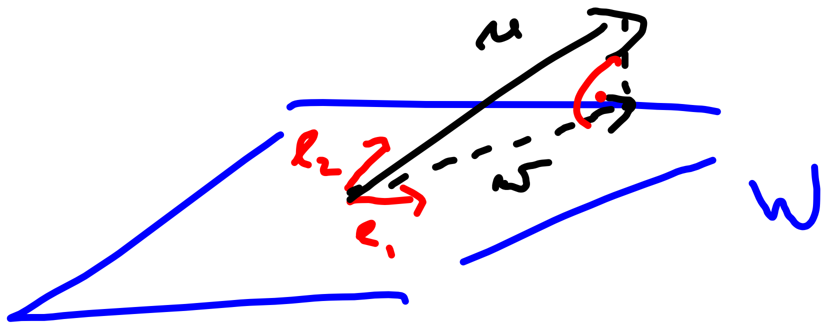
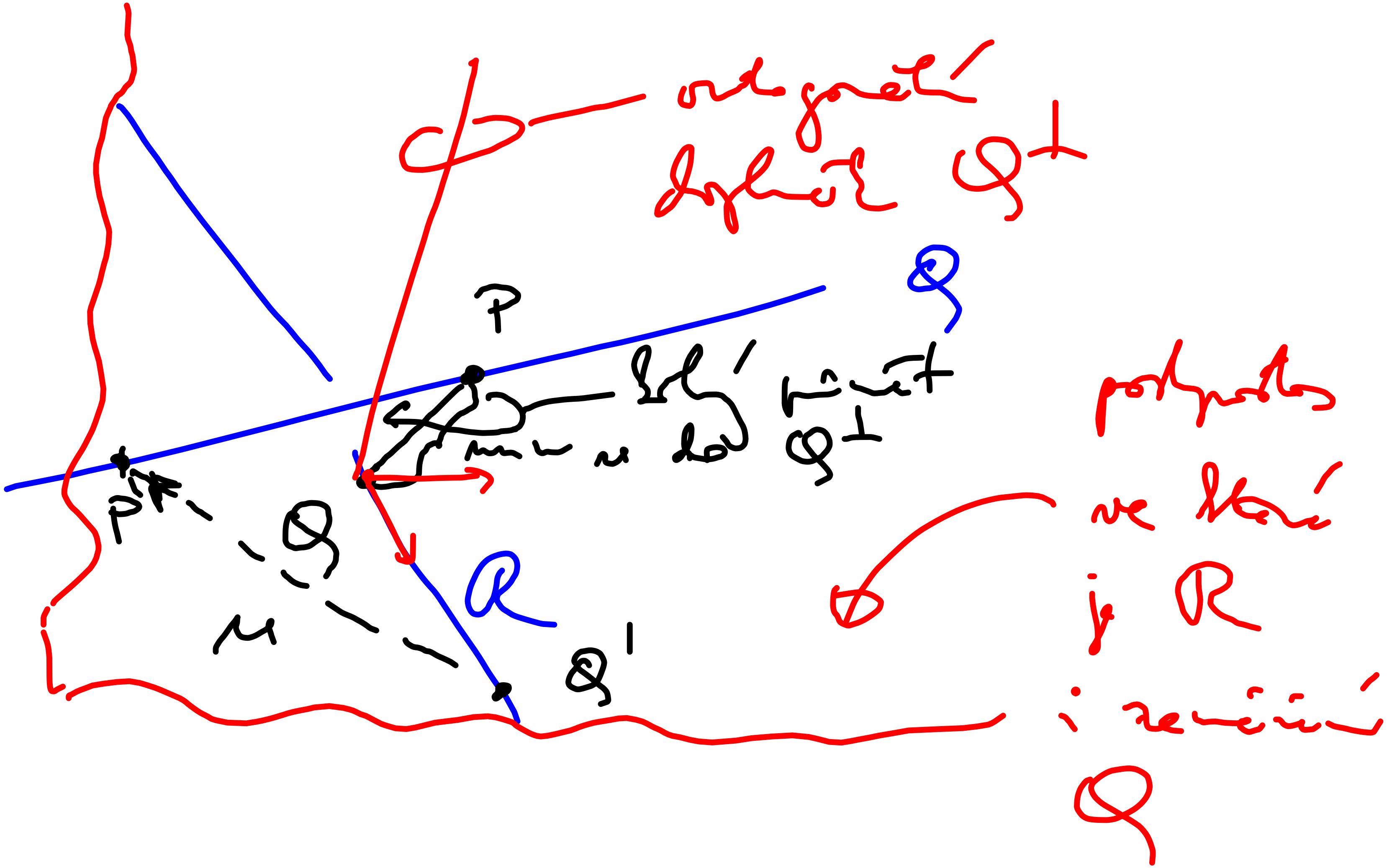
trojúhelníková nerovnost:



průmět do podprostoru $W = \langle e_1, \dots, e_k \rangle \subset V$





ortogonální doplněk Q⊥
P
Q
R
P'
Q'
M
podprostor
Q

Př.

$$p: X \equiv [1,-1,0] + t(-1,2,3)$$

$$q: X \equiv [2,5,-1] + t(-1,-2,1)$$

$$V = \langle (-1,2,3), (-1,-2,1) \rangle$$

$$V^{\perp} = \langle \omega \rangle \quad \begin{aligned} -x+2y+3z &= 0 \\ -x-2y+z &= 0 \end{aligned} \quad \Bigg| \quad \begin{aligned} -x+2y+3z &= 0 \\ +4y+2z &= 0 \end{aligned}$$

$\Rightarrow$ řeší $\omega = (4,-1,2)$ ✓

$$V = \left\langle \frac{(4,-1,2)}{\|\omega\|} \right\rangle$$

$$u = [1,-1,0] - [2,5,-1] = (-1,-6,1)$$

$$u \cdot \frac{\omega}{\|\omega\|} = \frac{(-1,-6,1)\cdot(4,-1,2)}{(16+1+4)^{1/2}} = \underline{\underline{\frac{4}{\sqrt{21}}}}$$ ✓

Osa leží v rovině dané p a n,

(spojnice realizující vzdálenost)

tj. v rovině dané $\alpha: X = [1,-1,0] + t(-1,2,3) + s(4,-1,2)$

Průnik α a přímky q je jeden bod

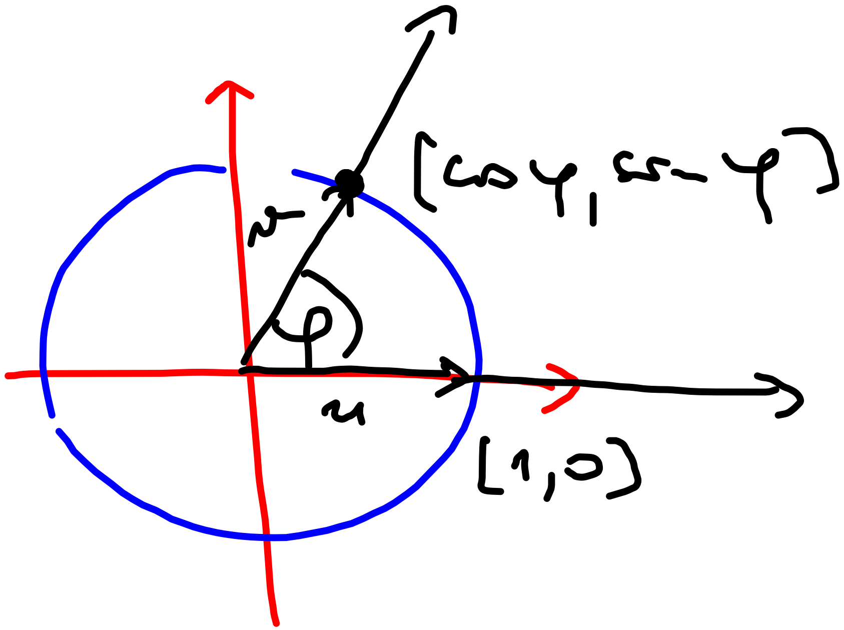
osy [illegible].

$$\begin{aligned}
x&: \quad 1 - t + 4s = 2 - r\\
y&: \quad -1 + 2t - 1s = 5 - 2r\\
z&: \quad +3t + 2s = -1 + r
\end{aligned}$$

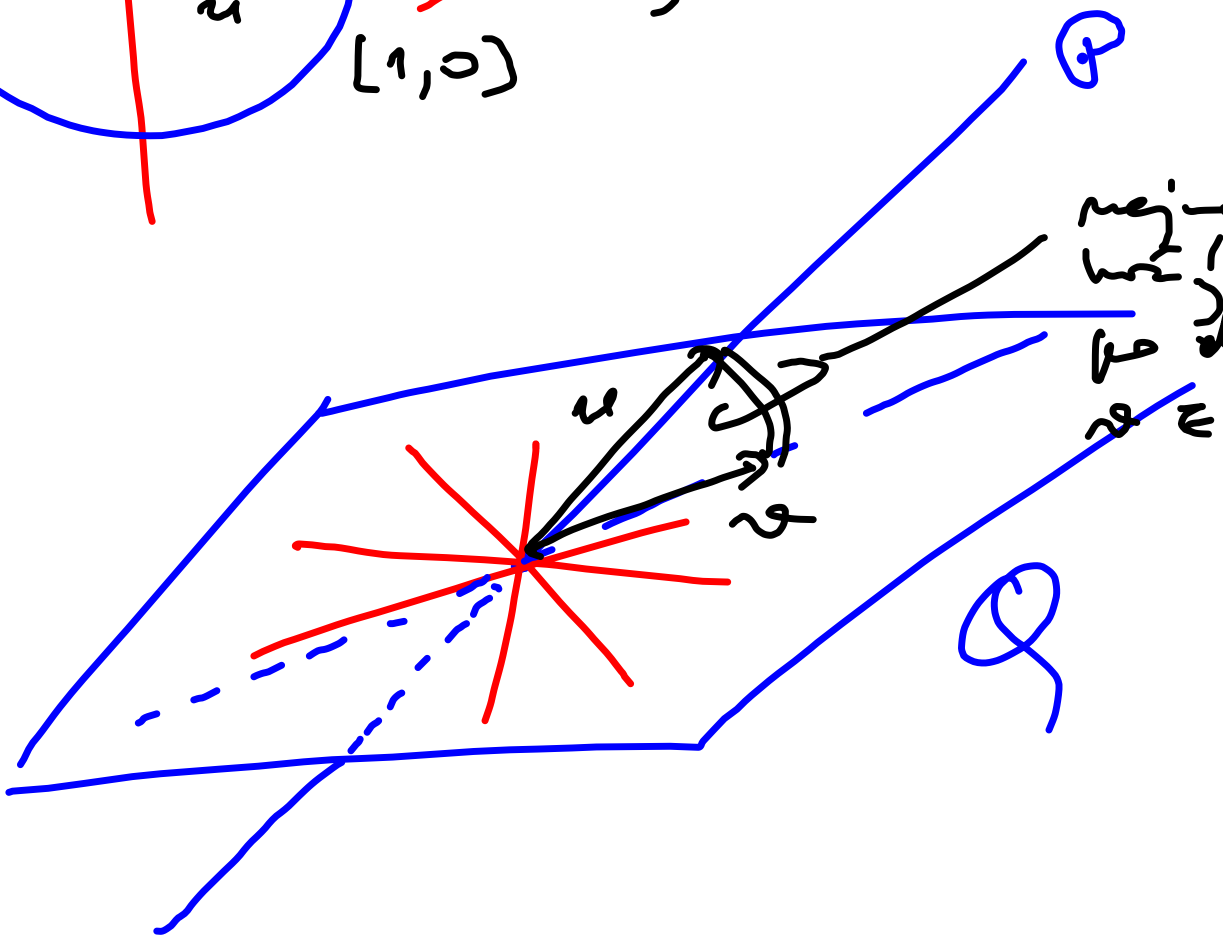
$$\left(\begin{array}{ccc|c} -1 & +4 & +1 & 1\\ +2 & -1 & +2 & 6\\ +3 & +2 & -1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -1 & -1\\ 0 & 7 & 4 & 8\\ 0 & 14 & 2 & 2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & -1\\ 0 & 7 & 4 & 8\\ 0 & 0 & -6 & -14 \end{array}\right) \Rightarrow \left[0, \frac{4}{21}, \frac{14}{6}\right]$$

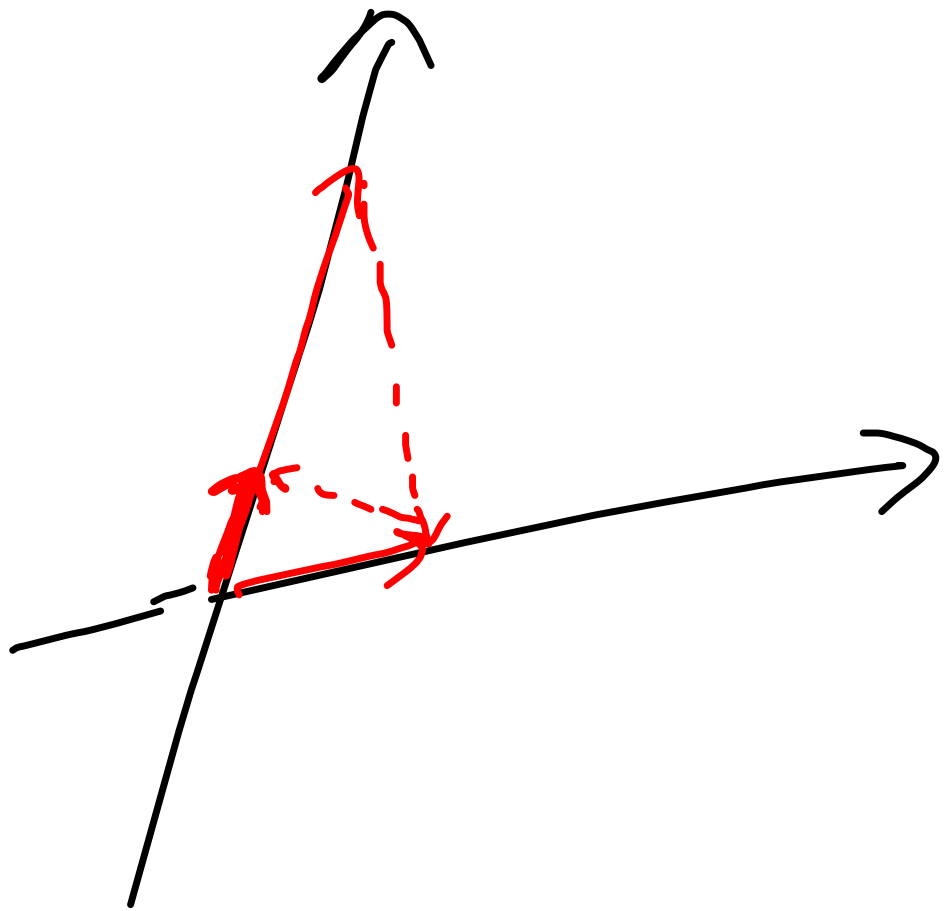
$$\frac{1}{7}\cdot\left(8 - \frac{4\cdot 14}{6}\right) = \frac{8}{7\cdot 6} = \frac{4}{21}$$

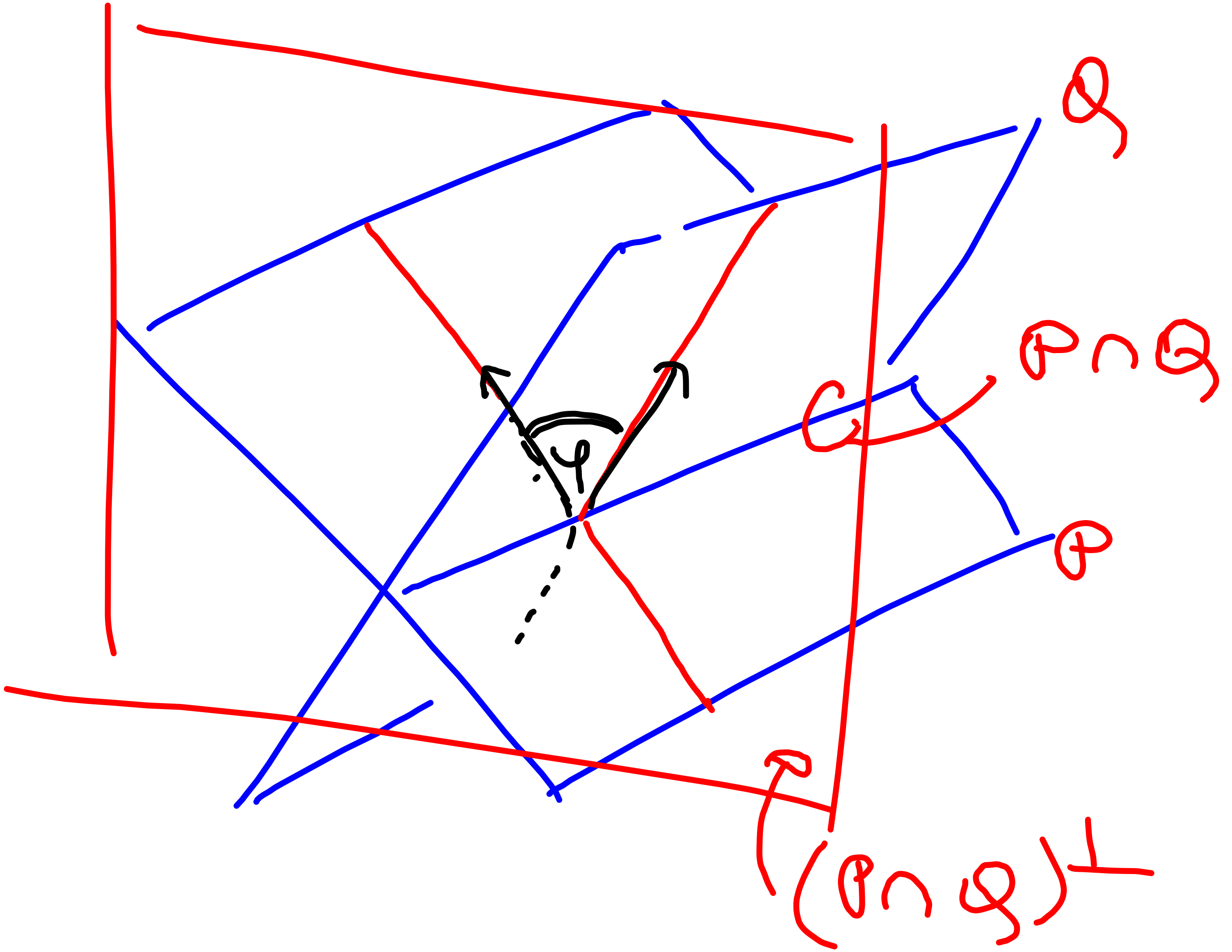


$$\frac{u}{\|u\|} \cdot \frac{v}{\|v\|} = \cos\varphi$$



nejmenší z úhlů [illegible] po dvojicích $u \in P$ $v \in Q$




Q
P ∩ Q
φ
P
(P ∩ Q)⊥

Př. 1) $[1,1,1] = P$

$\varrho: \ 1x + 2y + 3z - 3 = 0$

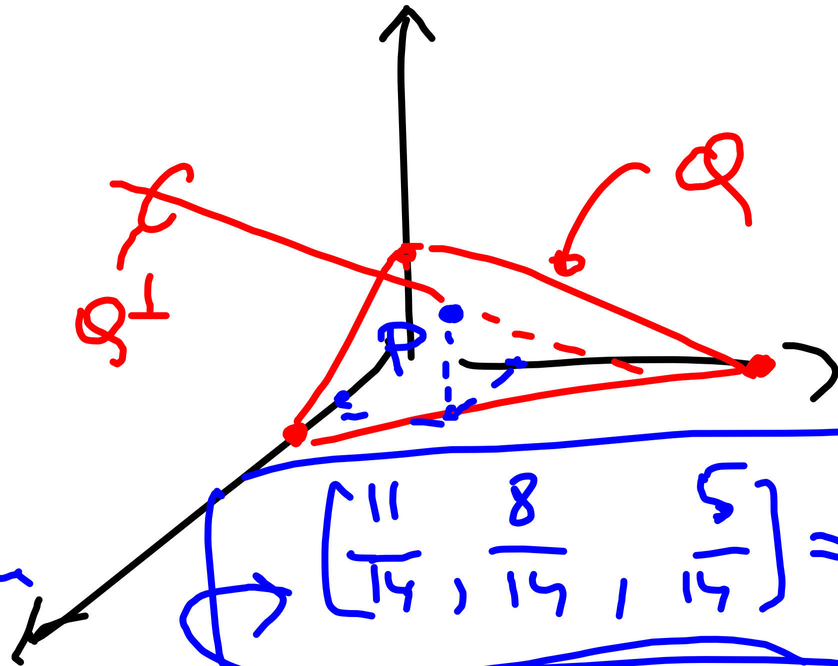
$\varrho^{\perp} = \langle (1,2,3) \rangle$, $(1,2,3) =: v$

$Q \in \varrho, \ Q = [3,0,0]$

$\boxed{P - Q} = (-2, 1, 1) =: u$

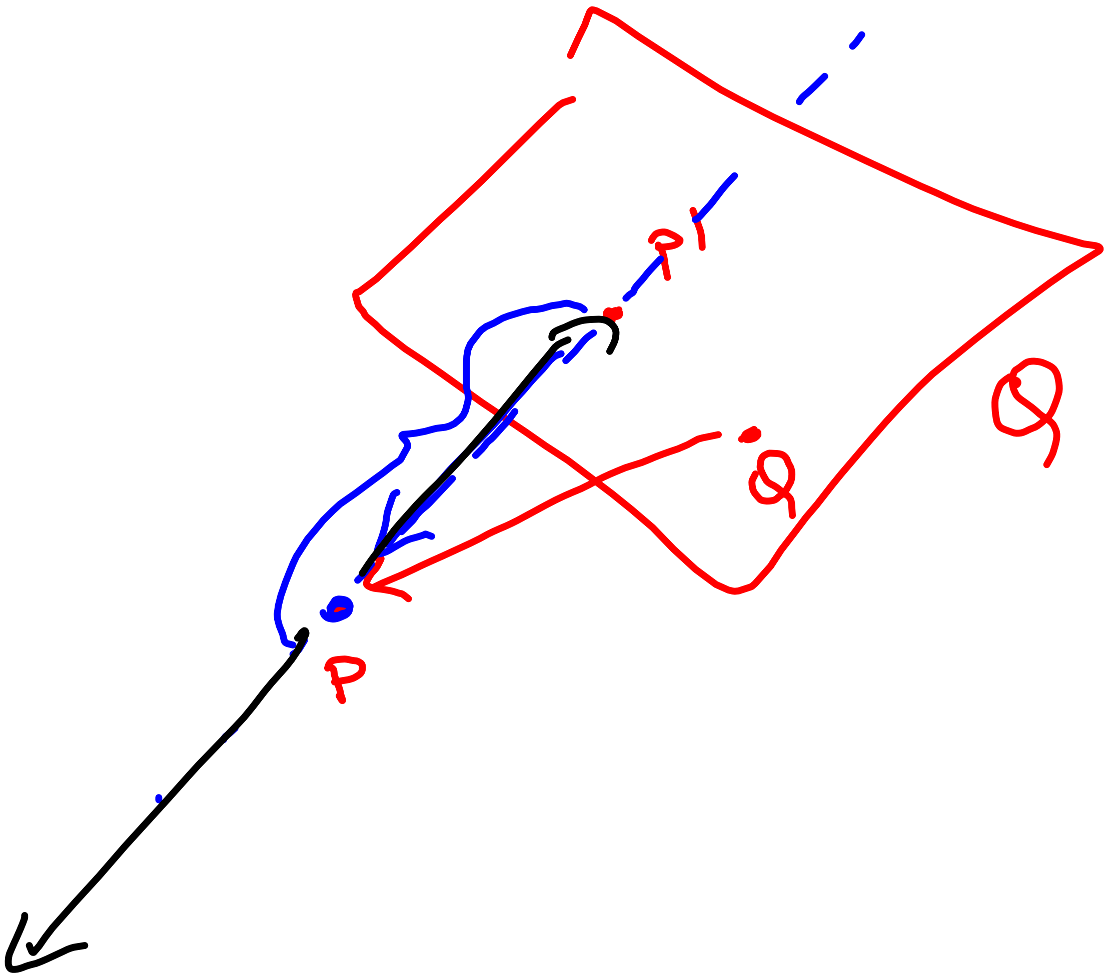
velikost kolmého průmětu u do $\varrho^{\perp}$ je:

$$u \cdot \frac{v}{\|v\|} = (-2\cdot 1 + 1\cdot 2 + 1\cdot 3)/\sqrt{1+4+9} = \frac{3}{\sqrt{14}}$$

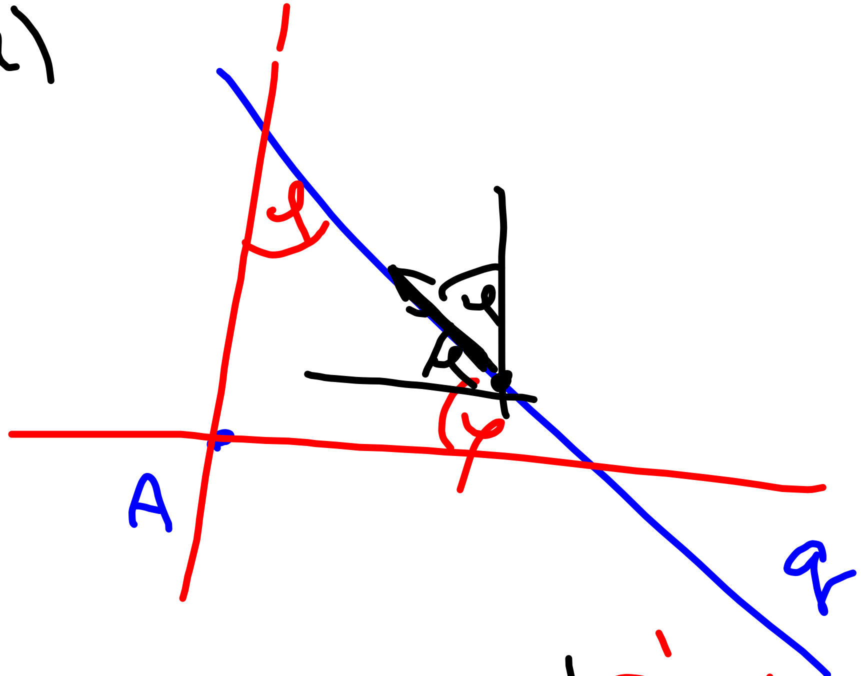


$$P - \frac{3}{\sqrt{14}} \cdot \frac{v}{\sqrt{14}} = [1 - 3/14, \ 1 - 6/14, \ 1 - 9/14] =$$

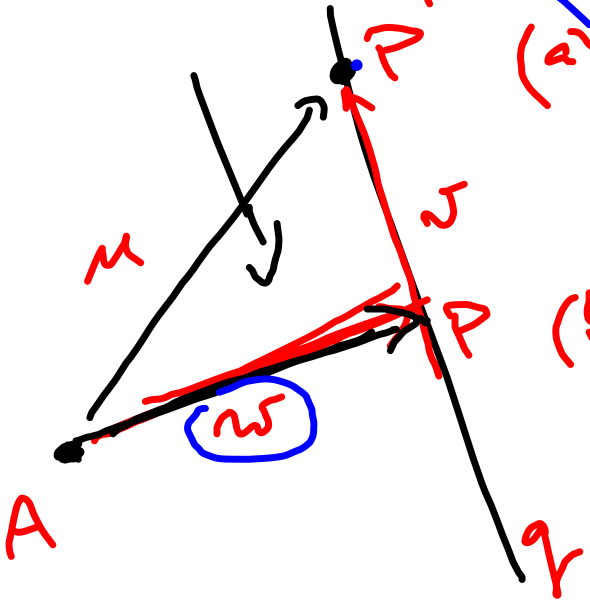
$$\left[\frac{11}{14}, \frac{8}{14}, \frac{5}{14}\right] = P'$$


P'
Q
Θ
P

2)



3)



(a) $q: \underline{ax + by + c = 0}$

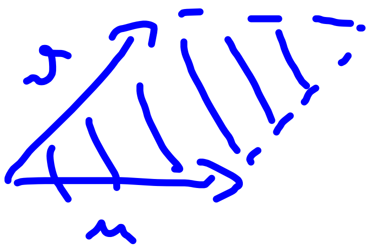
$\Rightarrow \langle (a,b) \rangle = q^{\perp}$

(b) $q: \underline{X = P' + t \cdot v'}$

najít $P' - A$ do $\langle v' \rangle$

$\Rightarrow v \Rightarrow w = u - v$

$A + v = P$



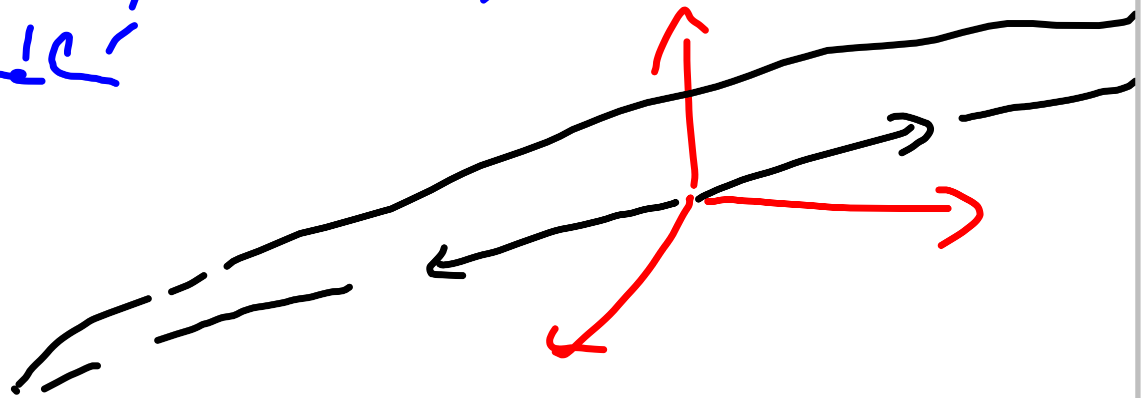
$$\mathrm{vol}(P_2(u,v)) = \det\begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix}$$

$$= \underline{\underline{u_1 v_2 - u_2 v_1}}$$

Př. $u = (u_1, u_2, u_3)$ $\quad v = (v_1, v_2, v_3)$

Pro jaké $t = (t_1, t_2, t_3)$, $\|t\| = 1$, bude

$P_3(u, v, t)$ maximální



$$\begin{vmatrix} u_1 & v_1 & t_1 \\ u_2 & v_2 & t_2 \\ u_3 & v_3 & t_3 \end{vmatrix} = t_1 \begin{vmatrix} u_2 & v_2 \\ u_3 & v_3 \end{vmatrix} - t_2 \begin{vmatrix} u_1 & v_1 \\ u_3 & v_3 \end{vmatrix} + t_3 \begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}$$

$=$ skalární součin t a $(u \times v)$

$\Rightarrow$ maximální pro t shodné s $(u \times v)$

$$\Rightarrow \quad t = \frac{u \times v}{\|u \times v\|}.$$