Timetabling at Purdue University

March 24, 2010

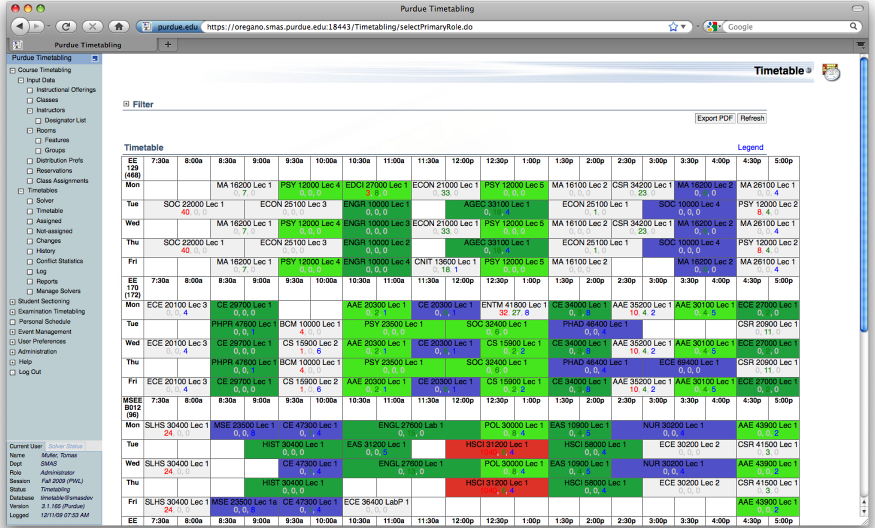
Part I: Classical Course Timetabling

Course timetabling problems

Coordinated decentralized timetabling for:

- a centrally timetabled large lecture problem
  - almost 900 classes timetabled into 55 rooms with up to 474 seats
- individually timetabled departmental problems
  - about 70 problems with 10 to 500 classes using departmental laboratory spaces and centrally managed classrooms allocated to departments based on expected class hours
- a centrally timetabled computer laboratory problem
  - about 200 classes timetabled into 31 rooms with 20 to 45 seats

GUI with generated timetable



Problem	Classes	Meetings per class	Hours per class	Classes per subpart	Students	Classes per students	Timetabled classes per student	Rooms	Room capacity (min-max)	Frequency (in %) (used slots)	Utilization (in %)	Distribution constraints per class
pu-spr07-llr	803	2.09	2.40	1.25	27881	3.15	0.00	55	40-474	67.77	62.54	0.69
pu-fal07-llr	891	2.07	2.32	1.26	30855	3.23	0.00	55	40-474	74.63	70.40	0.71
pu-spr07-ms	440	2.32	2.43	3.52	11992	1.11	3.13	25	24-51	84.43	76.18	2.74
pu-fal07-ms	525	2.35	2.40	4.45	14331	1.10	3.18	33	24-61	78.30	67.88	2.18
pu-spr07-cs	93	1.63	2.14	1.82	725	2.03	3.19	13	17-61	36.18	30.42	2.83
pu-fal07-cs	174	1.31	1.92	2.72	2002	1.57	4.00	13	22-61	52.19	44.93	2.49
pu-spr07-cfs	214	1.44	2.91	2.21	1610	1.94	2.94	29	10-71	41.52	26.70	1.79
pu-fal07-cfs	201	1.38	2.90	2.14	1936	1.75	3.17	28	10-71	39.73	23.14	3.28
pu-spr07-vpa	249	1.71	3.24	1.64	1836	2.17	2.47	47	10-45	34.34	28.80	2.06
pu-fal07-vpa	290	1.59	2.92	1.72	1747	2.22	2.42	41	10-45	40.39	32.81	1.26
pu-spr07-lab	443	1.25	1.97	4.82	8421	1.14	4.20	36	20-45	52.57	43.27	2.05
pu-fal07-lab	200	1.20	1.81	3.70	4835	1.08	4.49	31	20-45	27.19	23.21	3.29
pu-spr07-c8	2418	1.81	2.45	1.95	29514	4.16	0.00	213	10-474	55.27	56.35	1.76
pu-fal07-c8	2457	1.85	2.40	1.90	32399	4.10	0.00	208	10-474	56.83	61.29	1.74

Basic set of constraints

		Hard constraint	Soft constraint
Times for class	Time pattern	x	
	Individual times	x	x
Rooms for class	Individual buildings/rooms	x	x
	Individual room equipment	x	x
Resource constraints	Room	x	
	Instructor	x	
Students	Conflicts between two classes		x
Distribution constraints between classes	Time between classes	x	x
	Time precedences between classes	x	x
	Classes placed in similar times	x	x
	Same or different meeting days/times/rooms for classes	x	x

Model of course structure

Instructional Offering	Introduction to Actuarial Science					MATH 170 STAT 170
Configuration	Traditional			Computer-Assisted		
Subpart	Lecture			Lecture		
Parent		Recitation			Recitation	
Child					Laboratory	
Class	Lec1			Lec3		
Parent		Rec1	Rec2		Rec5	Rec6
Child					Lab1	Lab2
	Lec2			Lec4		
		Rec3	Rec4		Rec7	Rec8
					Lab3	Lab4

Course structure with classes as displayed in user interface

							Preferences			
	Demand	Mins Per Week	Limit	Manager	Date Pattern	Time Pattern	Time	Room	Distribution	Instructor
M E 263 M E 263H	98		96							
Lecture		150	96	LLR	Full Term	3 x 50 2 x 75		WTHR Computer		
Recitation		100	96	M E	Full Term	2 x 50		ME 120 ME 236 Classroom		
Laboratory		50	84-120	LAB	Even Wks	1 x 50		Windows XP		
Lec 1		150	96	LLR	Full Term	3 x 50 2 x 75		WTHR Computer		J. Smith C. Bing
Rec 1		100	48	M E	Full Term	2 x 50		ME 120 ME 236 Classroom	Back-To-Back M E 263 Rec 1 M E 263 Rec 2	J. Novak
Lab 1		50	14-20	LAB	Even Wks	1 x 50		Windows XP		
Lab 2		50	14-20	LAB	Even Wks	1 x 50		Windows XP		
Lab 3		50	14-20	LAB	Even Wks	1 x 50		Windows XP		
Rec 2		100	48	M E	Full Term	2 x 50		ME 120 ME 236 Classroom	Back-To-Back M E 263 Rec 1 M E 263 Rec 2	J. Novak
Lab 4		50	14-20	LAB	Odd Wks	1 x 50		Windows XP		
Lab 5		50	14-20	LAB	Odd Wks	1 x 50		Windows XP		
Lab 6		50	14-20	LAB	Odd Wks	1 x 50		Mac Os X		

Weighted constraint satisfaction problem

- $P = (V, \mathcal{D}, C, w_c, w_\theta)$
- set of variables V
- set of finite domains $\mathcal{D}$
  - each $v \in V$ takes a value from D_v such that $D_v \in \mathcal{D}$
- set of hard and soft constraints $C = C_h \cup C_s$
  - constraint weight w_c as a function associating each soft constraint $c \in C_s$ with its weight $w_c(c)$
  - assignment weight as a function w_θ associating the value d of each variable v with its weight $w_\theta(v/d)$
- An assignment ω for a weighted CSP $(V, \mathcal{D}, C, w_c, w_\theta)$ is a set of pairs v/d such that $v \in V, d \in D_v, D_v \in \mathcal{D}$ and each v appears at most once in ω.
- assignment ω complete if each $v \in V$ appears in ω, otherwise ω is called a partial assignment

Solution of weighted CSP

- Consider a constraint $c \in C$ defined on variables $X \subseteq V$ and denote $n_c = \|X\|$ and $scope(c) = X$.
- Assignment ω satisfies the constraint c iff x_i/d_i exists in ω for all variables $x_i \in scope(c)$ and $(d_1, \ldots d_{n_c}) \in c$ holds (written $\omega \vDash c$).
- An assignment ω is consistent if it satisfies all of the hard constraints $c \in C_h$ whose scopes have no unassigned variables.
- A complete assignment ω which satisfies all hard constraints is called a solution (alternatively a solution is a complete consistent assignment).

Initial problem

$$F_s\omega = \sum_{c \in C_s \wedge \omega \models \neg c} w_c(c) + \sum_{v/d \in \omega} w_\theta(v/d)$$

$$F_{\mathrm{wcsp}}\omega = (\|\omega\|, F_s\omega)$$

$$F_{\mathrm{wcsp}}\omega \leq_{\mathrm{wcsp}} F_{\mathrm{wcsp}}\eta \equiv ((\|\omega\| > \|\eta\|) \vee ((\|\omega\| = \|\eta\|) \wedge (F_s\omega \leq F_s\eta)))$$

An optimal solution of the initial problem is a solution σ with the minimal $F_{\mathrm{wcsp}}\sigma$.

Consider a consistent assignment ω with $F_{\mathrm{wcsp}}\omega = (\|\omega\|, F_s\omega)$ and a new assignment v/d such that v is not present in ω. Such an assignment may increase the violation of soft constraints by the value

$$\Delta F_s(\omega, v/d) = \quad w_\theta(v/d) + \sum_{c \in C_s \wedge v \in scope(c) \wedge \omega \not\models \neg c \wedge (\omega \cup \{v/d\}) \models \neg c} w_c(c)$$

Domain variable

- Each class is specified by a domain variable representing the desired values for meeting times (weeks and patterns of days the class should meet during the term, start times and duration of all meetings) and rooms.
- Domain variables will be denoted $v = (v_w, v_p, v_s, v_d, v_r)$ and their particular values $d = (d_w, d_p, d_s, d_d, d_r)$.
- Example:
  - Value $d =$ (11110000, MW, 7:30 am, 50, WTHR 200)
  - Domain variable $v = (v_w, v_p, v_s, v_d, v_r)$ with

    $(v_w, v_p, v_s, v_d) \in \{$(11111111, MWF, 7:30 am, 50), (11111111,MWF, 8:30 am, 50), (11111111, MWF, 9:30 am, 50), (11111111, TTh, 7:30 am, 75), (11111111, TTh, 9:00 am, 75)$\}$

    and $v_r \in \{$WTHR 200, CL50 224, EE 129, LILY 1105$\}$

Hard constraints

- No constraint propagation
- Consistency of the constraint detected when the last variable is assigned
- Weak for non-binary constraints
- However, non-binary constraints can be translated to simpler contraints to check their consistency
- Example: resource constraint
  - a set of binary constraints prohibiting the placement of each pair of classes into overlapping time periods
  - implementation: array of assigned classes over time

Soft constraints

- Soft unary constraints

$$w_\theta(v/d) = w_{\mathrm{time}} w_\theta((v_w, v_p, v_s, v_d)/(d_w, d_p, d_s, d_d)) + w_{\mathrm{room}} w_\theta(v_r/d_r)$$

- Student conflicts: soft constraint c between each two classes v_1 and v_2 with the weight $w_c(c) = s$ which is satisfied when the classes v_1 and v_2 do not overlap
- Soft distribution constraints with a weight $w_c(c)$