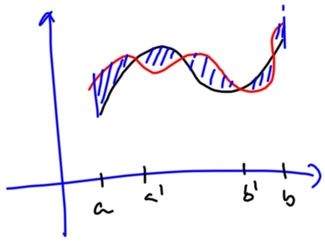


$f \in C[a,b]$

$g \in$

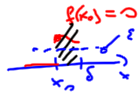
$$\int_a^b |f(x) - g(x)|^2 = \|f - g\|^2$$

na $[a,b]$

5 2-14:08

$\mathcal{S} = \{$po částech spojité, omezené na $[a,b]\}$

$$\langle f, g\rangle = \int_a^b \underbrace{f(x)\, g(x)}\, dx$$



$\|f\|^2 = \langle f, f\rangle$

skalární součin

(1) bilineární: $\langle f + h, g\rangle = \int_a^b (f + h)(x)\, g(x)\, dx$

$= \langle f, g\rangle + \langle h, g\rangle \qquad = \int_a^b f(x) g(x)\, dx + \int_a^b h(x) g(x)\, dx$

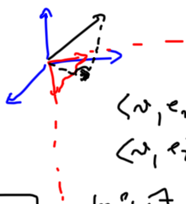
(2) symetrický $\langle f, g\rangle = \langle g, f\rangle$

(3) nedegenerovaný: $\|f\|^2 = 0 \Leftrightarrow f \equiv 0$:

$\int_a^b |f(x)|^2 dx = 0 \wedge f(x_0) \neq 0 \Rightarrow$ spor

5 2-14:14

v $\mathbb{R}^3$



$v = c_1 e_1 + c_2 e_2 + c_3 e_3$

$\langle v, e_1\rangle = c_1 \langle e_1, e_1\rangle$

$\langle v, e_2\rangle = c_2 \langle e_2, e_2\rangle$

průmět je "nejbližší"

nad $[0,1]$

$\mathbb{R}_2(x) = \langle 1, x, x^2\rangle$

$\boxed{e_1 = 1}$, $\quad e_2 = 1 + cx \qquad \int_0^1 (1 + cx)\, dx = 0$

$= \boxed{1 - 2x} \qquad = 1 + \frac{1}{2}c \Rightarrow c = -2$

$e_3 = 1 + c_1(1 - 2x) + c_2 x^2 \qquad e_3 \perp e_2,\ e_3 \perp e_1$

$\Rightarrow c_1, c_2 \Rightarrow \boxed{e_3}$

5 2-14:24

$$\int_0^1 (1 - 2x)^2 dx = \int_0^1 1 - 4x + 4x^2\, dx$$

$$= 1 - 2 + \frac{4}{3} = \frac{1}{3}$$

$\langle \mathbb{R}_2[x] \cup \{f\}\rangle \subset \mathcal{S}$

$\Rightarrow \underbrace{\langle f, e_1\rangle \cdot \|e_1\|^{-2} e_1} + \langle f, e_2\rangle \cdot \|e_2\|^{-2} e_2 + \ldots$

je nejlepší možné aproximace

$$\langle F, f_n\rangle = \int_a^b \left(\sum_{m=1}^{\infty} c_m f_m\right) \cdot f_n(x)\, dx$$

5 2-14:31

$F(x) = \sum_{n=1}^{\infty} c_n f_n(x)$, $\quad f_1, f_2, \ldots$ ortonormální

(2) $\sum_{n=1}^{\infty} c_n^2 \leq \|g\|^2$

(3) konvergence $\Leftrightarrow$ rovnost

5 2-14:45

$$\left\|g - \sum_{n=1}^{k} a_n f_n\right\|^2 = \int_a^b \left(g(x) - \sum_{n=1}^{k} a_n f_n(x)\right)^2 dx$$

$$= \int_a^b g^2(x)\, dx + \sum_{n=1}^{k} -2\, g(x)\, a_n f_n(x) + \ldots$$

$\|g\|^2$

5 2-14:53

$$f_n = \sin nx$$

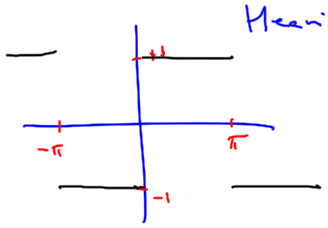
$$g_n = \cos nx$$

$$\int \underbrace{\sin x \cos x}_{\frac{1}{2}\sin 2x}\, dx = 0$$

$$\int_{-\pi}^{\pi} (\sin x)^2\, dx$$

5 2-15:16

Heaviside h



$$\int_{-\pi}^{\pi} h(x) \cos nx\, dx = 0 \quad \forall n$$

5 2-15:28