MB104 - 2 - A

(1) a) $X \sim N(\mu; \sigma^2)$, kde $\sigma^2 = 2500$

n měření ... $M \sim N(\mu, \frac{\sigma^2}{n})$; int. spolehlivosti α je $\left(M - \frac{\sigma}{\sqrt{n}} u_{1-\frac{\alpha}{2}}; M + \frac{\sigma}{\sqrt{n}} u_{1-\frac{\alpha}{2}}\right)$, $\alpha = 0{,}1$

Pož. přesnost $\frac{\sigma}{\sqrt{n}} u_{1-\frac{\alpha}{2}} \le 10$ $u_{0,95} = 1{,}65$

$\frac{50}{\sqrt{n}} 1{,}65 \le 10 \iff \sqrt{n} \ge 5 \cdot 1{,}65$

$n \ge 68{,}09.$

Je třeba min. 69 měření.

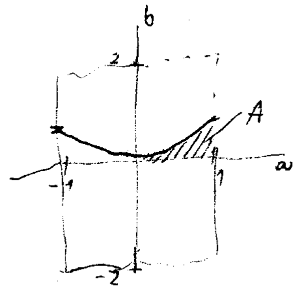
b) $M = 1339{,}1\overline{6}$ $S^2 = \frac{1}{5} \sum_{i=1}^{6} (X_i - M)^2 = 2114{,}1\overline{6}$

c) známe σ^2: $\left(M - \frac{\sigma}{\sqrt{n}} u_{1-\frac{\alpha}{2}}, M + \frac{\sigma}{\sqrt{n}} u_{1+\frac{\alpha}{2}}\right)$

$\sigma = 50$, $n = 6$ $u_{0,975} = 1{,}96 \Rightarrow (1339{,}1\overline{6} - 40{,}01; 1339{,}1\overline{6} + 40{,}01) \doteq (1299; 1379)$

(2) a) $x^2 + ax + b$, $|a| \le 1$, $|b| \le 2$, kořeny označme x_1, x_2.

Platí $a = -(x_1 + x_2)$ $b = x_1 \cdot x_2$



A ... reálné záporné kořeny $x_1, x_2 < 0 \iff a > 0 \wedge b > 0$

reálné kořeny $\iff a^2 - 4b \ge 0 \iff b \le \frac{a^2}{4}$

$\mu(A) = \int_0^1 \frac{a^2}{4} da = \left[\frac{a^3}{12}\right]_0^1 = \frac{1}{12}$

$\mu(\Omega) = 2 \cdot 4 = 8$

$P(A) = \frac{\mu(A)}{\mu(\Omega)} = \frac{1/12}{8} = \frac{1}{96}$

b) $h(x) = 9x^2 - 9x - 4 = (3x-4)(3x+1)$ [rozklad pomocí nalezení racionálních kořenů]

Pro např. $f(x) = (3x-4)^3 \cdot (3x+1)$

$g(x) = (3x-4) \cdot (3x+1)^3$

$h(x) = \frac{1}{125}\left[(-6x+13) g(x) + (6x+7) f(x)\right]$

(3) a) $\cos\frac{3\pi}{2} + i \sin\frac{3\pi}{2} = -i$ $\langle -i \rangle = \{-i, -1, i, 1\}$

b) $(x^2+1)(x^3+2)$

c) $x^{2011} + 2$ (Eisensteinovo kritérium $\Rightarrow$ ireducibilita)

d)

$2011 = 1 \cdot 1492 + 519$

$1492 = 2 \cdot 519 + 454$

$519 = 1 \cdot 454 + 65$

$454 = 6 \cdot 65 + 64$

$65 = 1 \cdot 64 + 1$

$1 = 65 - 64 = 65 - (454 - 6 \cdot 65) = 7 \cdot 65 - 1 \cdot 454 =$

$= 7(519 - 454) - 454 = 7 \cdot 519 - 8 \cdot 454 = 7 \cdot 519 - 8(1492 - 2 \cdot 519) =$

$= 23 \cdot 519 - 8 \cdot 1492 = 23 \cdot 2011 - 31 \cdot 1492$

Odtud $23 \cdot 2011 \equiv 1 \ (1492)$,

tedy $[2011]^{-1}_{1492} = [23]_{1492}$.

MB104 - zk 2 - B

1.) a) $n = 11$

$M \doteq 3{,}55$

$S^2 = \frac{1}{10} \sum (X_i - M)^2 = 0{,}025$

oboustranný: $\left(M - \frac{S}{\sqrt{n}} t_{1-\frac{\alpha}{2}}(n-1),\ M + \frac{S}{\sqrt{n}} t_{1-\frac{\alpha}{2}}(n-1) \right) = (3{,}55 - 0{,}11;\ 3{,}55 + 0{,}11)$

$t_{0{,}975}(10) = 2{,}228$ $\qquad = (3{,}44;\ 3{,}66)$

jednostranný $\left(M - \frac{S}{\sqrt{n}} t_{1-\alpha}(n-1), \infty \right) = (3{,}55 - 0{,}09; \infty) = (3{,}46; \infty)$

$t_{0{,}95}(10) = 1{,}8125$

Tvrzení tvrzení je opodstatněné, s pravděpodobností 95% je $\mu \geq 3{,}46$.

b) $\left(\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S^2}{\chi^2_{\alpha/2}(n-1)} \right) = (0{,}0137; 0{,}0634)$

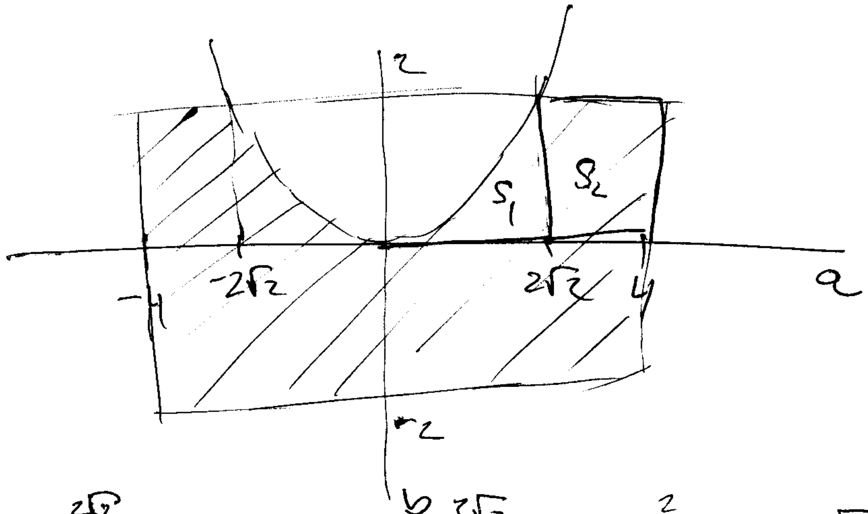
$\alpha = 0{,}1$

18,304 $\qquad$ 3,94

b

2

$$\frac{a^2}{4} = 2$$
$$a^2 = 8$$
$$a = \pm 2\sqrt{2}$$



$$|a| \le 4,\ |b| \le 2$$
$$a^2 - 4b > 0$$
$$a^2 > 4b$$
$$\frac{a^2}{4} > b$$

$$S_1 = \int_0^{2\sqrt{2}} \frac{a^2}{4}\,da = \frac{1}{4}\left[\frac{a^3}{3}\right]_0^{2\sqrt{2}} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{8}^{\,2} \cdot 2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

$$S_2 = (4 - 2\sqrt{2})2 = 8 - 4\sqrt{2}$$

$$\cancel{8}\, P = \frac{2 \cdot S_1 + 2 \cdot S_2 + 2 \cdot 8}{32} = \frac{2 \cdot \frac{4\sqrt{2}}{3} + 2(8 - 4\sqrt{2}) + 16}{32} =$$

$$= \frac{8\sqrt{2} + 48 - 24\sqrt{2} + 48}{96} = \frac{96 - 16\sqrt{2}}{96} = 1 - \frac{\sqrt{2}}{6}$$

3

a) $\xi_5 = \cos\frac{2\pi}{5} + i \sin\frac{2\pi}{5}$ $\qquad \langle \xi_5 \rangle = \{\xi_5^1; \xi_5^2, \xi_5^3, \xi_5^4, \xi_5^5 = 1\}$

b) neexistuje, každý polynom ~~nad~~ nad $\mathbb{C}$ stupně aspoň 2 má kořen a je reducibilní!

c) stačí najít kvadratický ireducibilní polynom nad $\mathbb{Z}_3$

$x, x+1, x-1$
$x^2, x^2+1, x^2-1,$
$x^2+x+1, x^2+x-1, x^2-x+1, x^2-x-1$

např. x^2+1 :

příklad takového polynomu je pak např. $(x^2+1)(x+1)^{2009}$

d)

$2011 = 1 \cdot 1492 + 519$
$1492 = 2 \cdot 519 + 454$
$519 = 1 \cdot 454 + 65$
$454 = 6 \cdot 65 + 64$
$65 = 1 \cdot 64 + 1$

$1 = 65 - 64 = 65 - (454 - 6 \cdot 65) = 7 \cdot 65 - 1 \cdot 454 =$
$= 7(519 - 454) - 454 = 7 \cdot 519 - 8 \cdot 454 = 23 \cdot 519 - 8 \cdot 1492 = 23 \cdot 2011 - 31 \cdot 1492$

Odtud $-31 \cdot 1492 \equiv 1\ (2011)$, tedy $[1492]_{2011}^{-1} = [-31] = [1980]_{2011}$.