



# Analyzing robustness of biological reaction systems

DTEDI  
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Sven Dražan



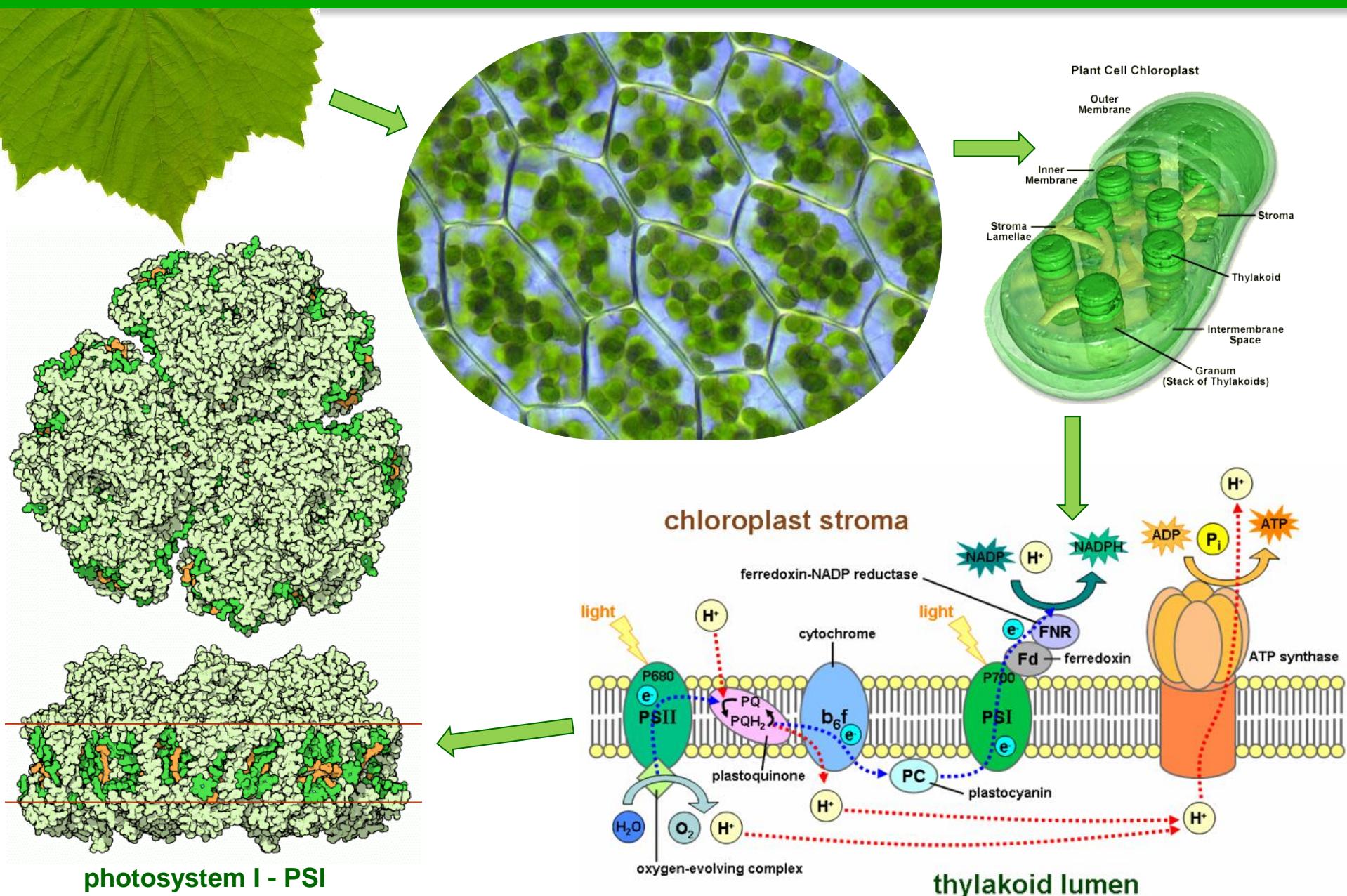
# Contents

- What is Robustness?
- Dynamic reaction systems
- Behavior and properties
- Computing robustness
- Thesis proposal

# Motivation / Bioreactors

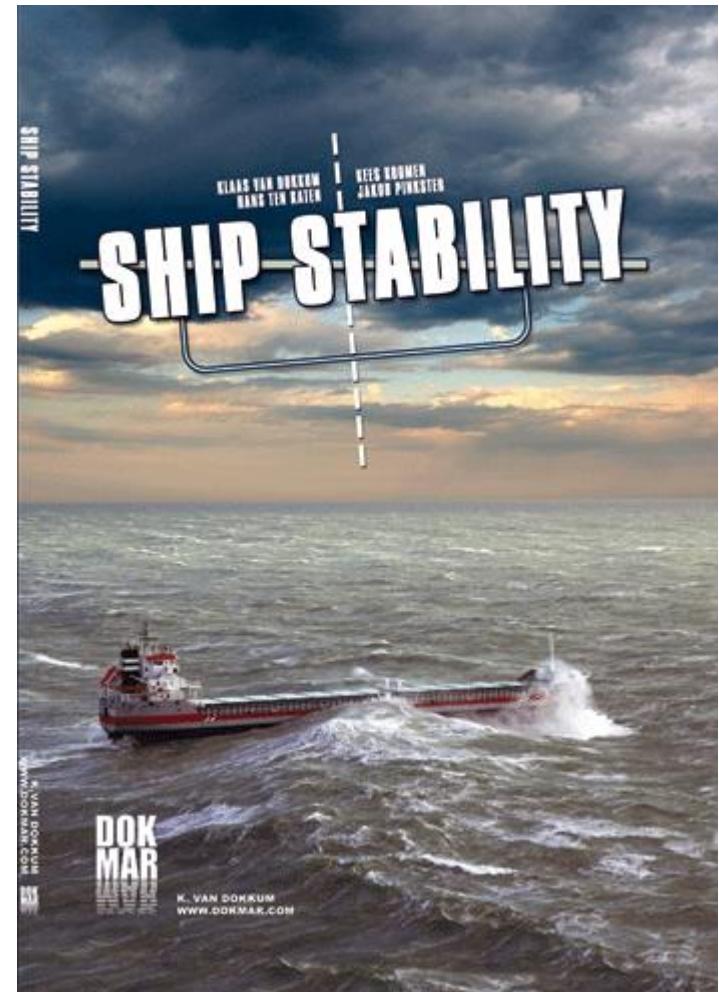
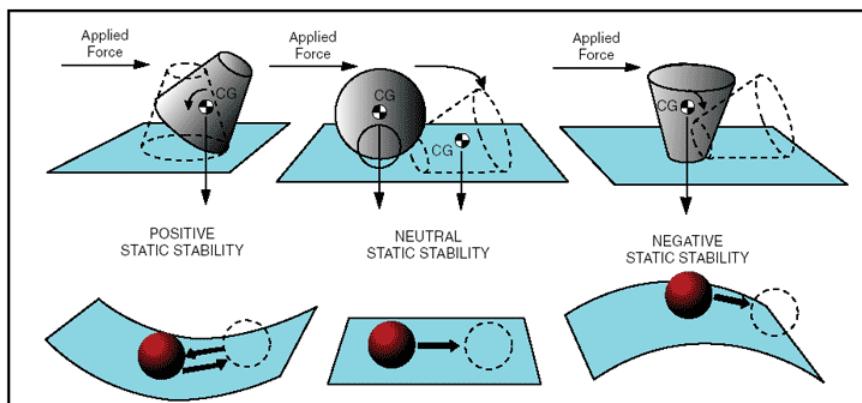


# Motivation / Photosynthesis



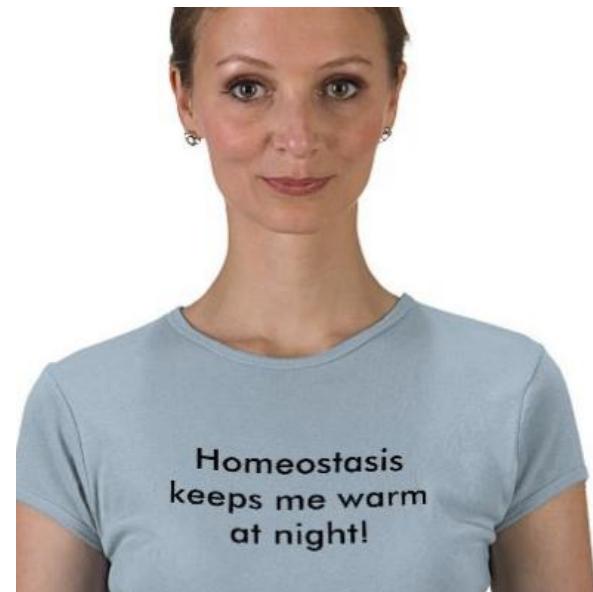
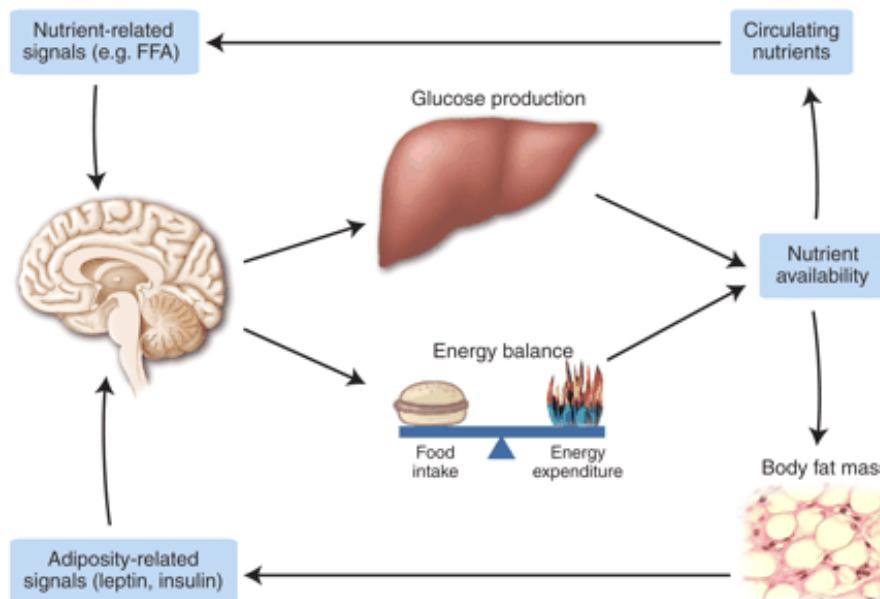
# What is robustness

Is **stability** robustness?



# What is robustness

## Is homeostasis robustness?



# What is robustness

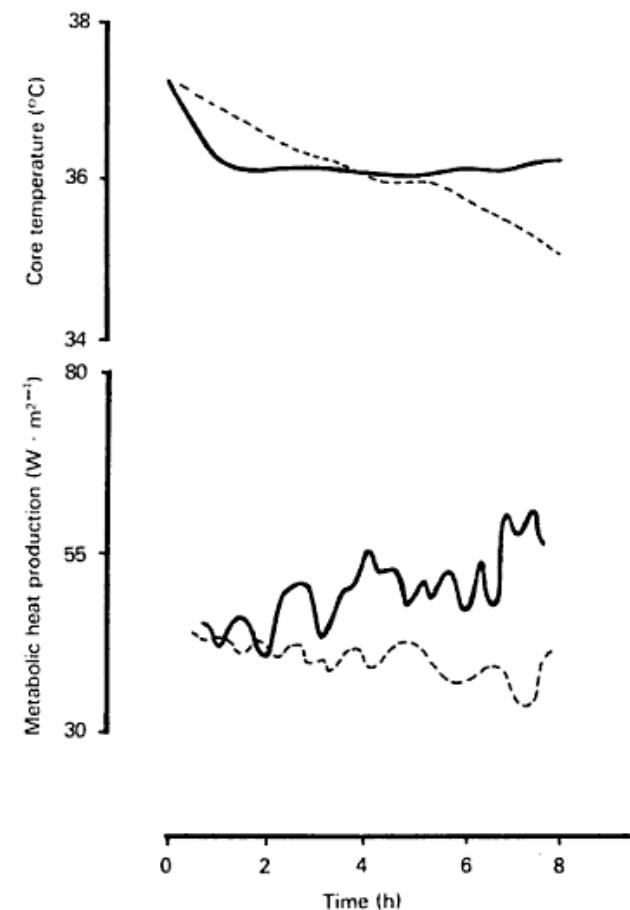
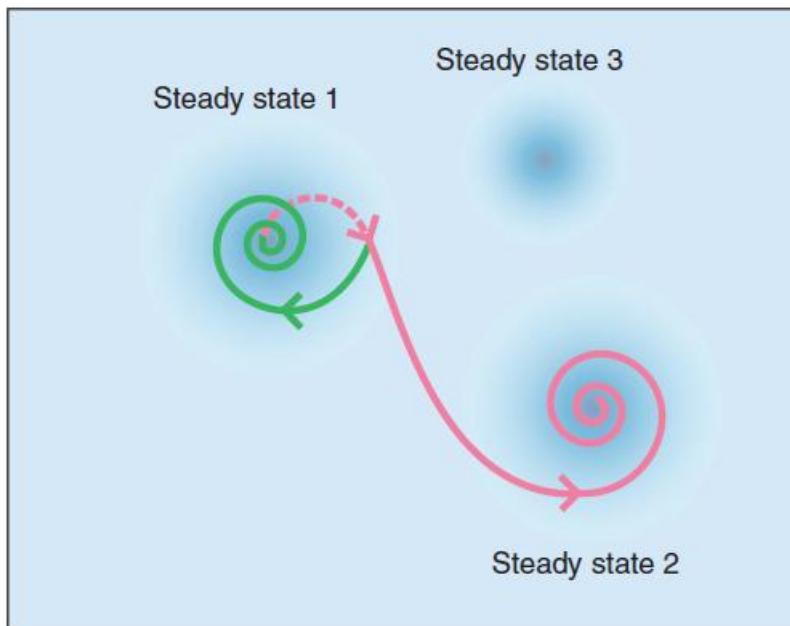


Fig. 5.4 Reduced body temperature in man. Response of a group of male Aborigines (---) and Europeans (—) to a night of moderate cold exposure. From Richards, S.A. (1973). *Temperature Regulation*, Wykeham Publications, Taylor & Francis: London.

# What is robustness



Is  
multistability or instability  
robustness?



# What is robustness?

Robustness is a property that allows a **system** to maintain its **function** against internal and external **perturbations**.

Kitano, 2004a

# What is robustness...

Robustness is a property that allows a **system** to maintain its **function** against internal and external **perturbations**.

Kitano, 2004a

function ~ behavior ~ property

$$R_{a,P}^s = \int_P \psi(p) D_a^s(p) dp$$

Robustness is a **property** that allows a **system** to maintain its **property** against internal and external **perturbations**.

Kitano, 2004a

$$R_{a,P}^s = \int_P \psi(p) D_a^s(p) dp$$

# What is robustness

A

p16	p26	p36	p46	p56	p66
p15	p25	p35	p45	p55	p65
p14	p24	p34	p44	p54	p64
p13	p23	p33	p43	p53	p63
p12	p22	p32	p42	p52	p62
p11	p21	p31	p41	p51	p61
p10	p20	p30	p40	p50	p60

Degree of perturbation

Features that are perturbed

B

p16	p26	p36	p46	p56	p66
p15	p25	p35	p45	p55	p65
p14	p24	p34	p44	p54	p64
p13	p23	p33	p43	p53	p63
p12	p22	p32	p42	p52	p62
p11	p21	p31	p41	p51	p61
p10	p20	p30	p40	p50	p60

Degree of perturbation

Features that are perturbed



$D(p) \neq 0, f(p)/f(0) > 0.8$



$D(p) \neq 0, 0.8 \geq f(p)/f(0) > 0.6$



$D(p) \neq 0, 0.6 \geq f(p)/f(0) > 0.4$

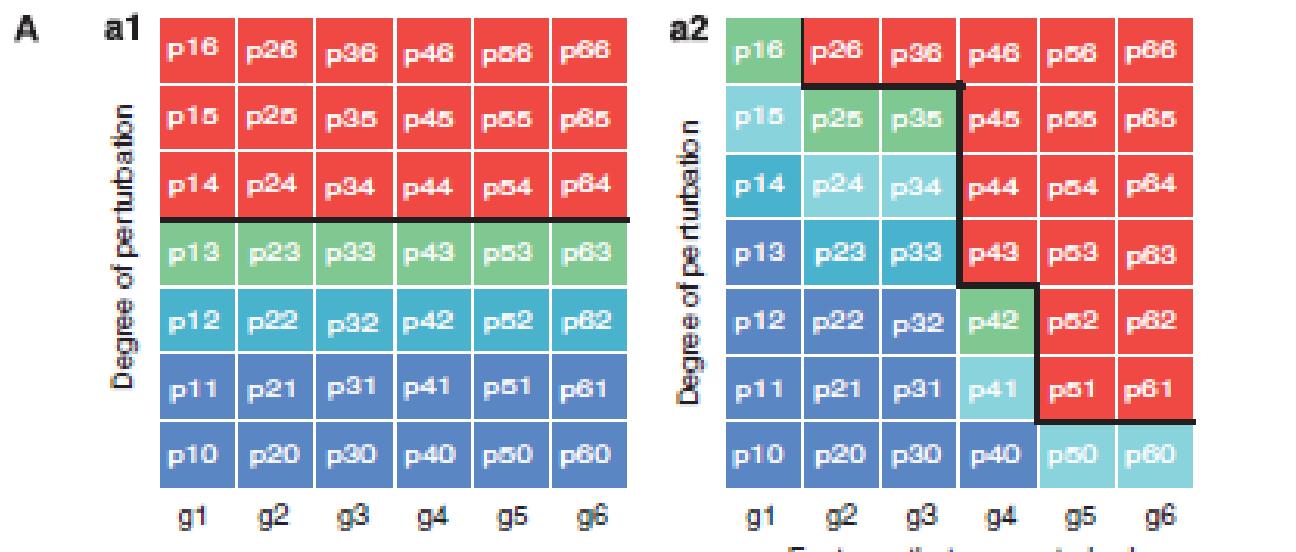


$D(p) \neq 0, 0.4 \geq f(p)/f(0) > 0.0$

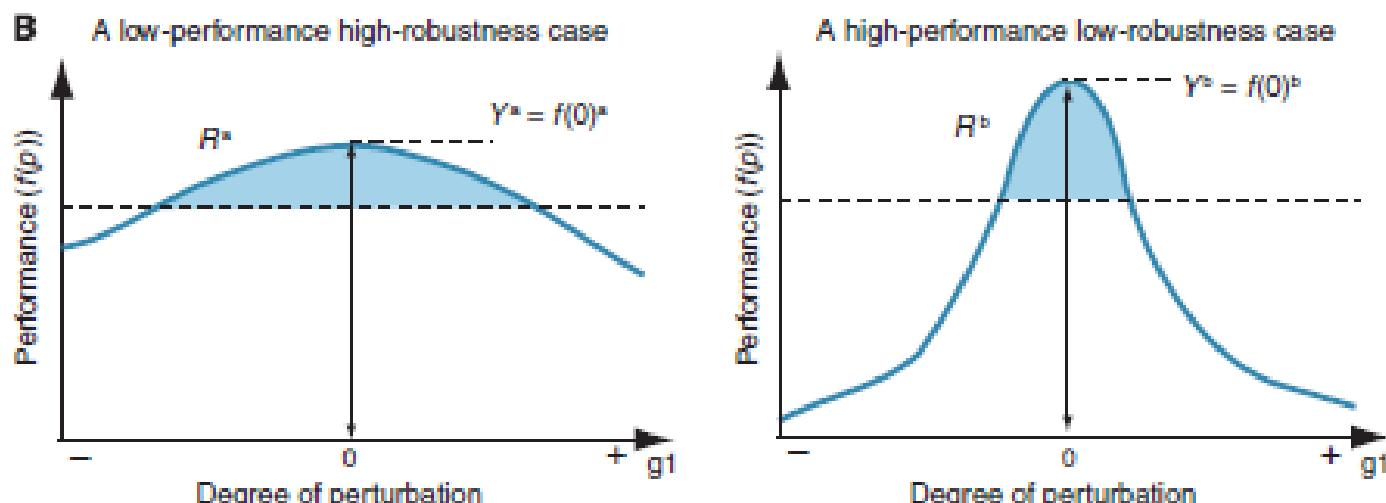


$D(p)=0$

# Robustness and tradeoffs



Robustness–fragility trade-off

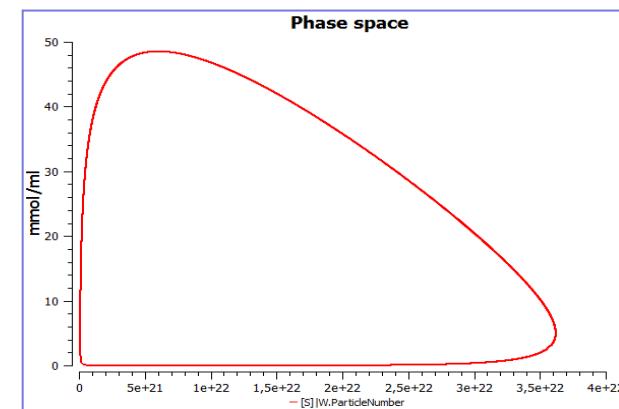
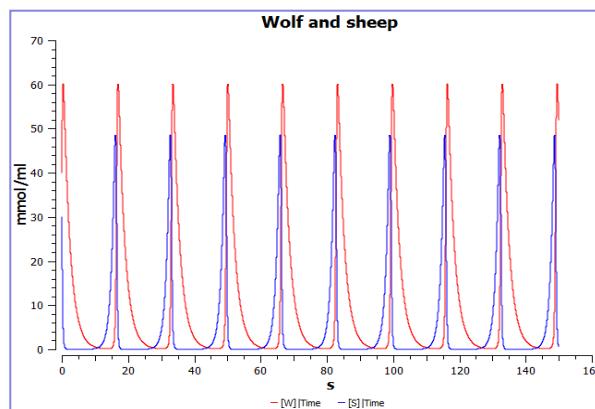
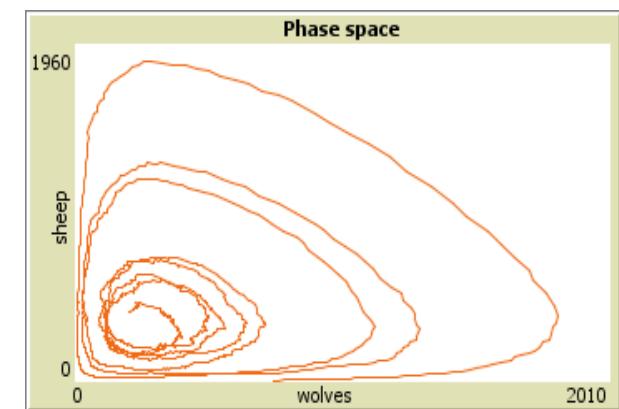
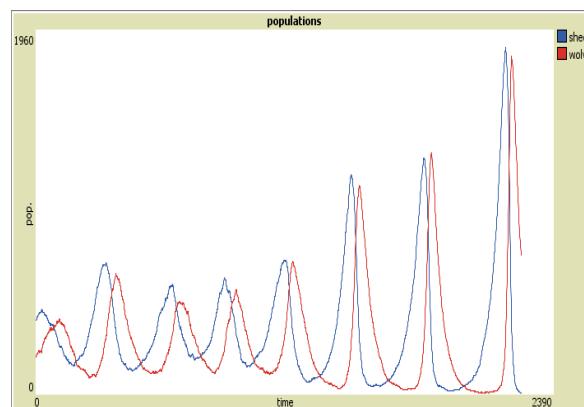
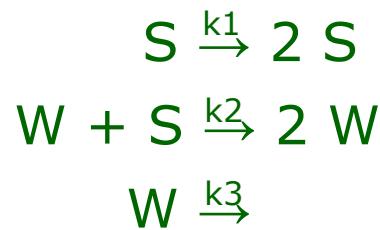


# What is a dynamic reaction system



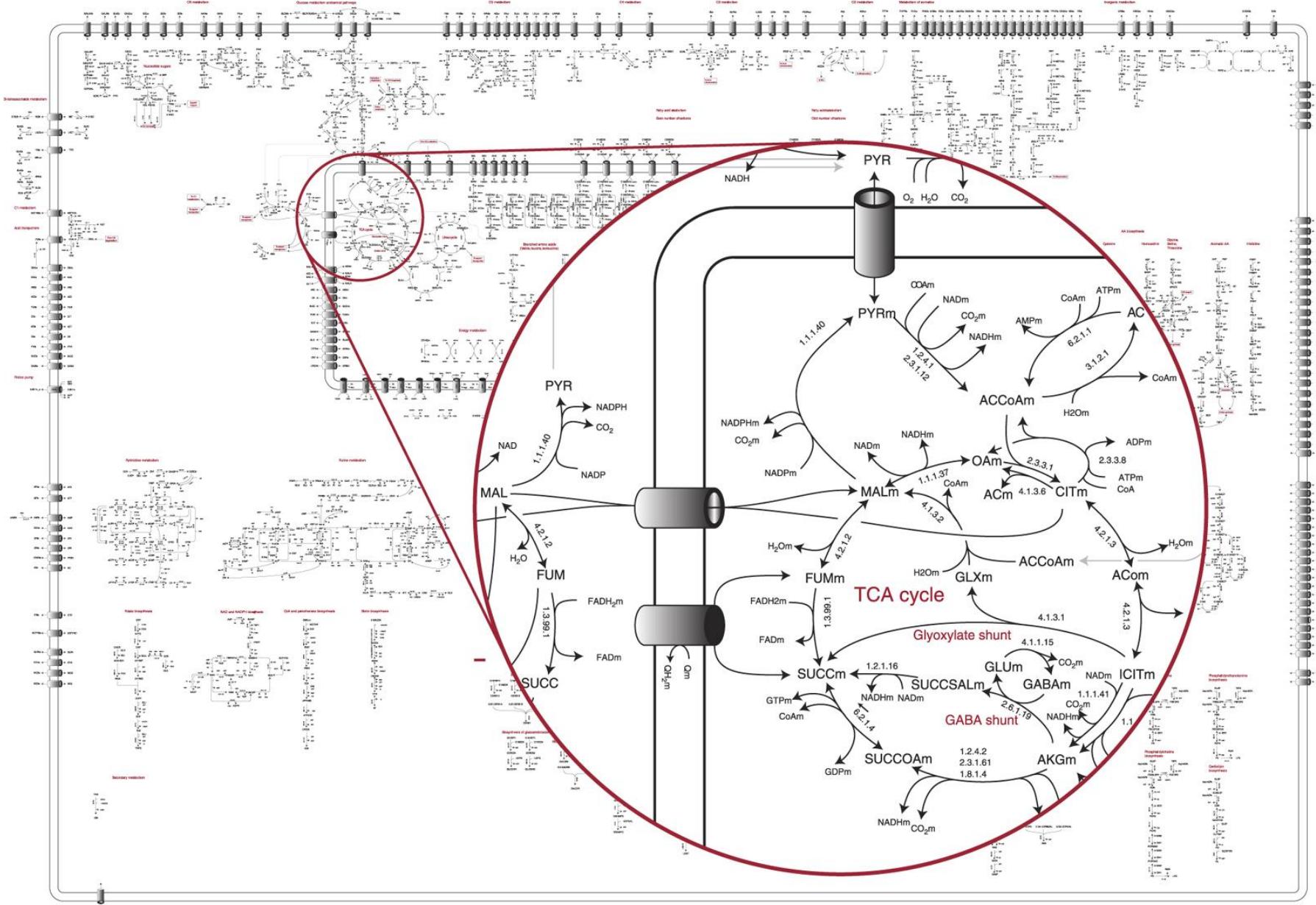
## Demo

# Dynamic reaction systems – Example

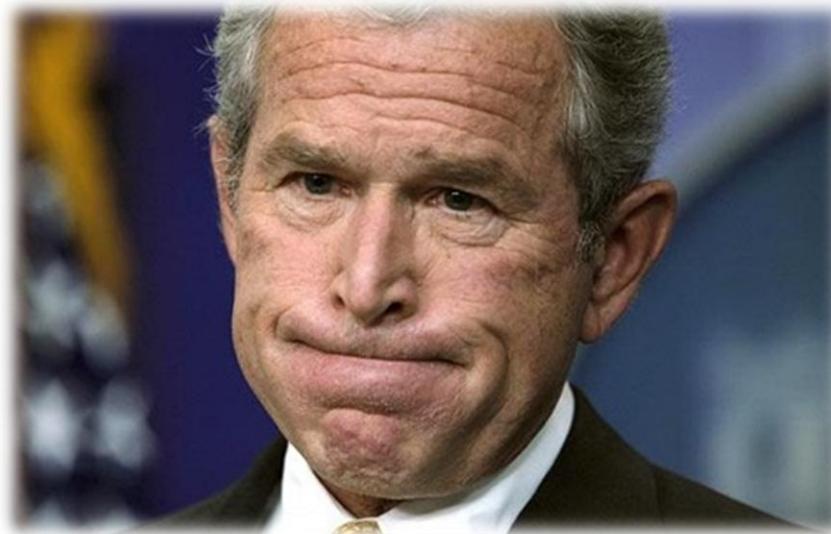


$$\begin{aligned}\frac{dS}{dt} &= k_1 \cdot [S] - k_2 \cdot [W] \cdot [S] \\ \frac{dW}{dt} &= k_2 \cdot [W] \cdot [S] - k_3 \cdot [W]\end{aligned}$$

# Dynamic reaction systems – Bigger example



# What is a behavior?

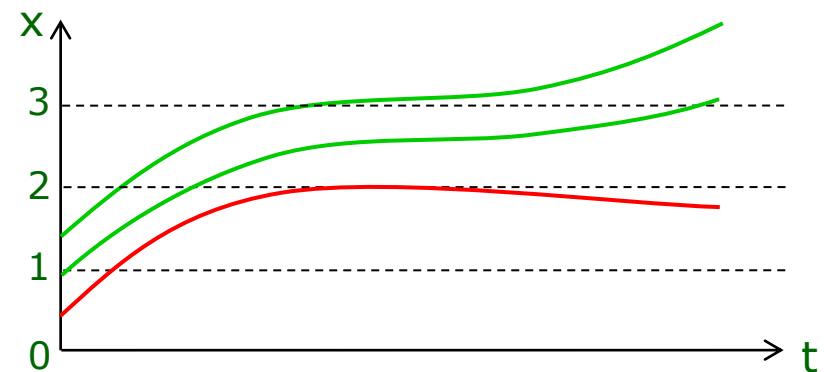


98.8% common DNA

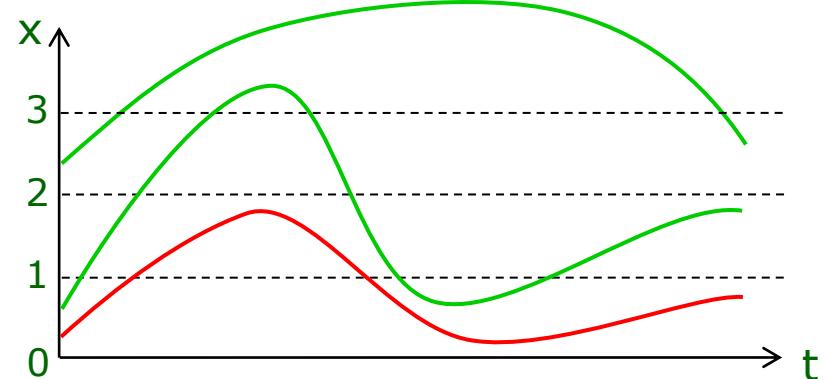
Any difference in behavior?

# What is a behavior ~ property

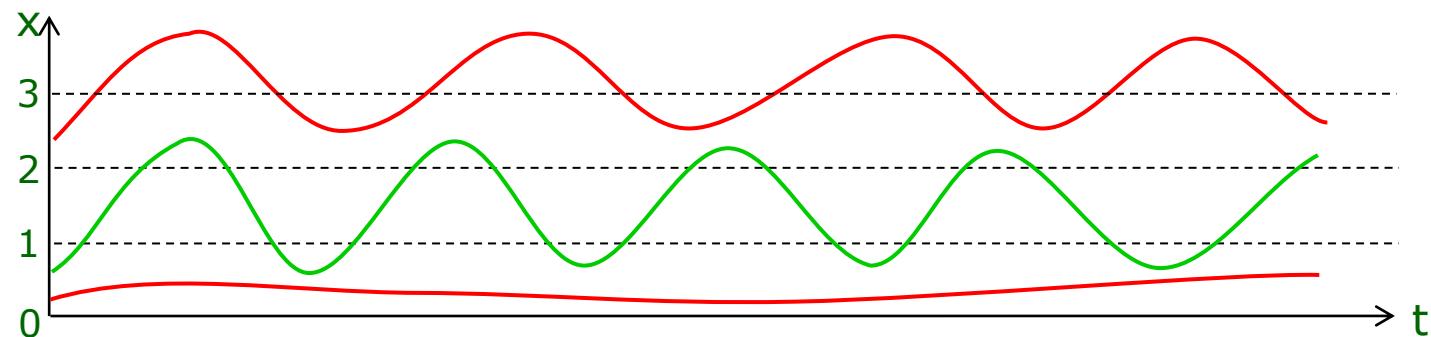
Reachability -  $F(x > 3)$



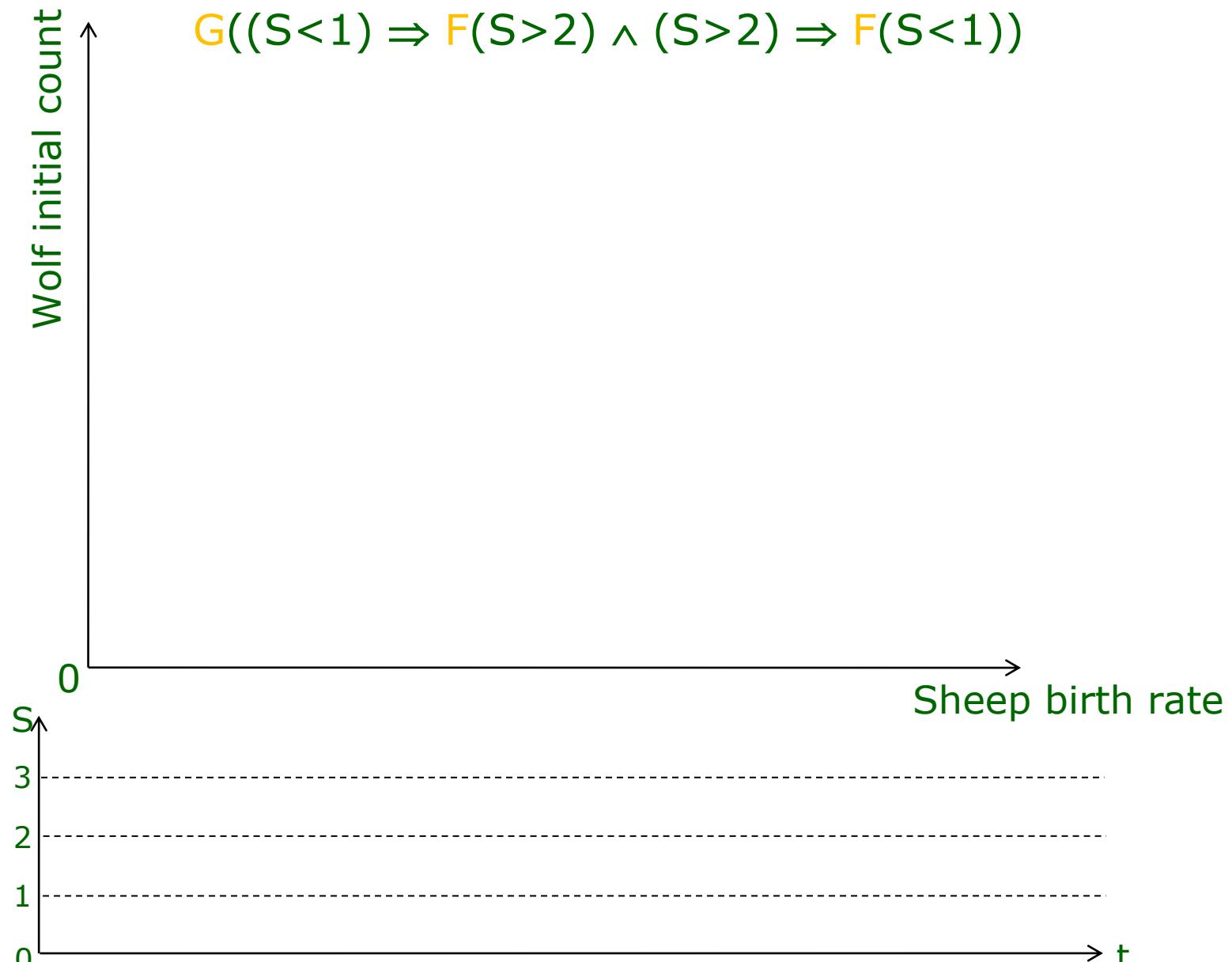
Response -  $G((x < 1) \Rightarrow F(x > 3))$



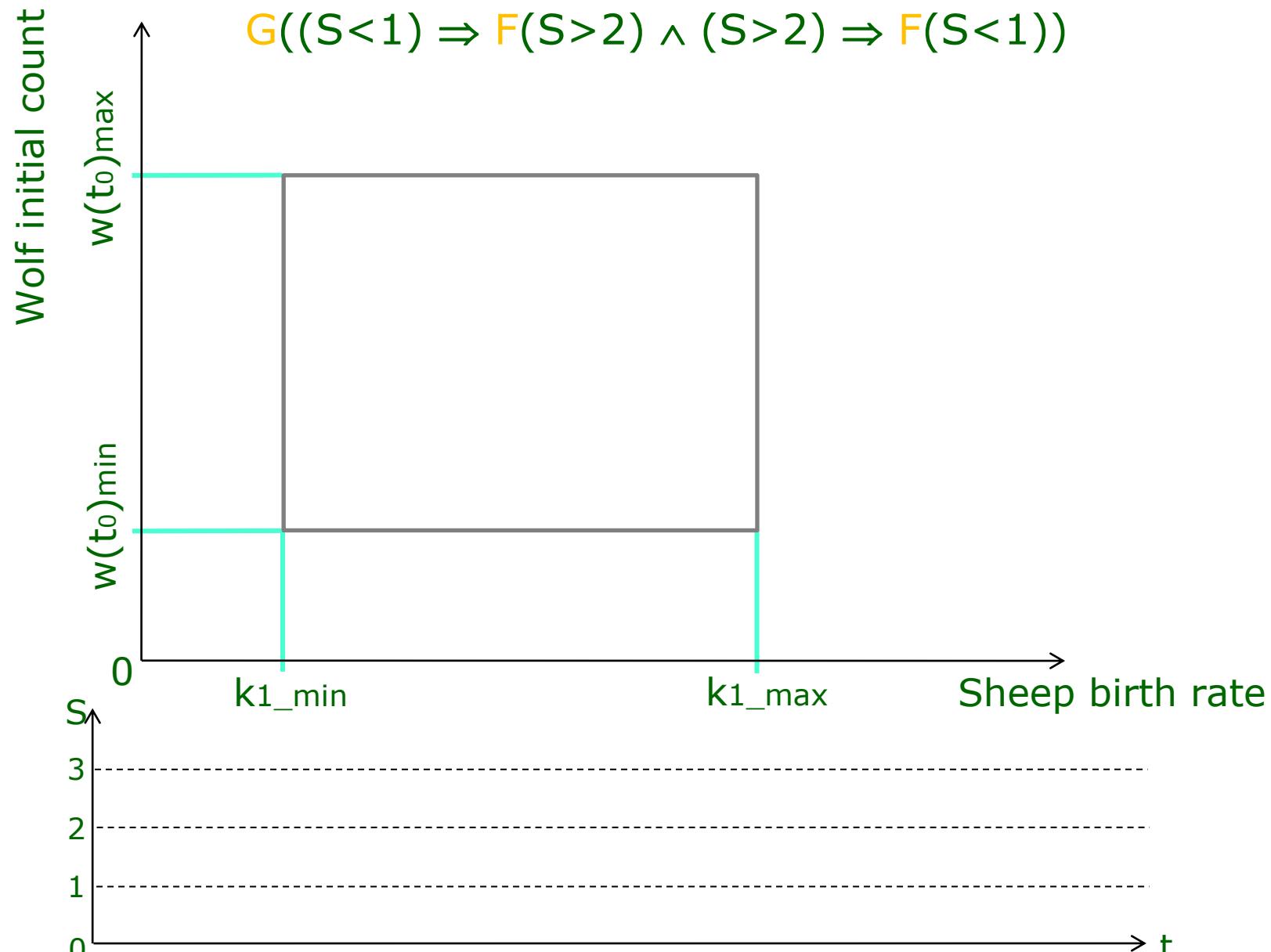
Oscillation -  $G((x < 1) \Rightarrow F(x > 2) \wedge (x > 2) \Rightarrow F(x < 1))$



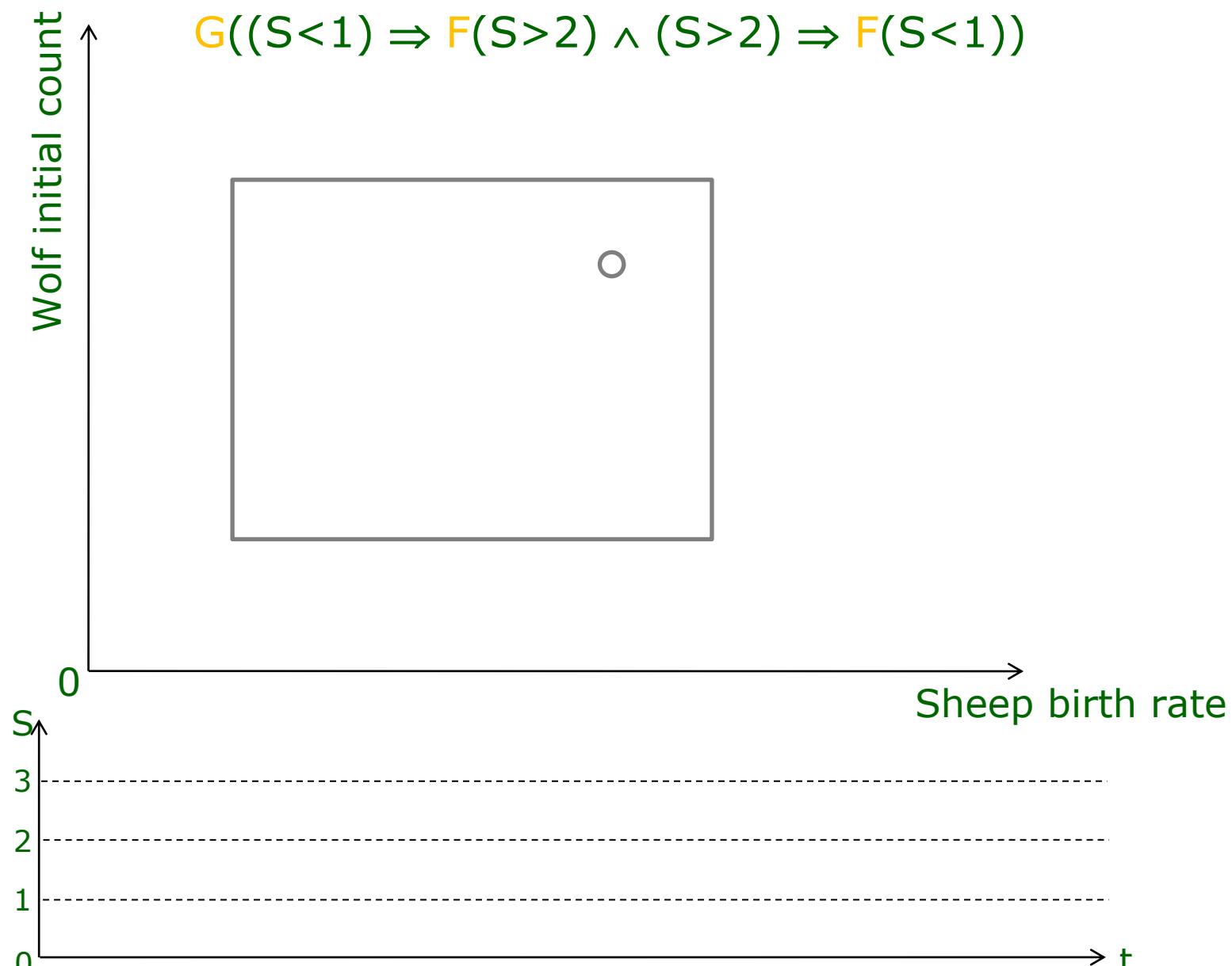
# Computing robustness



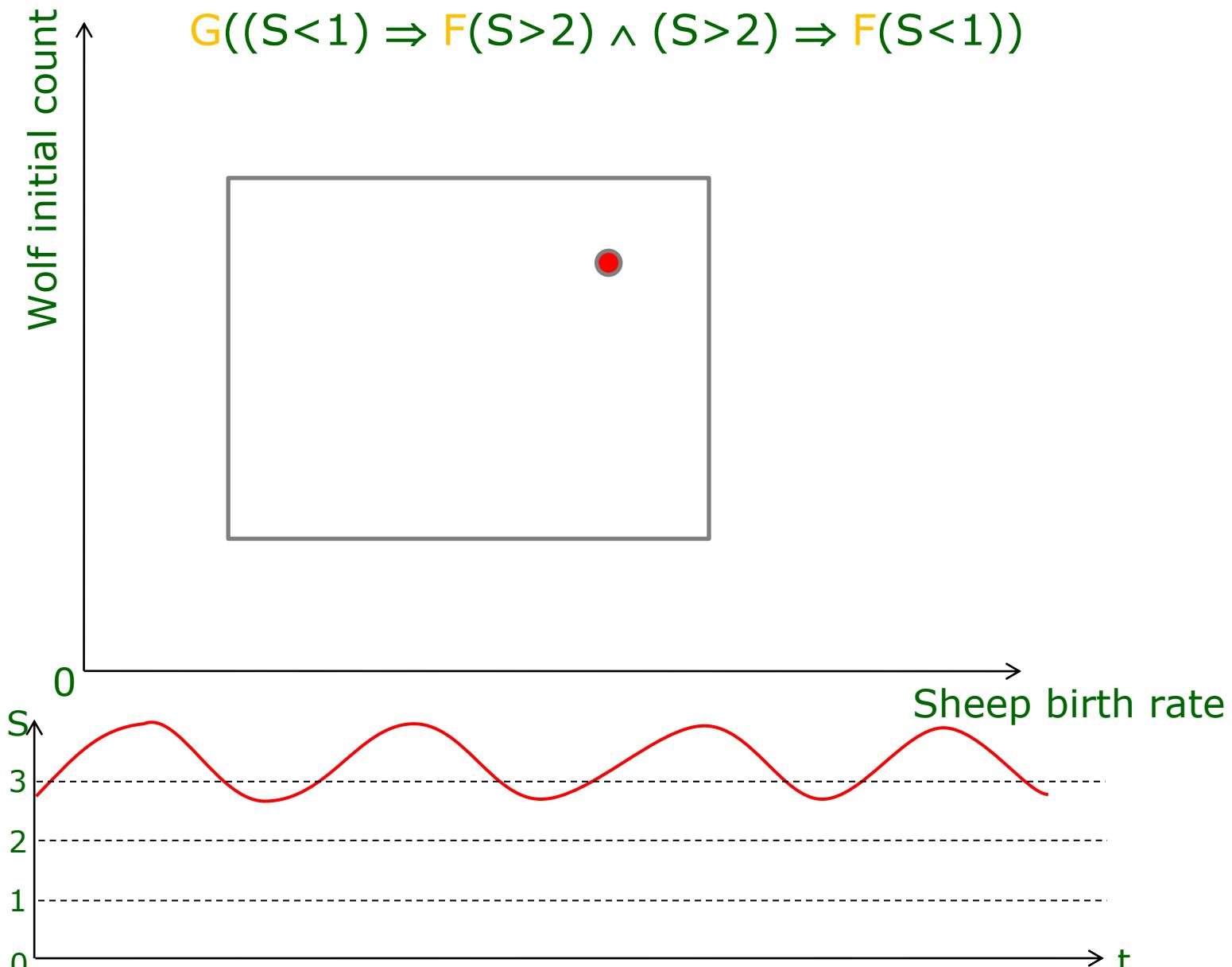
# Computing robustness



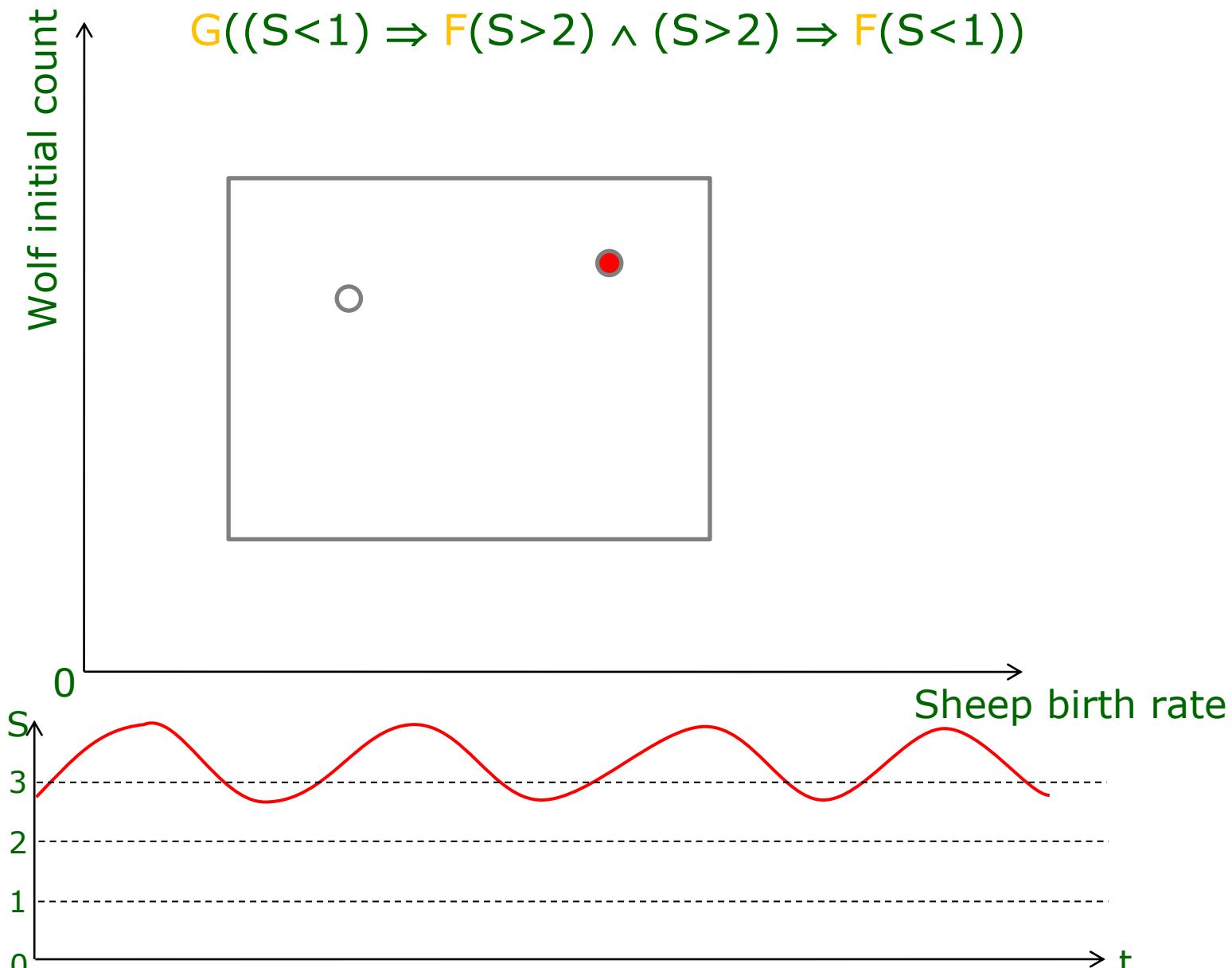
# Computing robustness



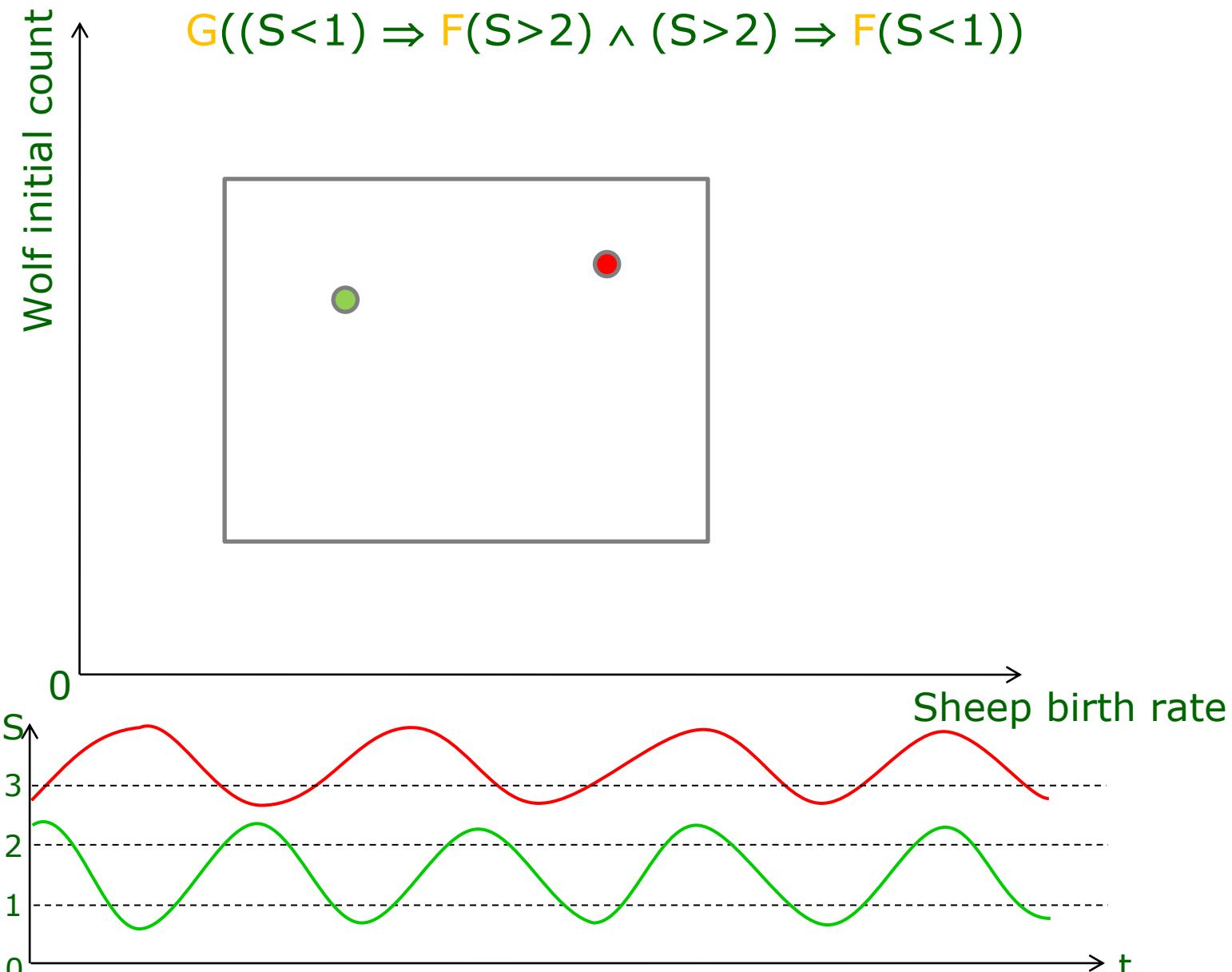
# Computing robustness



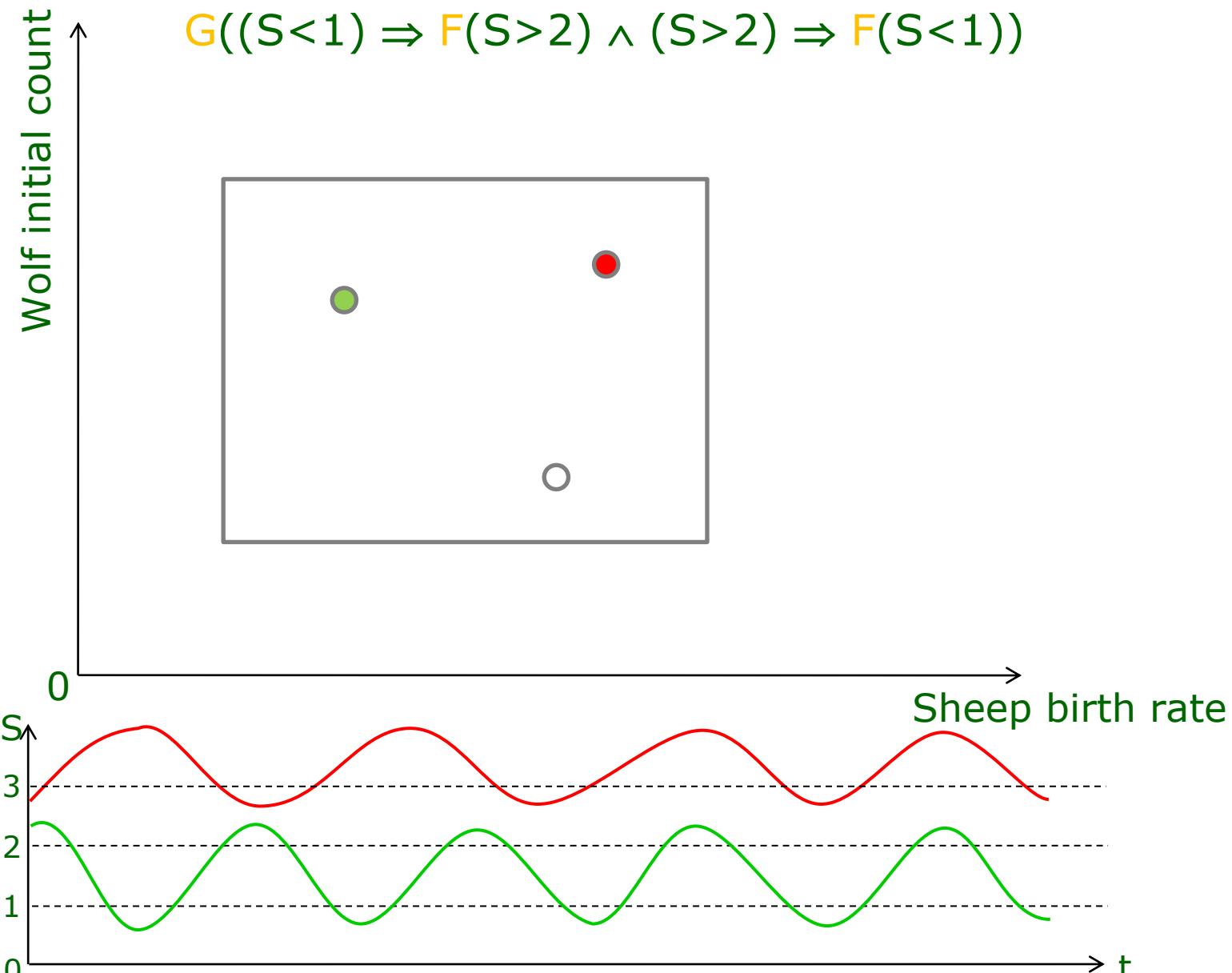
# Computing robustness



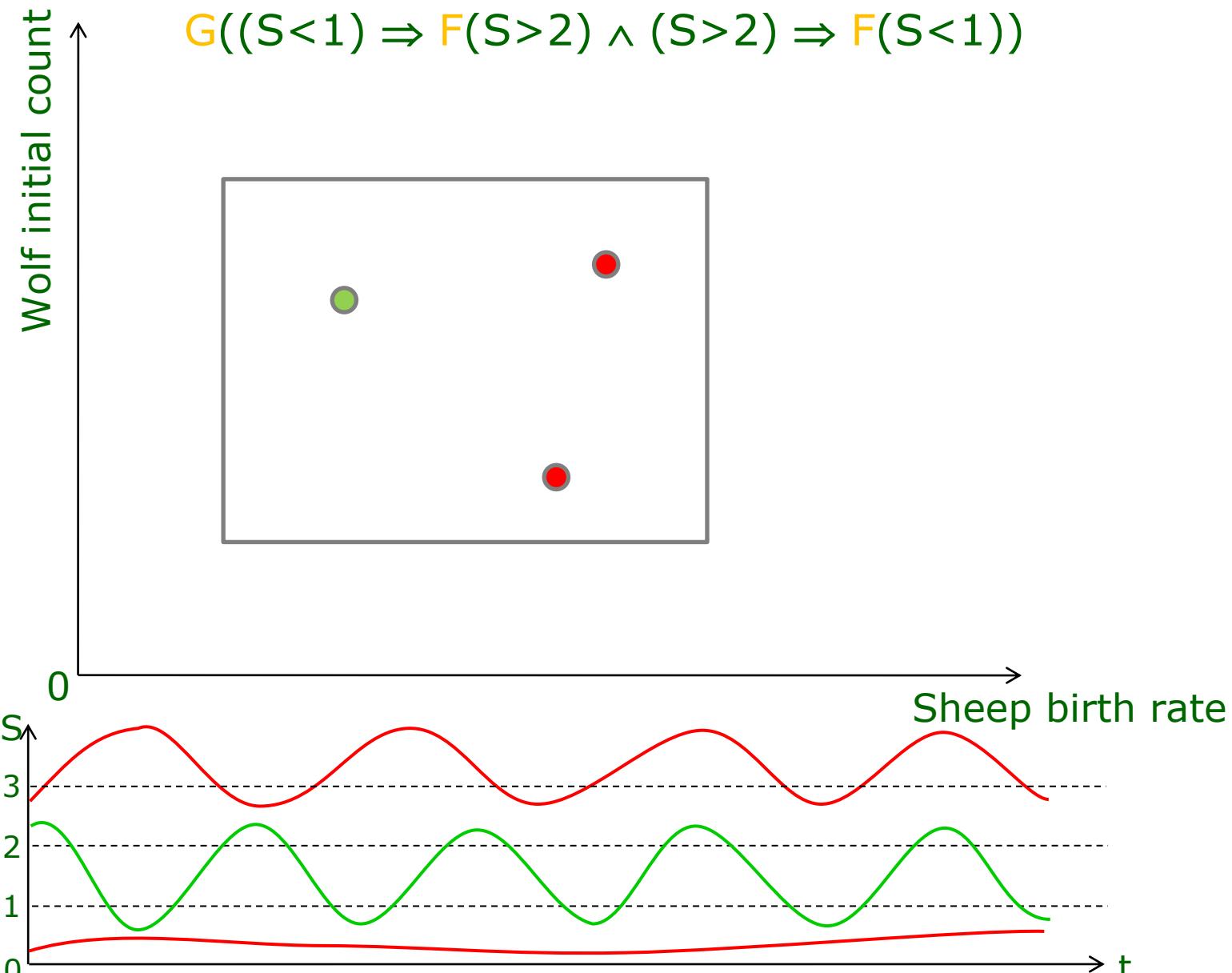
# Computing robustness



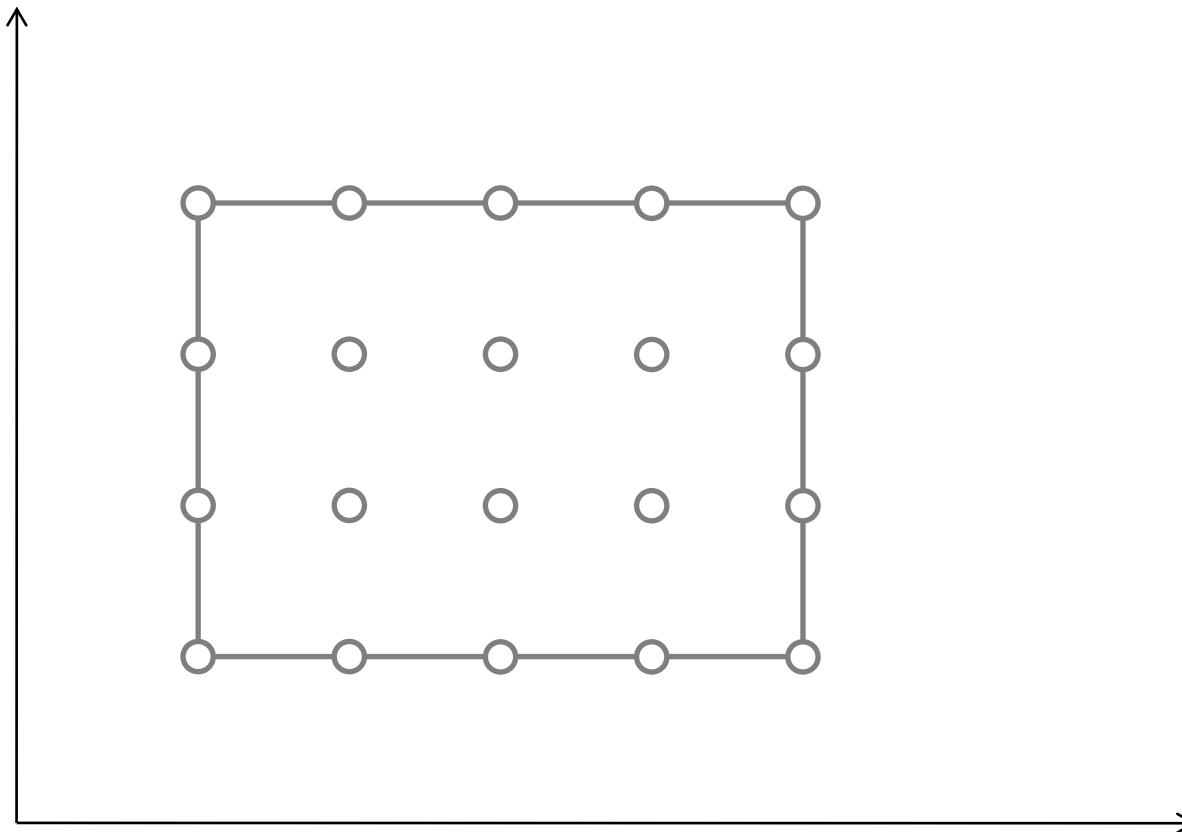
# Computing robustness



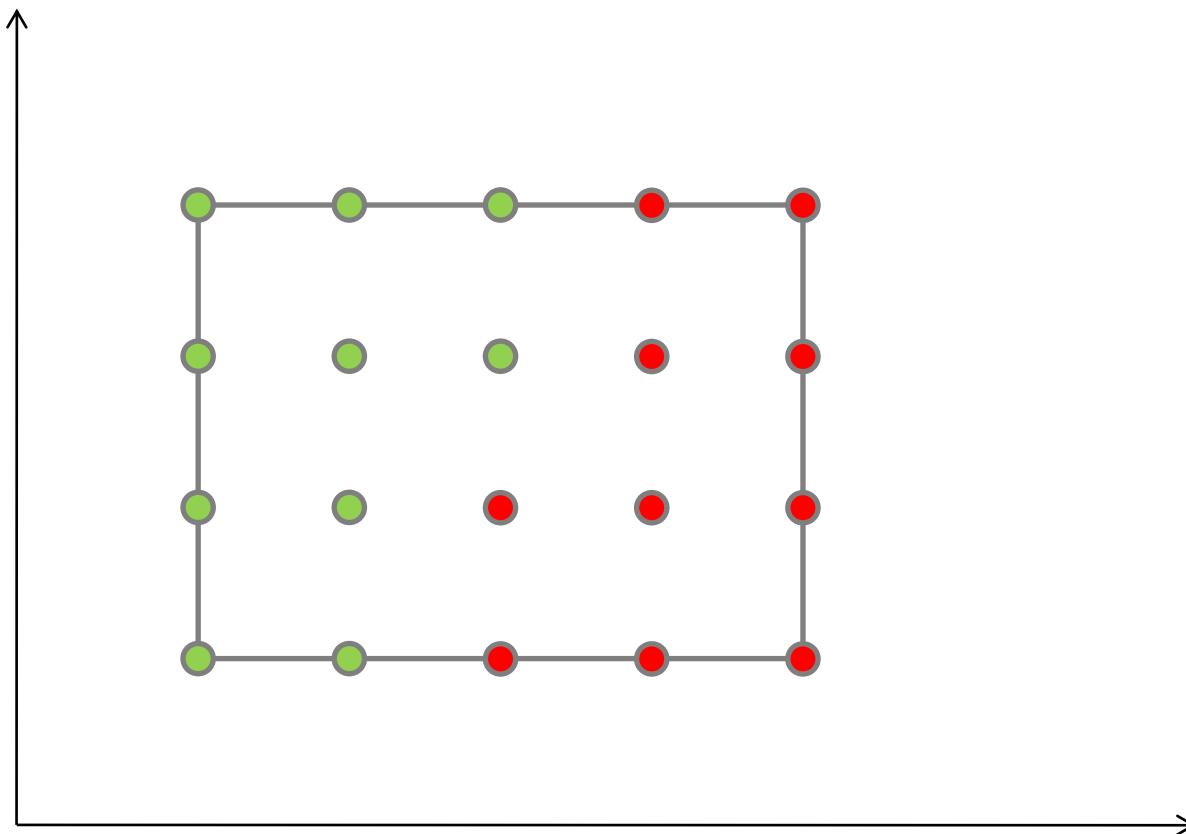
# Computing robustness



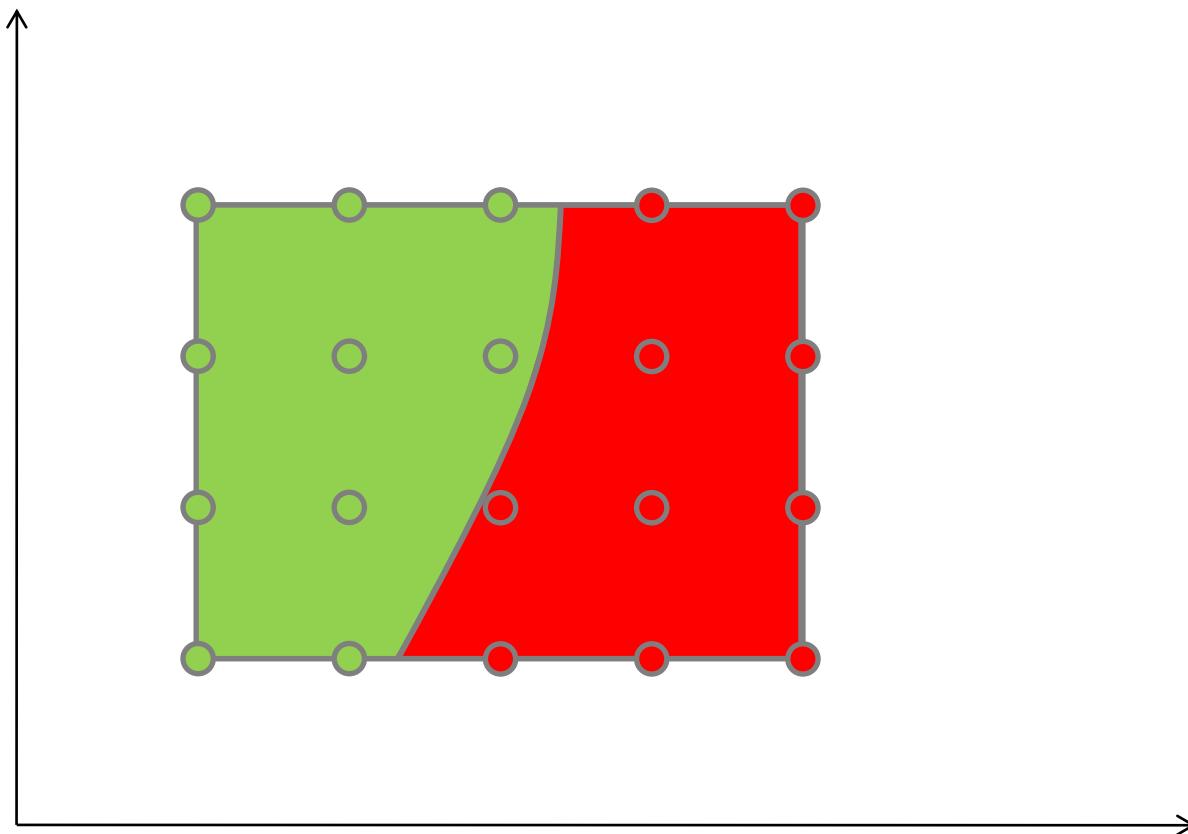
# Computing robustness – continuity problem



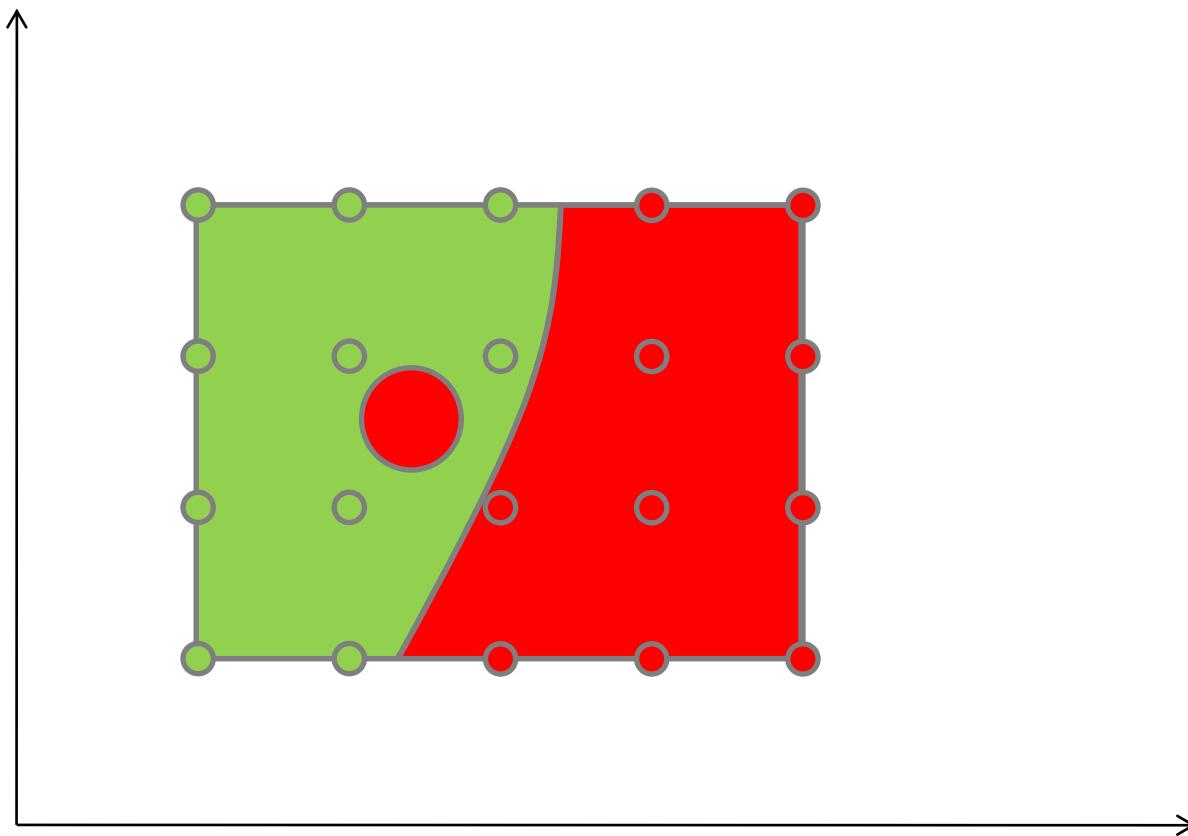
# Computing robustness – continuity problem



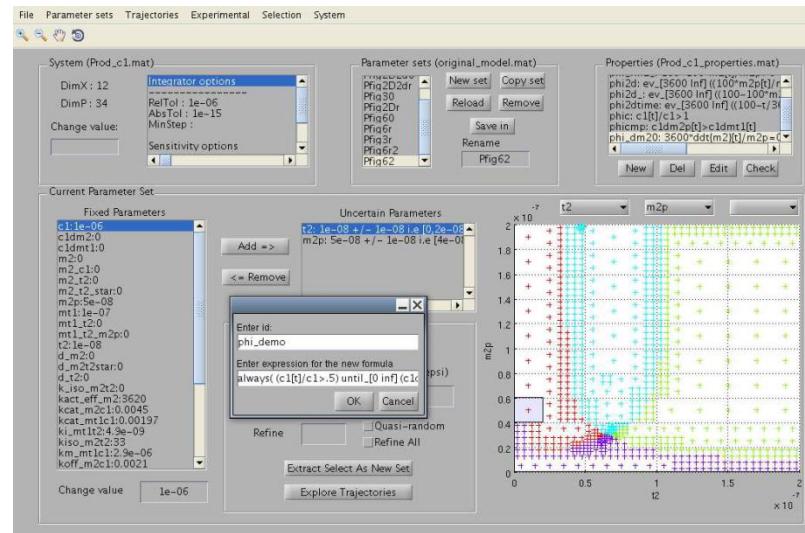
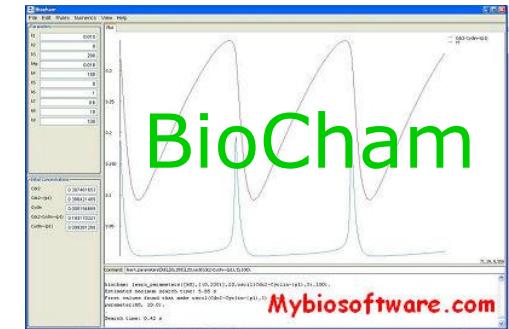
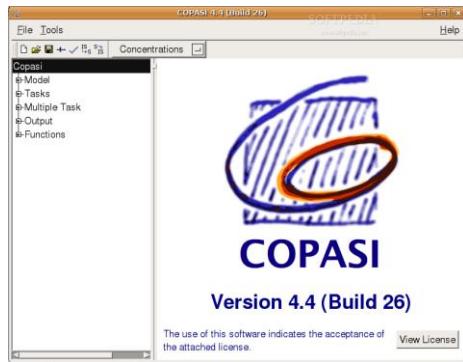
# Computing robustness – continuity problem



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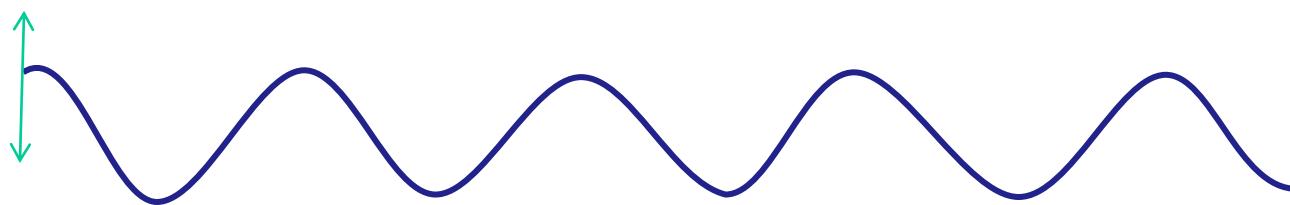


# Current approaches

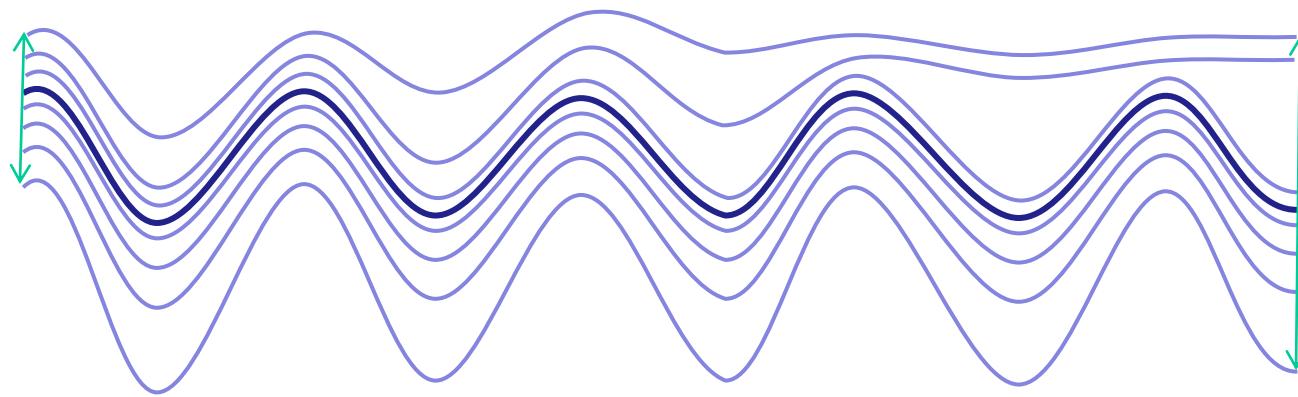


Breach (Donzé A. and Maler O. 2010)

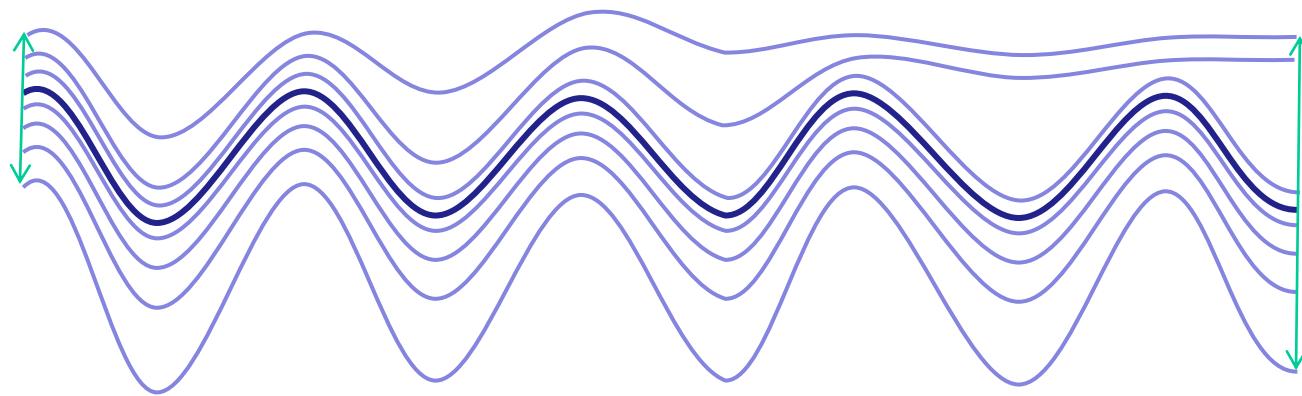
# How does a neighborhood behave?



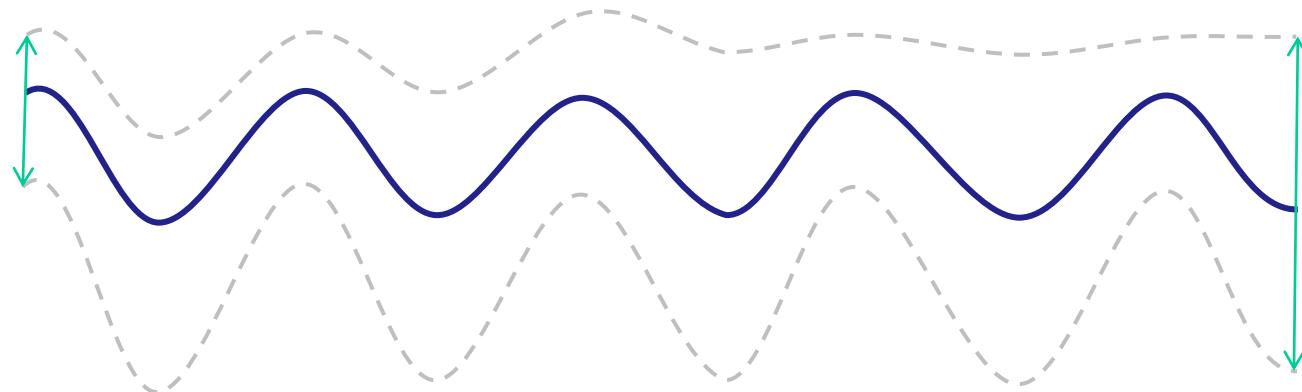
# How does a neighborhood behave?



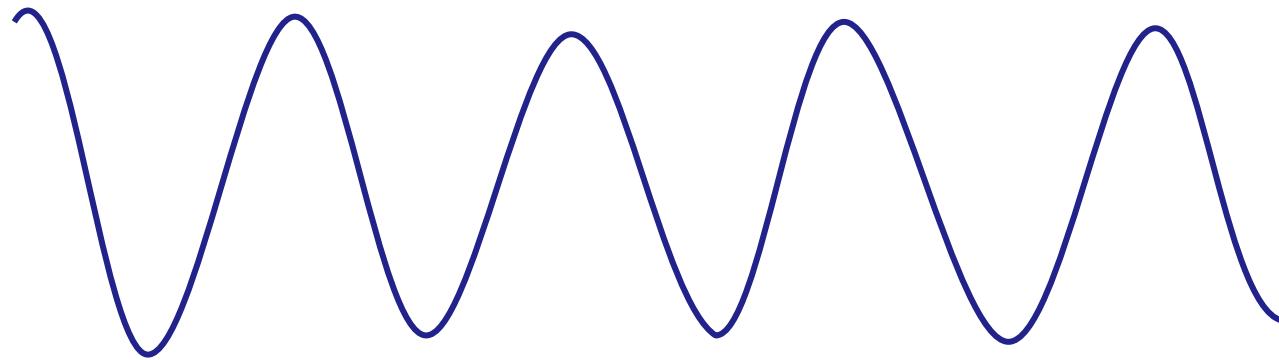
## How does a neighborhood behave?



## Sensitivity analysis

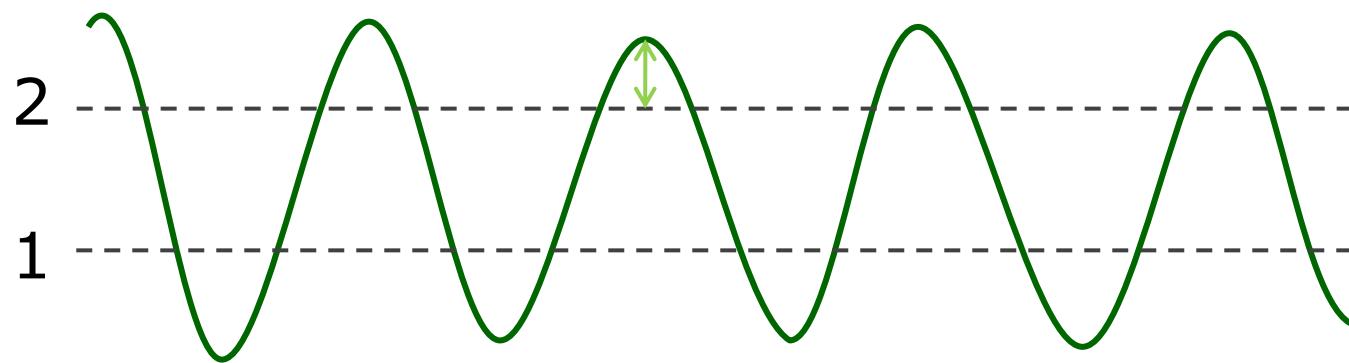


## Local property robustness

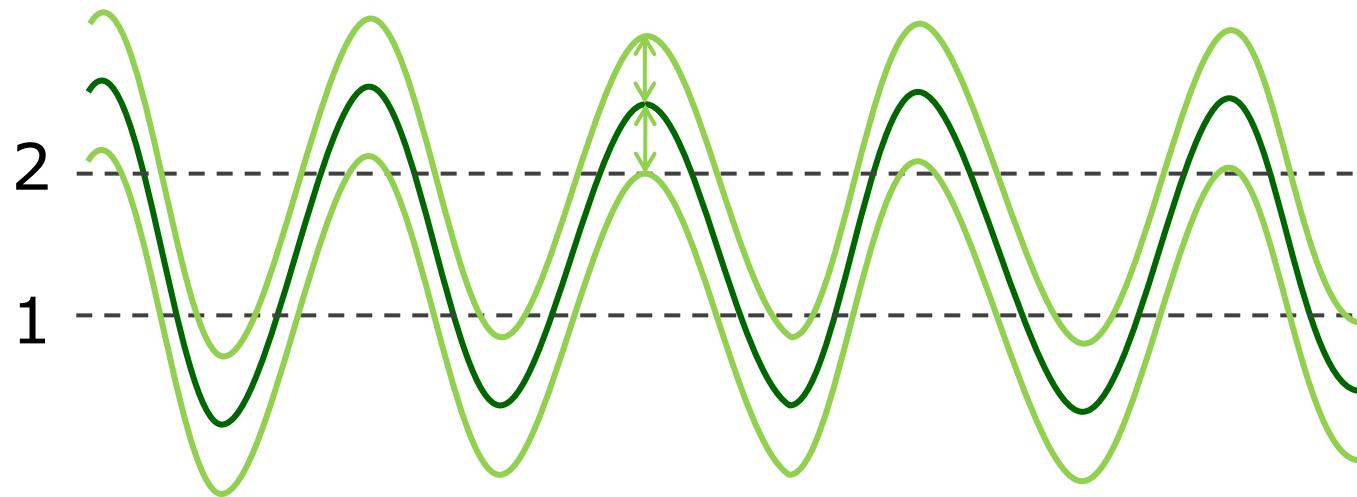


$$G((S < 1) \Rightarrow F(S > 2) \wedge (S > 2) \Rightarrow F(S < 1))$$

## Local property robustness

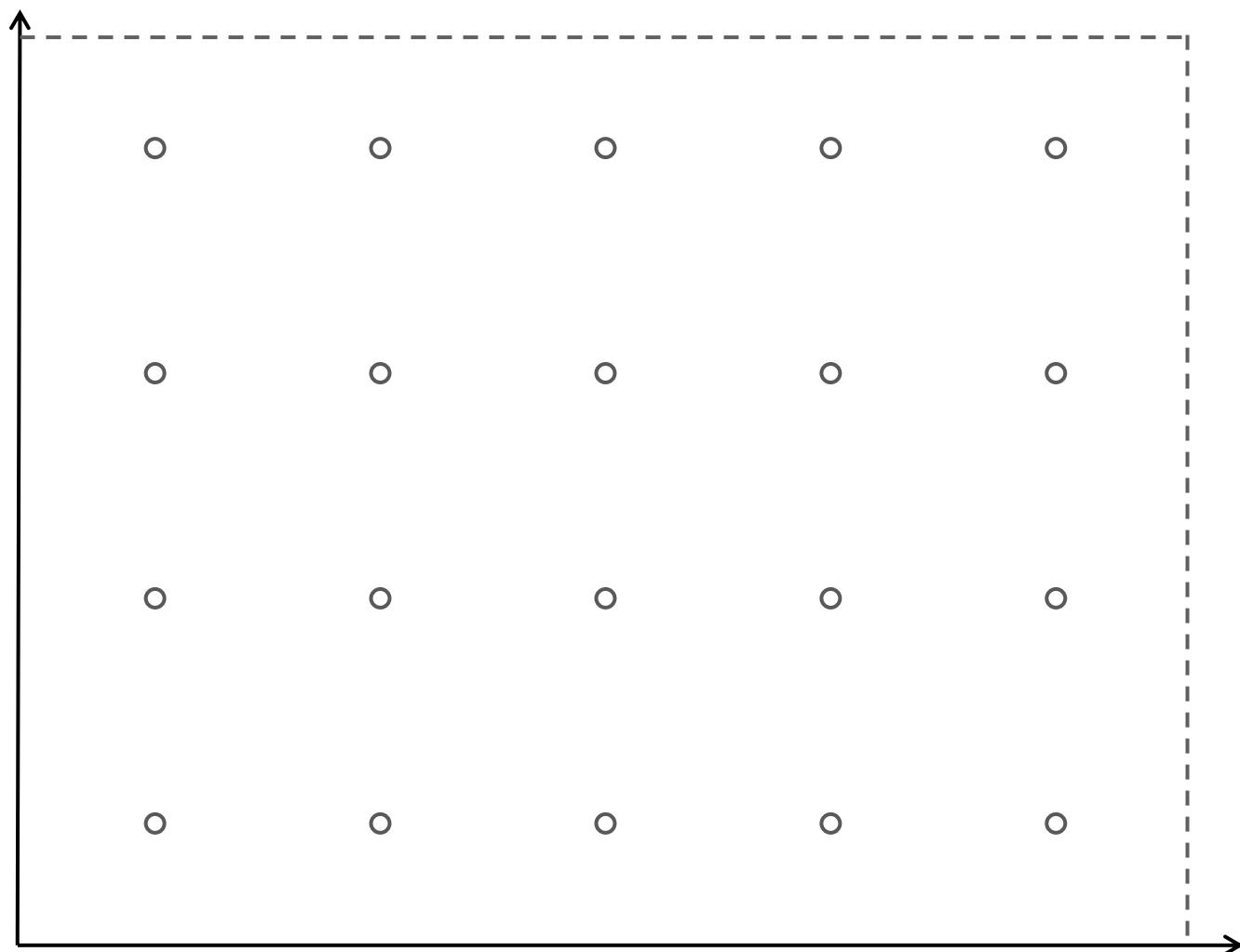

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## Local property robustness

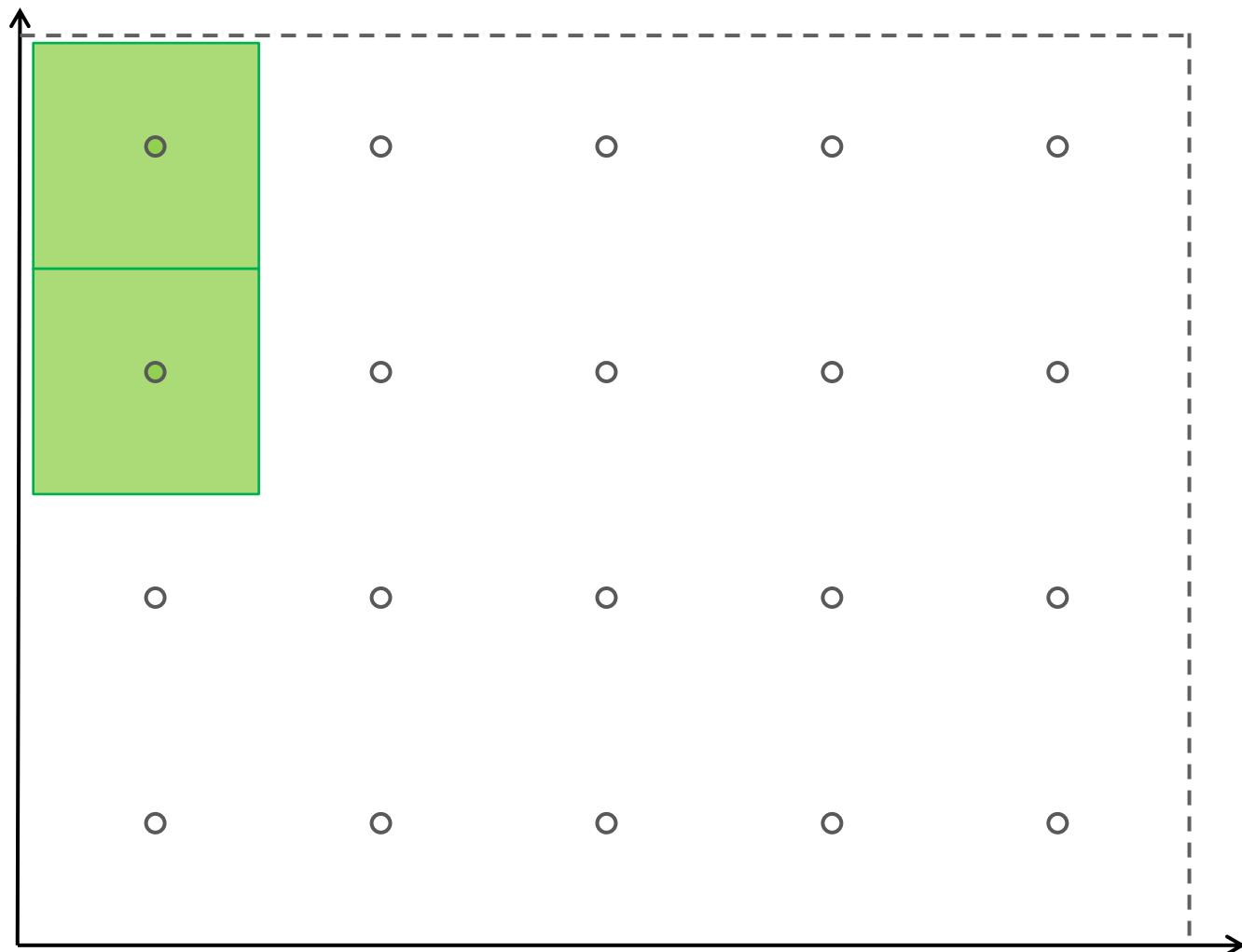


$$G((S < 1) \Rightarrow F(S > 2) \wedge (S > 2) \Rightarrow F(S < 1))$$

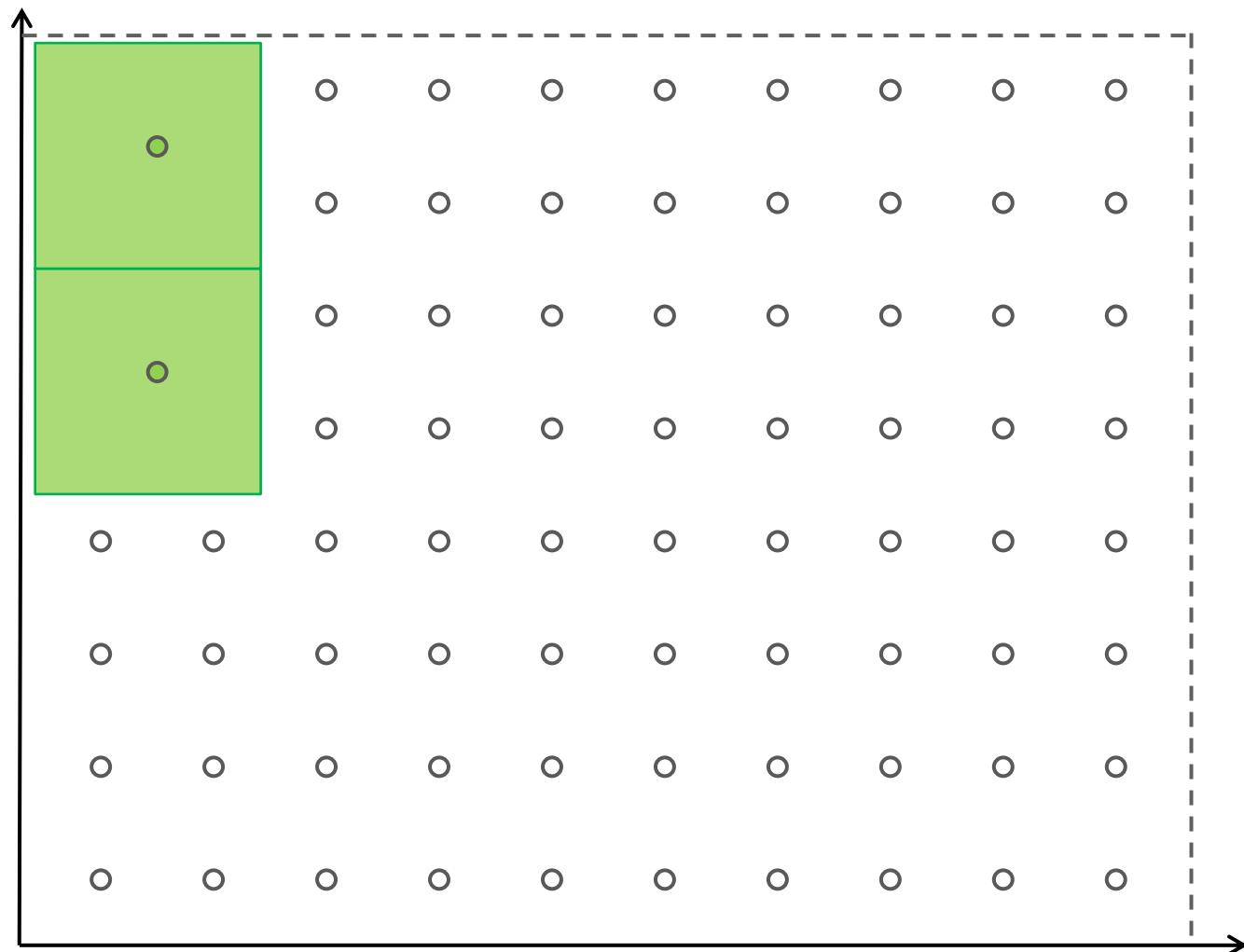
## Hierarchical refinement



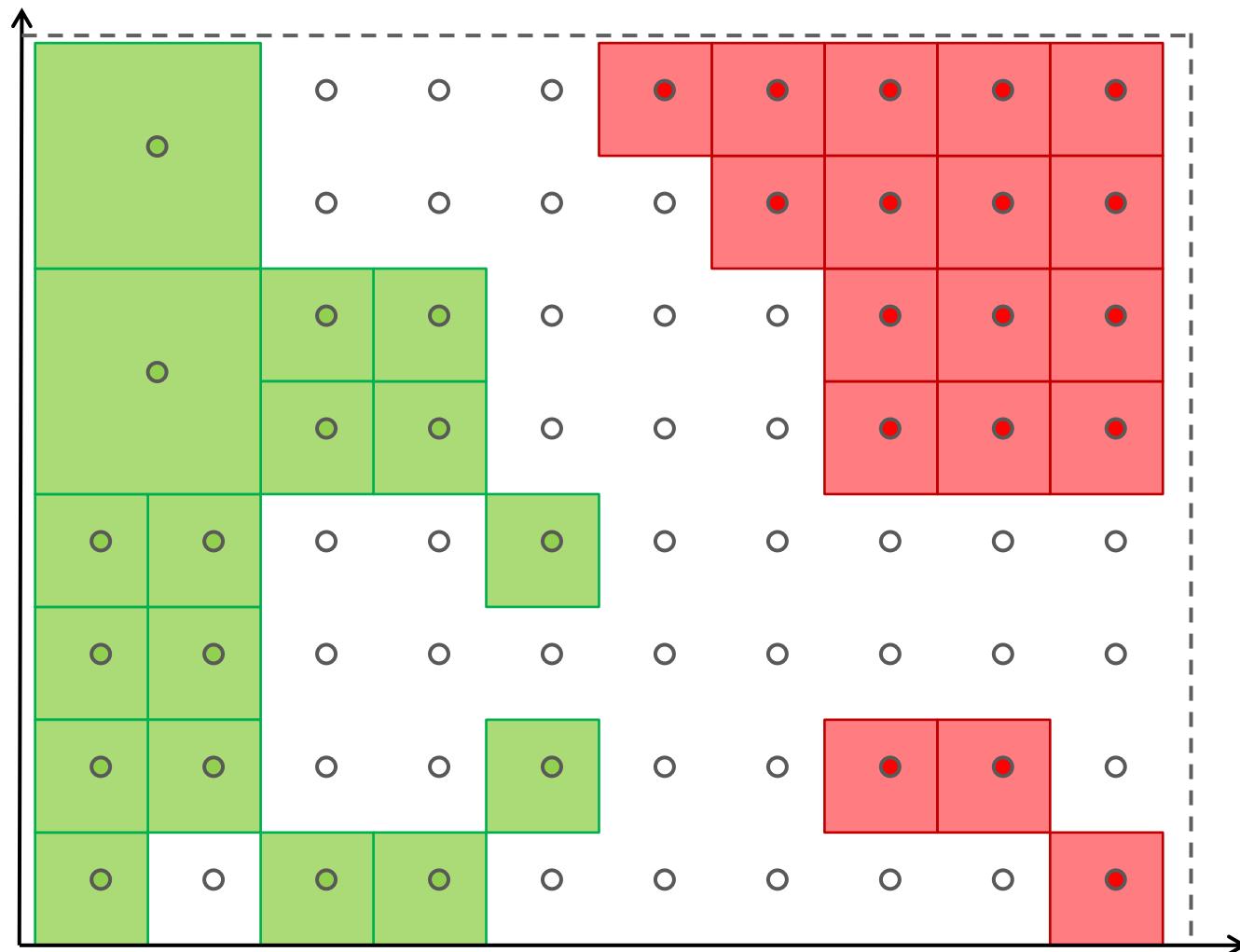
## Hierarchical refinement



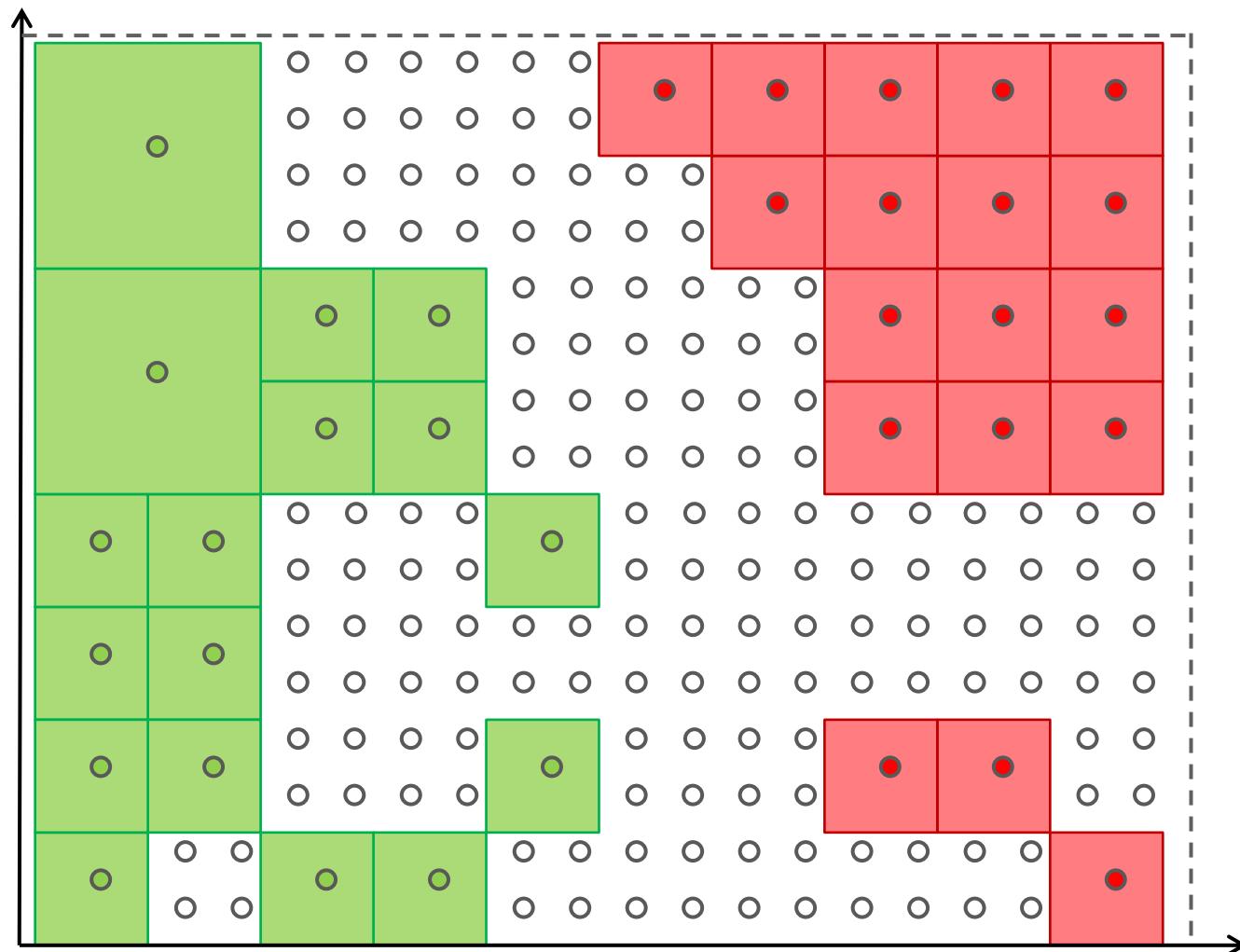
## Hierarchical refinement



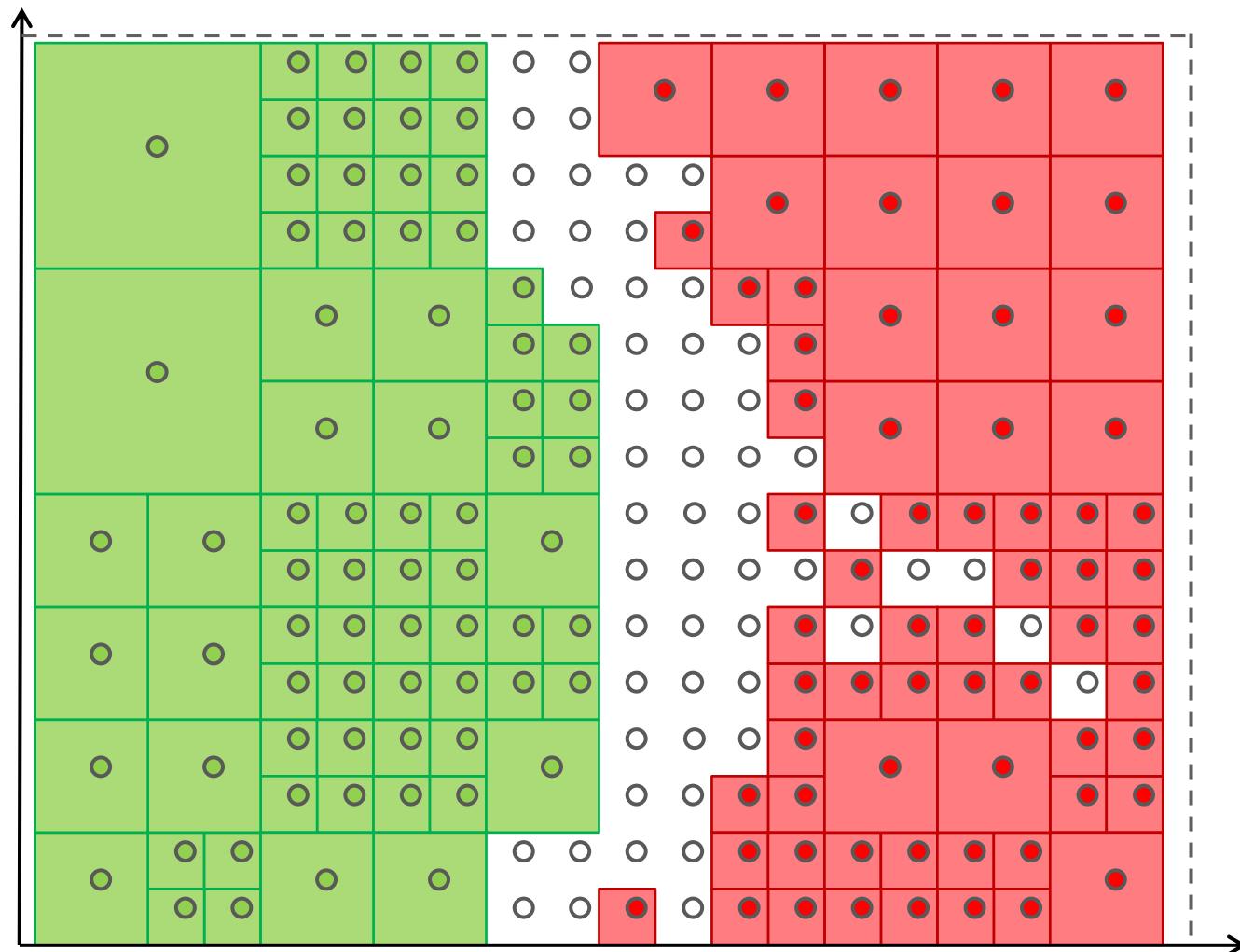
## Hierarchical refinement



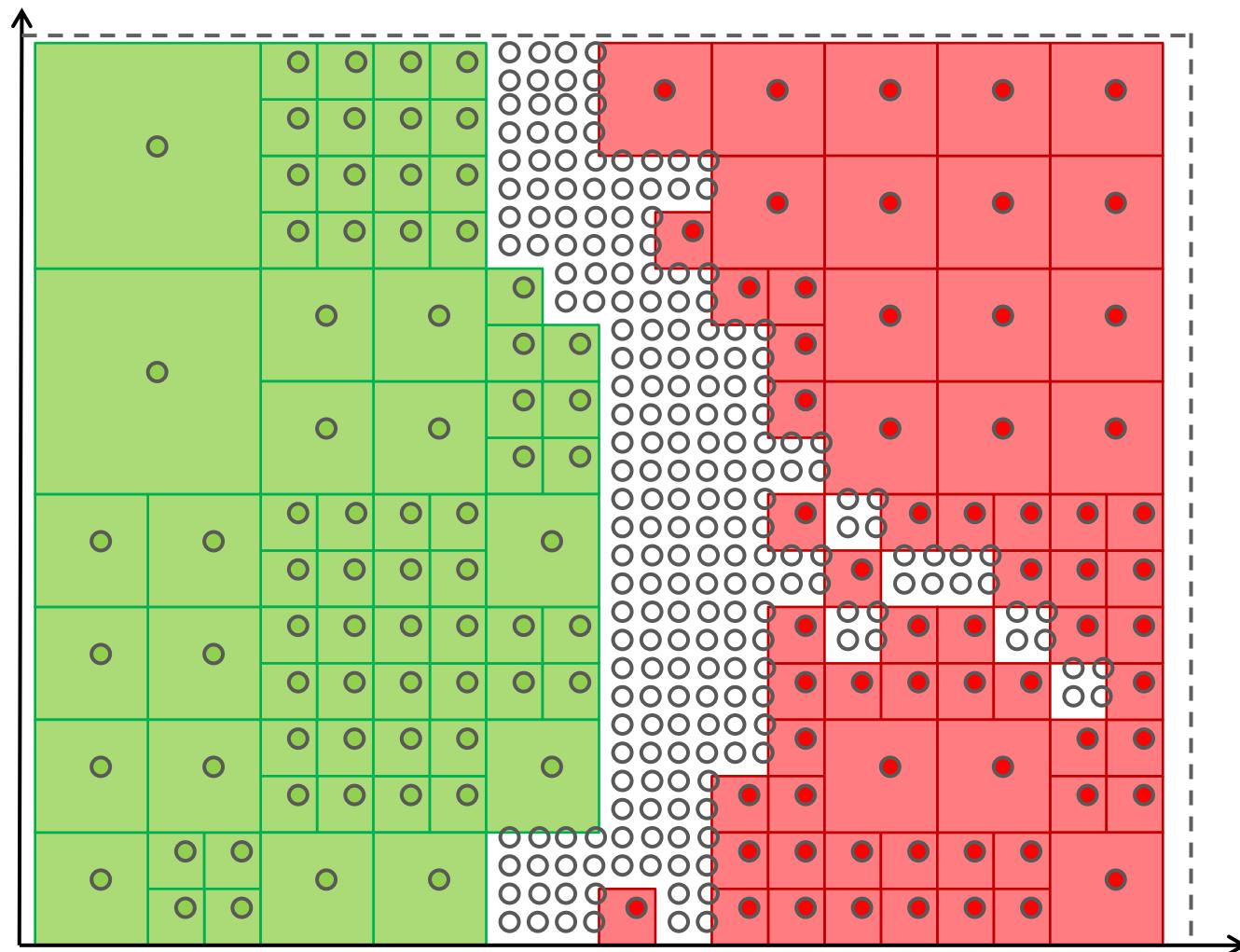
## Hierarchical refinement



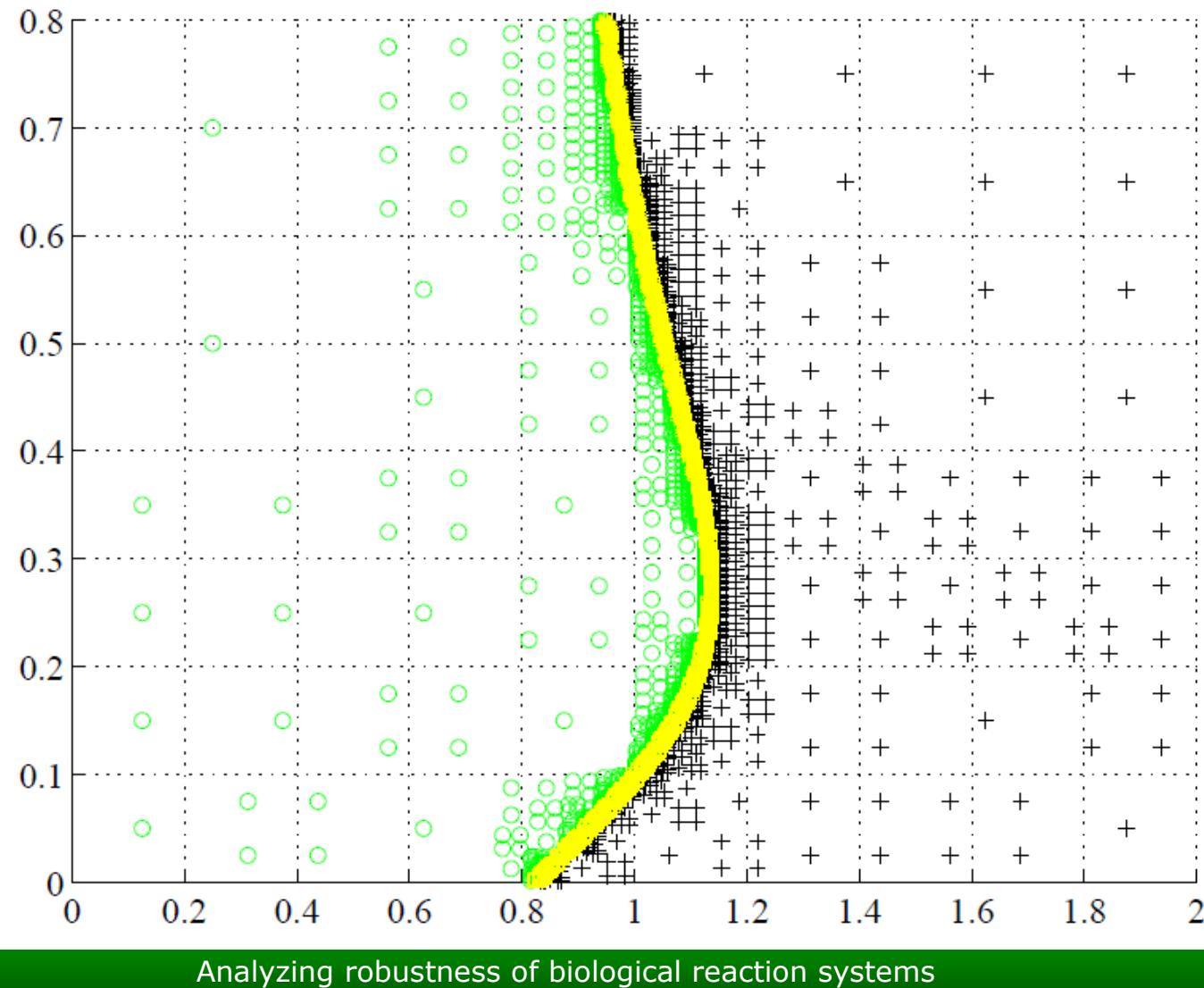
## Hierarchical refinement



## Hierarchical refinement

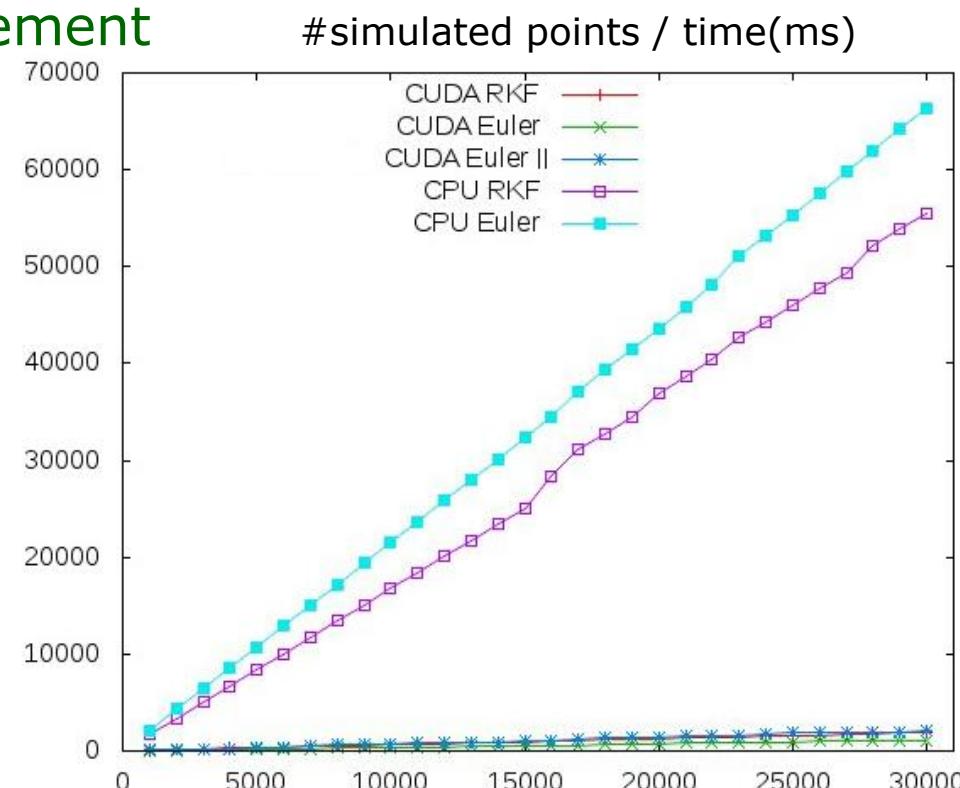


## Hierarchical refinement

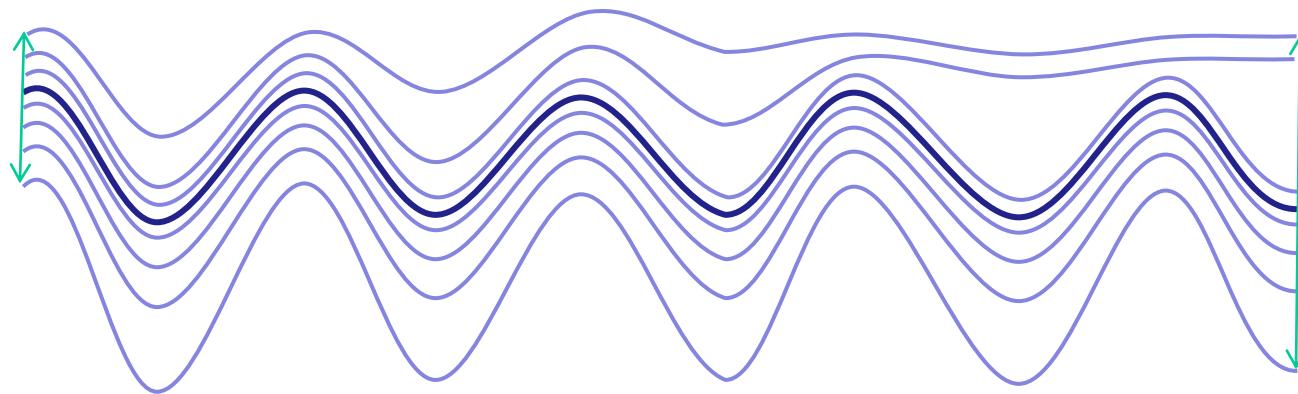


# Current results / Proposal

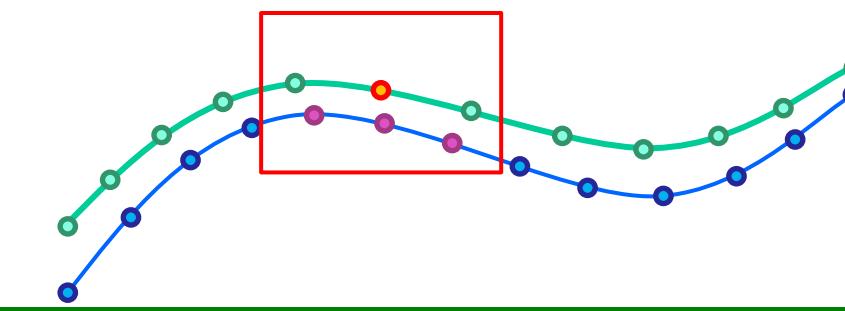
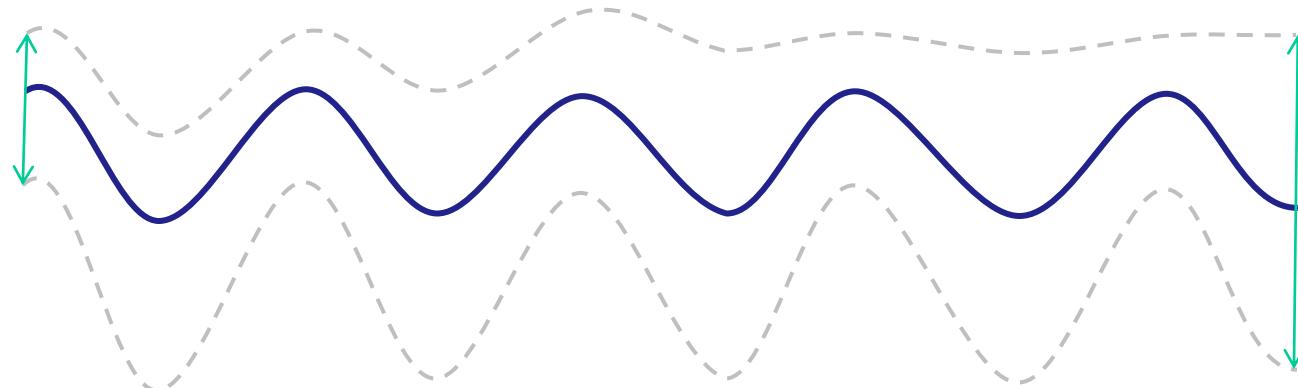
- Parallelization – faster / bigger models
- GPUs (CUDA)
  - Many small processors
  - Small local memory
- Distance checking instead of sensitivity
- Different hierarchical refinement



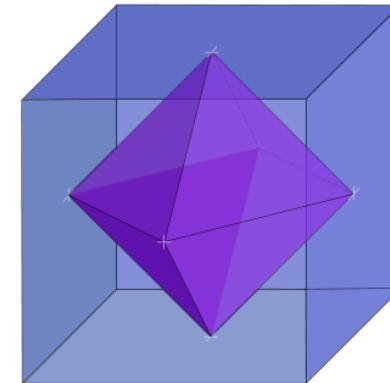
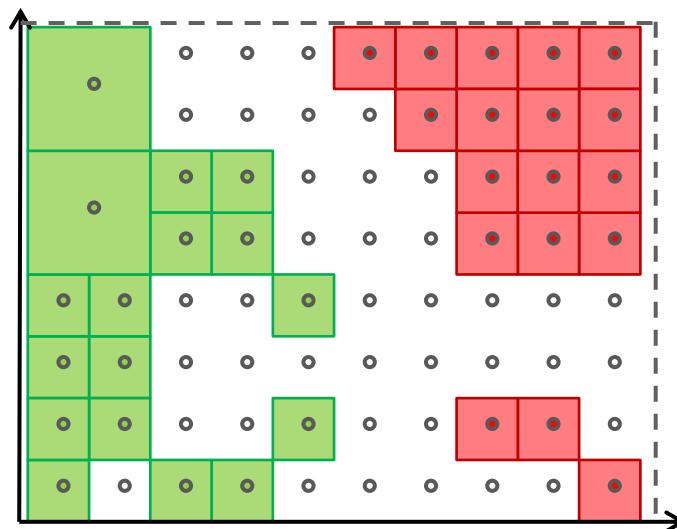
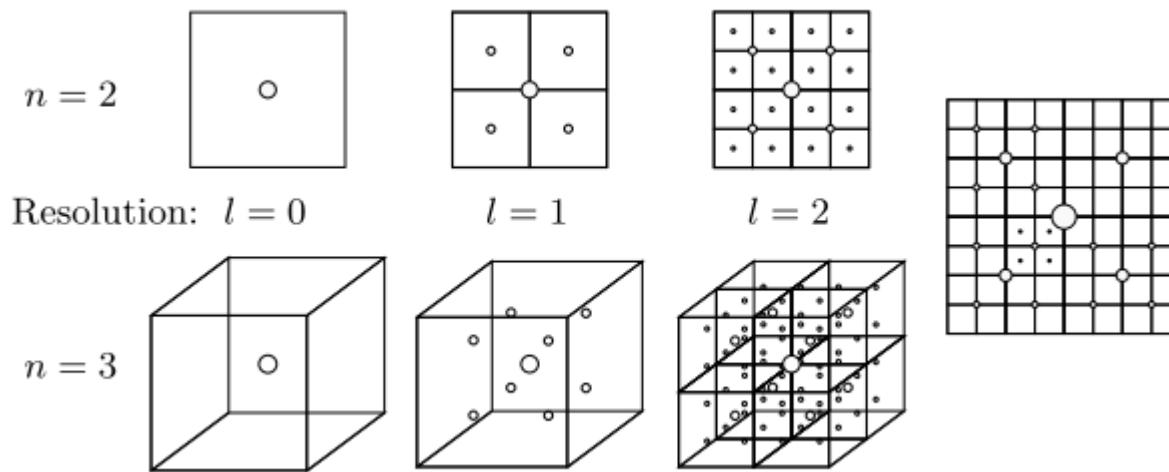
# Current results / Proposal / Distance checking



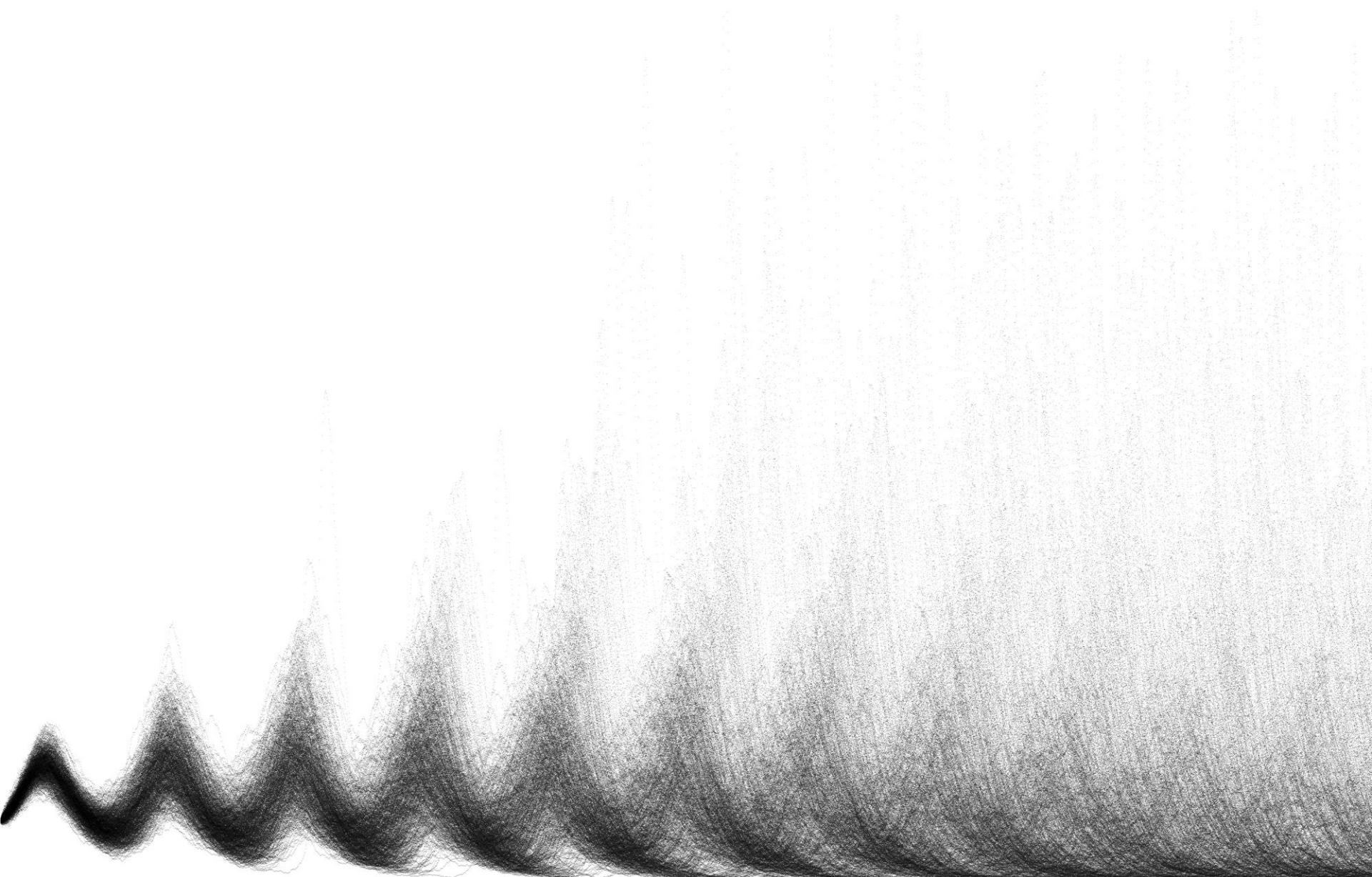
## Sensitivity analysis



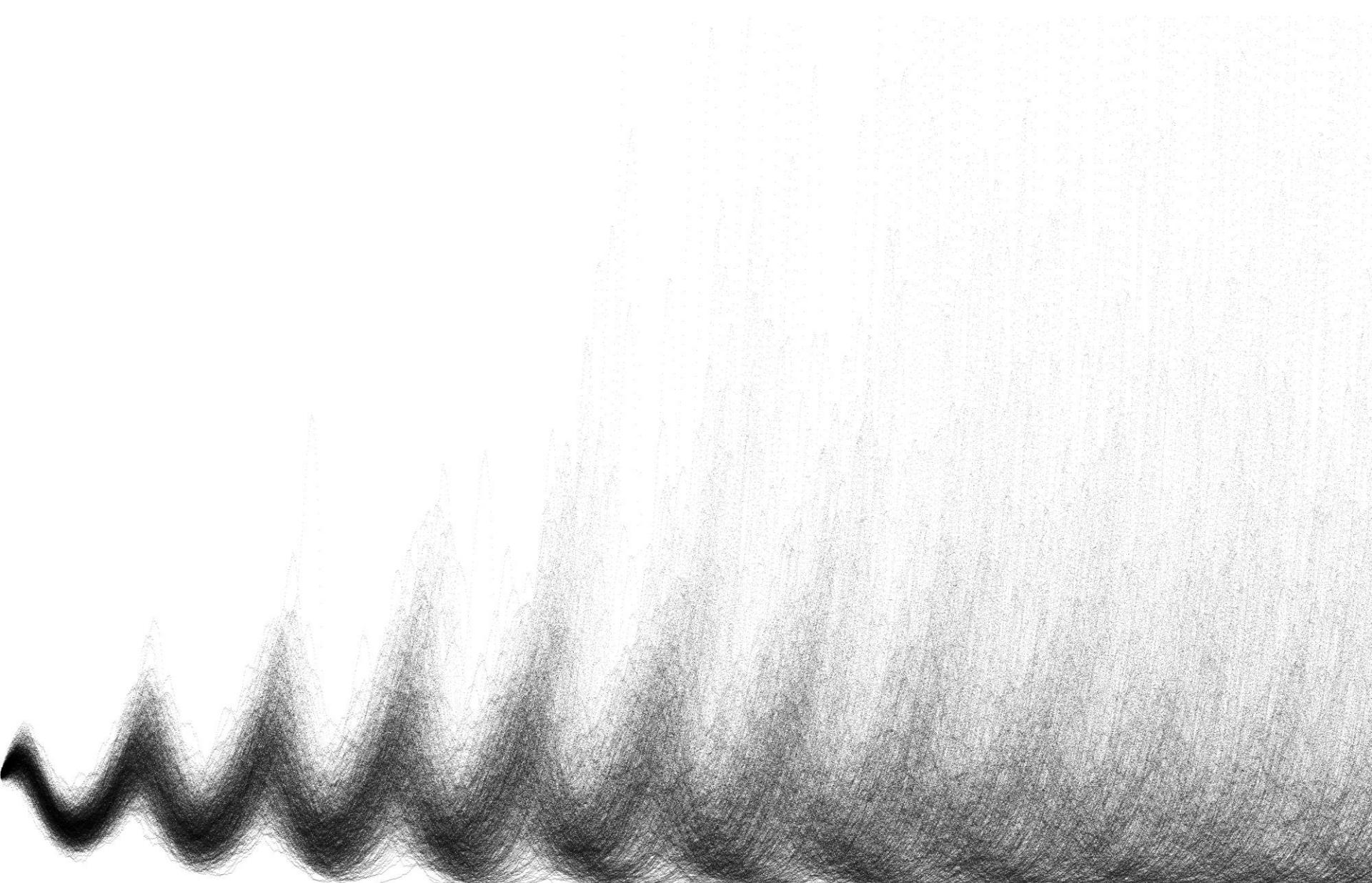
# Current results / Proposal / 2n-hierarchical refinement



# Robustness of Stochastic systems



# Robustness of Stochastic systems



# Summary and Conclusion

## What have you seen

- What is robustness
- Models of biological systems
- Expressing properties
- Current approaches to robustness of continuous systems
- Innovations in computing robustness of continuous systems
- Robustness of Stochastic systems

Questions and comments are welcome



Thank you for your attention.

# Sources

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  - [http://en.wikipedia.org/wiki/Snowshoe\\_Hare](http://en.wikipedia.org/wiki/Snowshoe_Hare)
  - <http://en.wikipedia.org/wiki/Lynx>
- **Other**
  - Jan Papoušek's Bachelor thesis: [https://is.muni.cz/auth/th/325494/fi\\_b/thesis.pdf](https://is.muni.cz/auth/th/325494/fi_b/thesis.pdf)
  - Human vs. Chimp: <http://www.sciencedirect.com/science/article/pii/S0002929707640968>