Modelling of Hollow Structures by Shell Elements

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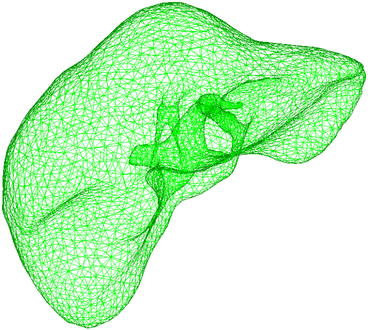
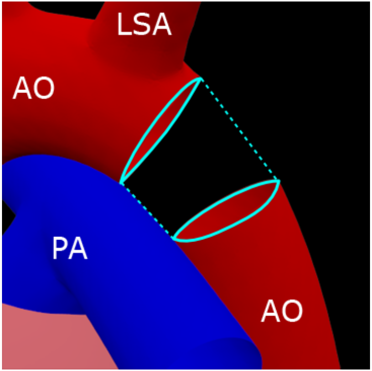
Content

Problem: Real-time Simulations

- Modelling of thin or hollow structures in 3D simulation of deformations
- Thickness in one dimension is significantly smaller than in the other two dimensions
- The work focuses on *real-time* simulations
- Main motivation are surgical simulators (but not limited to)
- Example use: deformation modelling, cutting & stitching

Examples of Application



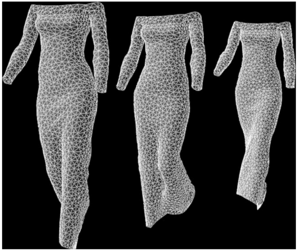


Available Methods

- Mass-Springs model
- Bending models
- Shell elements

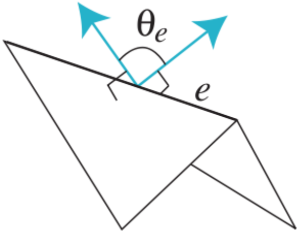
Mass-Springs Model

- Mesh of points interconnected by springs
- Numerically inexpensive
- Lacks physical accuracy
- Problem of ”magical” constants
- Often used in cloth modelling



Bending Models

- Purely geometrically based energy functions
- Bending energies usually based on angles
- The energy functions are physically motivated but not yet correct
- Often have severe limitations (e.g. explicit time integration) or are not always unconditionally stable



Shell Elements

- Continuum mechanics
- Formulation is physically based
- System is parametrized by physical constants (Young's modulus, Poisson's ratio, . . .)

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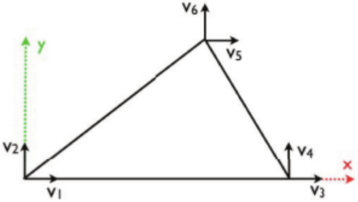
Deformation in the plane of the element.

Described by the plate theory.

Basically two types:

- only deformation by translation
- deformation by translation and rotation (*drilling*)



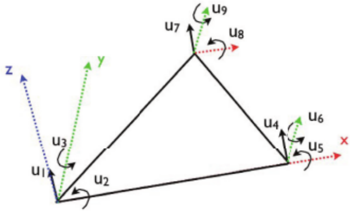
Shell Elements II

shell element = elastic membrane + bending plate

Off-plane deformations

Again basically two types:

- Thin plates: Kirchhoff-Love
- Thick plates: Mindlin (Mindlin-Reissner)



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The complexity grows quickly with number of elements.

Is There a Solution?

Observation

We usually don’t need so many elements for physical model as we need for visualisation/collisions/interaction.

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We exploit the bending property of the shells:

1. Coarse mesh for physical model
2. Finer model for the rest

Finer mesh is then mapped onto the bent surface of the element.

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Finer mesh is then mapped onto the bent surface of the element. To do that we need to know the geometry of deformed shell element.

Custom Shell Model

Not an original idea, O. Comas used a simple cubic polynomial to describe element geometry.
The solution has several flaws:

- No continuity between elements (only at nodes)
- No drilling degree of freedom
- Issue with non-symmetric deformations

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Our solution uses Bézier triangles to describe the geometry.

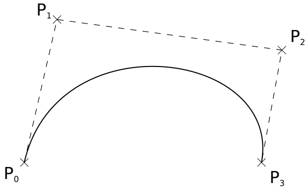
Bézier Curve

Parametric curve defined by a set of $n + 1$ control points

$$\mathbf{B}(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathbf{P}_i \ , \ t \in [0, 1]$$

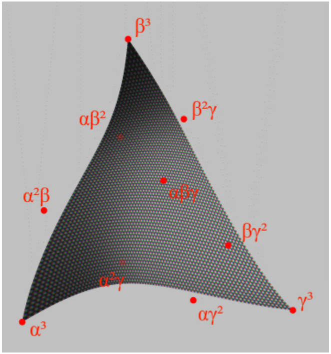
For $n = 3$:

$$\mathbf{B}(t) = (1-t)^3 \mathbf{P}_0 + 3(1-t)^2 t \mathbf{P}_1 + 3(1-t) t^2 \mathbf{P}_2 + t^3 \mathbf{P}_3$$



Bézier Triangle

Bézier triangle is a generalization to higher dimension



Our Shell Model

Our model solves all the issues of previous model:

- C^1 continuity across elements
- Drilling degree of freedom
- Symmetric deformations of the element

First tests show improvement in physical accuracy.

Bézier triangles are not new to graphics industry. This gives us some ready to use algorithms e.g. for subdivision.

Future Work

1. Techniques for remeshing/adaptivity on-the-fly
    - necessary for cutting
    - refinement in places with high curvature
2. Further improvements to the model to increase speed/accuracy

Thank you for your attention!