Classwork N°.2
due to 2nd March 2012

1. Combinatory base : **S** and **K**

   Proof of the completeness of the S-K basis

   Question: Calculate the following combinatorial expressions using the rules:

   S*xyz:=xz(yz)*

   K*xy:=x*

   I*x:=x*

   (1) **S K K** x

   =Kx(Kx)

   =x

   (2) **S (K S) K** x y z

   =KSx(Kx)yz

   =S(Kx)yz

   =Kxz(yz)

   =x(yz)

   (3) **S (K (S I)) K** x y

   =K(S I) x (K x) y

   =S I (K x)y

   =I y (K x y)

   =y (K x y)

   =(y x)

   (4) **S (K (S I)) (S (K K) I)** x y

=K(SI)x(S(KK)I x)y

=S I (S (K K)) I x)y

=I y (S (K K) I x y)

=y (S (K K) I x y

=y (K K x (I x) y)

=y (K(I x)y)

=y (I x)

=(y x)

2. Abstraction algorithm

T[] may be defined as follows:

1'| *T*[*v*] => *v*

2'| *T*[(E1 E2)] => (*T*(E1) *T*(E2))

3'| *T*[λx.E] => (**K** *T*[E])

4'| *T*[λx.x] => **I**

5'| *T*[λx.λy.E] => *T*[λx.*T* [λy.E]]

6'| *T*[λx.(E1 E2)] => (**S** *T*[λx.E1] *T*[λx.E2])

<u>**Question:**</u> Convert the lambda term *λx.λy.(yx)* to a combinator:

a)

T[λx.λy.(yx)]

=*T*[λx.T[λy.(yx)] (by 5)

=*T*[λx.(S T[λy.(y)] T[λy.x])] (by 6)

=*T*[λx.(S I T[λy.x])] (by 4)

=*T*[λx.(S I (Kx))] (by 3 and 1)

=(S *T*[λx.(S I)] T[λx. (Kx))] (by 6)

=(S K (S I)] T[λx. (Kx))] (by 3)

=(S K (S I)) (S T[λx. K] T[λx.x])) (by 6)

=(S K (S I)) (S (K K) T[λx.x])) (by 3)

=(S K (S I)) (S (K K) I)) (by 4)

b) λx.λy.(xy)

=T[λx.λy.(xy)]

=T[λx.T[λy.(xy)]] (by 5)

=T[λx.(S T [λy.x] T [λy.y])] (by 6)

=T[λx.(S T [λy.x] I)] (by 4)

=K S (T[λy.x] I) (by 3)

=K S K x I (by 3)

=K S x (by e.-K)

=S (by e.-K)