Classwork N°3
due to 9th March 2012

1. Normal form

When a combinatorial expression X cannot be anymore reduced by reaching to an expression x, we say that x is the normal form of X. Give the normal form of the given combinatorial expressions. The β-reduction rule of the basic combinators is given in the following:

$$\mathbf{B}xyz \geq_\beta x(yz)$$

$$\mathbf{C}xyz \geq_\beta x(zy)$$

$$\mathbf{S}xyz \geq_\beta xz(yz)$$

$$\mathbf{I}x \geq_\beta x$$

$$\mathbf{K}xy \geq_\beta x$$

$$\mathbf{W}xy \geq_\beta xyy$$

$$\mathbf{\Phi}xyzu \geq_\beta x(yu)(zu)$$

$$\boldsymbol{\psi}\, xyzu \geq_\beta x(yz)(yu)$$

(1) ***WK****x*

(2) ***BCC****xyz*

(3) ***CSI****fx*

(4) ***SS(KI)****fx*

(5) ***B(BS)B****fxyz*

(6) ***BB(BB)****fgxgy*

(7) ***S(BBS)(KK)****xyz*

(8) ***B(BW(BC))(BB(BB))****fgxy*

(9) ***Φ(Φ(ΦB))B(KK)****fgxy*

Please comment the definitions that you could find by reducing the given combinators. For example, is the definition **[W≡SS(KI)]** *an acceptable definition according your calculus?*