IA165

Combinatory Logic for Computational Semantics

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Juyeon Kang

qkang@fi.muni.cz

B410, Faculty of Informatics, Masaryk University, Brno, Czech Rep.

- Tom is mortal → is-mortal(Tom)
- Dick is mortal → is-mortal (Dick)
- Fido is mortal → is mortal (Fido)

----------------> Everything is mortal → is-mortal(everything)

→ "?" is-mortal(x)

Quantification_Introduction1

- **Universal quantifier**

  The expression: ∀x P(x), denotes the universal quantification of the atomic formula P(x).

  - ∀ is called the universal quantifier, and x means all the objects x in the universe. If this is followed by P(x) then the meaning is that P(x) is true for every object x in the universe.
  - For example, "*All cars have wheels*" could be transformed into the propositional form, ∀x P(x), where:
    - P(x) is the predicate denoting: x *has wheels*, and
    - the universe of discourse is only populated by *cars*.

- Socretes is handsome → is–handsome(Socretes)
- Tom is handsome → is–handsome (Tom)
- Harry is handsome → is handsome (Harry)

––––––––––––––––> Something is handsome → is–handsome(something)

→ "?" is–handsome(x)

Quantification_Introduction2

- <u>Existential quantifier</u>

  The expression: $\exists x P(x)$, denotes the existential quantification of $P(x)$.

  - "There exists an x such that $P(x)$" or "There is at least one x such that $P(x)$".
  - $\exists$ is called the existential quantifier, and x means at least one object x in the universe. If this is followed by $P(x)$ then the meaning is that $P(x)$ is true for at least one object x of the universe.
  - For example, "*Someone loves you*" could be transformed into the propositional form, $\exists x P(x)$, where:
    - $P(x)$ is the predicate meaning: x *loves you*,
    - The universe of discourse contains (but is not limited to) all living creatures.

Quantification_Preliminary work1

- <u>Quantifiers:</u> "universal" and "existential"

  Natural language quantifiers have traditionally been categorised as either type <a> or type <b> quantifiers.

  <a>: Quantifiers of type <a> are properties of sets and are expressed through pronouns like *nothing, everybody* or *no one*. They combine with a verb phrase to form a sentence:

Everybody enjoyed the party.

<b>: Quantifiers of type <b> are binary relations between sets and are expressed through determiners like *some*, *all* or *no*. They combine with a noun phrase (the restriction of the quantifier) and a verb phrase (its scope) to form a sentence:

All guests enjoyed the party.

Quantification_Preliminary work2

- Theories of quantification

a. Fregean teories with bound variables

1. Classical theory in First-Order Language
2. Montague's quantification expressed in Church's λ-Calculus

b. Fregean theory without bound variables

3. Illative theory expressed in Curry's Combinatory Logic

Examples

a) Fregean analysis of Quantifiers in First-order language

b) Logical representations of quantifiers using Church's λ-calculus

Everybody is pretty

a) $(\forall x)[\text{is-pretty'}(x)]$

b) $(\lambda P.((\forall x)[P(x)])(\text{is-pretty'}))$

Every girl is pretty

a) $(\forall x)[\text{girl'}(x) \Rightarrow \text{is-pretty'}(x)]$

b) $(\lambda P.\lambda Q((\forall x)[P(x) \Rightarrow Q(x)])(\text{girl's})(\text{is-pretty'}))$

Some is pretty

a) $(\exists x)[\text{is-pretty'}(x)]$

b) $(\lambda P.((\exists x)[P(x)])(\text{is-pretty'}))$

Some girl is pretty

a) $(\exists x)[\text{girl'}(x) \,\&\, \text{is-pretty'}(x)]$

b) $(\lambda P.\lambda Q((\exists x)[P(x) \,\&\, Q(x)])(\text{girl's})(\text{is-pretty'}))$

Quantification_Formal analysis

- Illative quantifiers in CL framework
  - Illative operators "represent" classical quantifiers inside Curry's Combinatory Logic formalism.
  - Illative operators are adjoined to the "pure" applicative formalism and their actions are defined, by means of elimination and introduction rules in Gentzen's Natural Deduction style, without using bound variables.

- Illative universal quantifiers: Π_1 and Π_2
  - Π_1 f: every is f
  - Π_2 fg: every f is g

→ These are propositions

The two quantifiers Π_1 and Π_2 are not independent since it is possible to define Π_2, inside Combinatory Logic, from Π_1 by the following relation between operators:

Definition of the universal quantifier

$$[\ \Pi_2 =_{def} ((B(CB_2)\Phi) => \Pi_1)\ ]$$

- This relation shows that the restricted illative quantifier Π_2 is defined by means of a Combinator $B(CB^2)\Phi$ that combines the implication operator => with the quantifier Π_1.

1/ $\Pi_2 fg$	hyp.
2/ [Π_2 =def $(\mathbf{B}(\mathbf{CB}^2)\mathbf{\Phi})$ => Π_1]	def. de Π_2
3/ $((\mathbf{B}(\mathbf{CB}^2)\mathbf{\Phi})$ => $\Pi_1)$ fg	rempl. 2., 1.
4/ $(\mathbf{CB}^2)(\mathbf{\Phi}$ =>) Π_1 fg	[e-**B**]
5/ $\mathbf{B}^2\ \mathbf{\Pi}_1$ $(\mathbf{\Phi}$ =>) fg	[e-**C**]
6/ $\mathbf{\Pi}_1$ $(\mathbf{\Phi}$ => fg)	[e-$\mathbf{B}^2$]

The elimination rule [e-Π_2] is deduced from [e-Π_1]:

1/ Π_1 (Φ => fg)	hyp.
2/ fx	hyp.
3/ (Φ => fg) x	[e-Π_1],1.
4/ =>(fx)(gx)	[e-Φ],3.

Definitions of the [e-Π_2] and [e-Π_1]:

$$\frac{\Pi_1 f}{fx}\,[e\text{-}\Pi_1] \qquad \frac{\Pi_2 fg \qquad f(x)}{g(x)}\,[e\text{-}\Pi_2]$$

Modus ponens

$$\frac{P \rightarrow Q\ ,\ P}{Q}$$

Comment: whenever an instance of "P → Q" and "P" appear by themselves on lines of a logical proof, "Q" can validly be placed on a subsequent line.

- Illative existential quantifiers: Σ_1 and Σ_2
  - "$\Sigma_1 f$" ("there is a f") → These are propositions
  - "$\Sigma_2 fg$" ("there is a f which is g")

- Expression of Σ_2 in terms of & (conjunction) and Σ1 :

Definition of the existential quantifier

$$[\ \Sigma_2 =_{def} (B(CB^2)\Phi)\ \&\ \Sigma_1\ ]$$

- Examples

Jane is pretty	→	(C*Jane)(is-pretty)
Everybody is pretty	→	Π_1(is-pretty)
Every girl is pretty	→	(Π_2(girl))(is-pretty)
Somebody runs	→	Σ_1(runs)
Some girl is pretty	→	(Σ_2(girl))(is-pretty)
Every boy love some girl	→	(Π_2(boy))(love(Σ_2(girl)))

$[\ \Pi_2 =_{def} ((B(CB^2)\Phi) \Rightarrow \Pi_1)\ ]$

- *Every man like itself*

1/ (every man) (like itself)

2/ Π_2 man (like itself)

3/ $((B(CB^2)\Phi) \Rightarrow \Pi_1)$ man (like itself)

4/ $(CB^2)(\Phi \Rightarrow) \Pi_1$ man (like itself)

5/ $B^2\ \Pi_1 (\Phi \Rightarrow)$ man (like itself)

6/ $\Pi_1 ((\Phi \Rightarrow)$ man (like itself))

7/ $((\Phi \Rightarrow)$ man (like itself)) x

8/ => (man *x*) ((like itself) *x*)

Definitions of the [e-Π_2] and [e-Π_1]:

$$\frac{\Pi_1 f}{fx}\ [e\text{-}\Pi_1] \qquad \frac{\Pi_2 fg \quad f(x)}{g(x)}\ [e\text{-}\Pi_2]$$

$$[\ \Sigma_2 =_{def} (B(CB^2)\Phi)\ \&\ \Sigma_1\ ]$$

- *Some girl is pretty* → *there is (exist at least one) a girl who is pretty*

1/ (some(girl))(is-pretty)

2/ $(\Sigma_2$(girl))(is-pretty)

3/ $((B(CB^2)\Phi)\ \&\ \Sigma_1$(girl)) (is-pretty)

4/ $((CB^2)\ (\Phi\ \ \&)\ \Sigma_1$(girl)) (is-pretty)

5/ $(B^2\ \Sigma_1\ (\Phi\ \ \&)$ (girl)) (is-pretty)

6/ $\Sigma_1\ ((\Phi\ \ \&)$ (girl) (is-pretty))

7/ $((\Phi\ \ \&)$ (girl) (is-pretty)) x

8/ & (girl(x)) ((is-pretty) x)

Definitions of the [e-Σ_2] and [e-Σ_1]:

$$\frac{\Sigma_1 f}{fx}\ [e\text{-}\Sigma_1] \qquad \frac{\Sigma_2 fg \qquad f(x)}{g(x)}\ [e\text{-}\Sigma_2]$$

Next week...

- Continue about the application of the combinators to natural language analysis: **Revision**