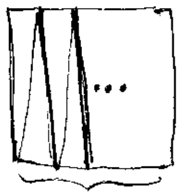
One square and an odd number of triangles

- Dividing square into an even number of triangles is quite easy
  (we want these triangles to have the same area)



2n triangles

n „separations"

- D. s. into an odd number may be tricky, <u>impossible even</u> !!

  When we divide a rectangle into n triangles ($n = 2k+1$ $k \in \mathbb{Z}$)
  then all triangles should have area $\frac{1}{n}$. But there is always
  at least <u>one with other area</u>

Valuation

$f : \mathbb{Q} \to \mathbb{R}_0^+$

- $f(x) = 0 \iff 0$
- $f(x \cdot y) = f(x) \cdot f(y)$
- $f(x+y) \leq f(x) + f(y)$

i.e. absolute value	x
$\lvert 0 \rvert = 0$ ( )	
$\lvert 2 \cdot (-3) \rvert = \lvert 2 \rvert \cdot \lvert -3 \rvert$	
$\lvert x+y \rvert \leq \lvert x \rvert + \lvert y \rvert$	

<u>Primes fact.:</u> $n = 2^{a_1} \cdot 3^{a_2} \cdot 5^{a_3} \cdot 7^{a_4} \ldots$

for $n \in \mathbb{Q}$ there might be some negative exponents $\frac{2}{15} = 2^1 \cdot 3^{-1} \cdot 5^{-1}$

$x = 2^{k} \frac{a}{b}$ $\quad |x|_2 = 2^{-k}$ valuation

$|1|_2 \Rightarrow 2^{1} \cdot \frac{1}{2} \Rightarrow 1$

$|2|_2 = \frac{1}{2}$

$|6|_2 = \frac{1}{2}$

! to make this satisfy <u>valuation axioms</u>, we have to define
$|0|_2 \overset{\text{def}}{=} 0$

$x = 2^k \frac{a}{b}$ $\quad y = 2^{\ell} \frac{c}{d}$

$x \cdot y = 2^{k+\ell} \frac{ac}{bd}$

$\begin{bmatrix} ac \text{ coprime } 2 \\ bd \text{ --- } 2 \end{bmatrix}$

$\circ \to |x \cdot y|_2 = 2^{-k-\ell} = 2^{-k} \cdot 2^{-\ell} = |x|_2 \cdot |y|_2$

o We should now show that this valuation satisfies valuation axiom 3 (triangle inequality)

But why won't we show and prove sth stronger? (Yes, why not?!)

T: $|x+y|_2 \leq \max\{|x|_2, |y|_2\}$ „non-Archimedian property"

Proof: $x = 2^k \frac{a}{b}$
$y = 2^\ell \frac{c}{d}$ Say $k \geq \ell$ $|x|_2 = 2^{-k} \leq 2^{-\ell} = |y|_2$

$|x+y|_2 = \left| 2^k \frac{a}{b} + 2^\ell \frac{c}{d} \right|_2 = \left| 2^\ell \cdot \left(2^{k-\ell} \frac{a}{b} + \frac{c}{d} \right) \right|_2$ (multiplication a.)

$= |2^\ell|_2 \cdot \left| 2^{k-\ell} \frac{a}{b} + \frac{c}{d} \right|_2 = |2^\ell|_2 \cdot \left| \frac{2^{k-\ell} ad + bc}{bd} \right|_2$

$= 2^{-\ell} \underbrace{\left| \frac{p^{k-\ell} ad + bc}{bd} \right|_2}_{\leq 1} \leq 2^{-\ell} = \max\{|x|_2, |y|_2\}$

for any prime?

□QED

Even stronger claim

$|x+y|_2 = \max\{\ldots\} \iff |x|_2 \neq |y|_2$

Say $|x|_2 < |y|_2 \Rightarrow |y|_2 = |(x+y) - x|_2 \leq \max\{|x+y|_2, |-x|_2\} =$

$|x|_2$ is the same
$|-x| = |-1x| = 1|x| \ldots$

$= |x+y|_2 \leq \max\{|x|_2, |y_2|\} = |y|_2$

□QED

Now, we would like to „valvate" points. These are „real numbers" though.

So we will „extend" our valuation function to work ~~f~~ with $\mathbb{R}$

$f: \mathbb{R} \to \mathbb{R}_0^+$

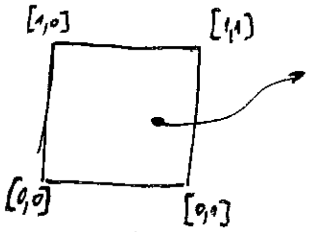
$\vdots$

(advertisement)

to be continued 2 pages later !!

(continuation)

Important: $\left|\frac{1}{2}\right|_2 > 1$



<u>Coloring</u>
red
green
blue
(RGB)

Lets say every number in the square have two real coordinates $x,y \in \langle 0, 1 \rangle$

<u>We will color the points</u> (since we like colours and points and there's nothing else to do)

$|x|_2$ maximal $\leftarrow$ red

$|1|_2 \le |x|_2 \ge |y|_2 \leftarrow$ blue

$|x|_2 < |y|_2 \ge |1|_2 \leftarrow$ green

$|x|_2 < |1|_2 > |y|_2$

Now, welcome ~~on~~ our dearly beloved <u>matrices</u>!
(% Sarcasm detected)

$$\det\begin{pmatrix} x_b & y_b & 1 \\ x_g & y_g & 1 \\ x_r & y_r & 1 \end{pmatrix} = x_b y_g + x_r y_b + x_g y_r - x_r y_g - x_b y_r - x_g y_b = D$$

$$|D|_2 = |x_b y_g + \ldots - x_r y_g - \ldots|_2$$ from <u>non-Arch.</u> this will be the maximum value from these 6 terms

$\rightarrow$ we want to show it's $\boxed{x_b y_g}$

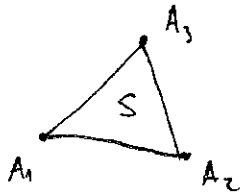
... well, we showed it somehow

$$|D|_2 = |x_b y_g 1| = |x_b|_2 |y_g|_2 \cdot |1|_2 \ge 1 \cdot 1 \cdot 1 \ge 1 \ge 0$$

This means, three points of different colours cannot <u>lie</u> ... on a single line.
Yeah, I meant lay !!

$|2k| < |1|_2 = 1 \mid k \in \mathbb{Z}$

$$|2k+1|_2 = \max\{|2k|_2, |1|_2\} = 1 \Rightarrow \left|\frac{1}{2k+1}\right|_2 = |1|_2 = 1$$



$$S = \frac{1}{2}\left|\det\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}\right|$$

"rainbow triangle"

$$|S|_2 = \left|\frac{1}{2}\right|_2 \left|\det\begin{pmatrix} \cdots \end{pmatrix}\right|_2$$

Now we will prove that $\frac{1}{n}$ ~~cann~~ cannot be the area of driangle. by contradiction.

$S = \frac{1}{n}$

$|S|_2 = 1$ but $\underline{1 = |S|_2 = \left|\frac{1}{2}\right|_2 \left|\det\left(\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}\right)\right|_2 > 1}$ is contradiction

"<u>One does not simply</u> exceed a value of one"

$\square$QED I guess

- <u>Another point of miew</u> using more colours and a lot of cool pictures of triangles, coloured points, meadows, unicorns... ... ... <u>is not here</u>.

<u>ERROR 404i</u>: Proof not found.