$\bar{z} = \frac{1}{z}$ $\quad z = a+bi$ $\quad \bar{z} = a-bi$

$a-bi = \frac{1}{a+bi}$

$a-bi = \frac{1\cdot(a-bi)}{(a+bi)(a-bi)}$

$a-bi = \frac{a-bi}{a^2-b^2i^2}$

$a-bi = \frac{a-bi}{a^2+b^2}$

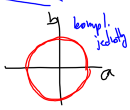
$a-bi = \frac{a}{a^2+b^2} - i\cdot\frac{b}{a^2+b^2}$

$a = \frac{a}{a^2+b^2}$ $\quad -b = \frac{-b}{a^2+b^2}$

$\Updownarrow$ $\quad \Downarrow$

$a^2+b^2=1$ $\quad a^2+b^2=1$

$\{a+bi \mid a^2+b^2=1\}$



2 22-17:50

$(3+4i)x-(2+2i)x = 3-5i+px$

$(3+4i)x-(2+2i)x-px = 3-5i$

$x(3+4i-2-2i-p) = 3-5i$

$x(1+2i-p) = 3-5i$

i) $1+2i-p=0$, $p=1+2i$, $x\cdot 0 = 3-5i$, $\emptyset$

ii) $1+2i-p\neq 0$, $p\neq 1+2i$, $x = \frac{3-5i}{1+2i-p}$

p	k
$1+2i$	$\emptyset$
$\mathbb{C}\setminus\{1+2i\}$	$\left\{\frac{3-5i}{1+2i-p}\right\}$

$x(1+2i-p) = 0$

$1+2i-p=0$, $p=1+2i$, $0x=0$, $\mathbb{C}$ $\quad \}\ x=0$

2 22-18:12

$p\in\mathbb{R}$

$\boxed{\frac{p-i}{1+pi}}$ je ryze imaginární

$$\frac{p-i}{1+pi}\cdot\frac{1-pi}{1-pi} = \frac{(p-i)(1-pi)}{1+p^2} =$$

$$= \frac{p-p-i-p^2i}{1+p^2} = i\,\frac{-1-p^2}{1+p^2}$$

$\frac{a+bi}{c+di}\cdot(c-di)$

2 22-18:17

$(i-2)\cdot x - 2iy = 1$

$x=a+bi$

$y=a-bi$

$(i-2)\cdot(a+bi) - 2i(a-bi) = i$

$-2a-b+ai-2bi-2ai-2b = i$

$\alpha+\beta i = \gamma+\delta i$, $\alpha=\gamma$, $\beta=\delta$

$-2a-3b=0$

$-a-2b=1$

$\Downarrow$ $a=-2b-1$

$-2(-2b-1)-3b=0$

$4b-3b+2=0$

$b=-2$

$a=-2b-1$

$a=3$

$3-2i$

$3+2i$

2 22-18:22

z_1, z_2;

$z_1 = |z_1|\cdot(\cos\varphi_1 + i\sin\varphi_1)$

$z_2 = |z_2|\cdot(\cos\varphi_2 + i\sin\varphi_2)$

$z_1\cdot z_2 = |z_1|(\cos\varphi_1+i\sin\varphi_1)\cdot|z_2|\cdot(\cos\varphi_2+i\sin\varphi_2)$

$= |z_1|\cdot|z_2|\left(\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\cdot\sin\varphi_2 + i(\sin\varphi_1\cos\varphi_2 + \cos\varphi_1\sin\varphi_2)\right)$

$= |z_1|\cdot|z_2|\left(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)\right)$

$z_1^2 = |z_1|^2\cdot(\cos 2\varphi_1 + i\sin 2\varphi_1)$

$z^n = |z|^n\cdot(\cos n\varphi + i\sin n\varphi)$

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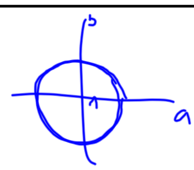
2 22-18:29

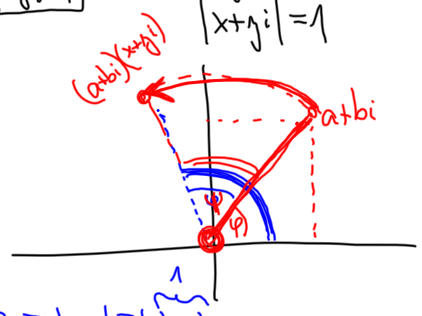
$(a+bi)\cdot(x+yi)$



$\arg(x+yi)=\varphi$

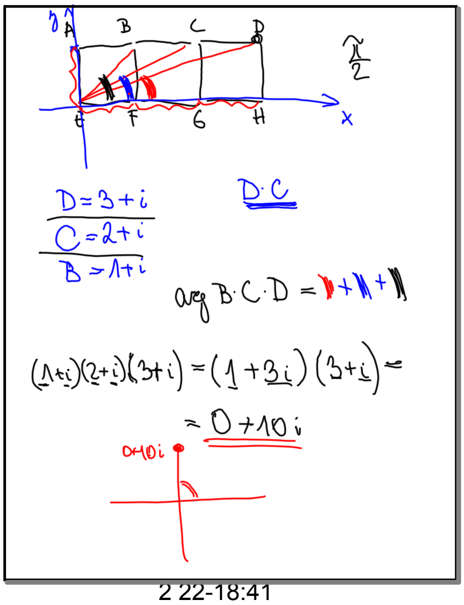
$\sqrt{x^2+y^2}=1$

$|x+yi|=1$

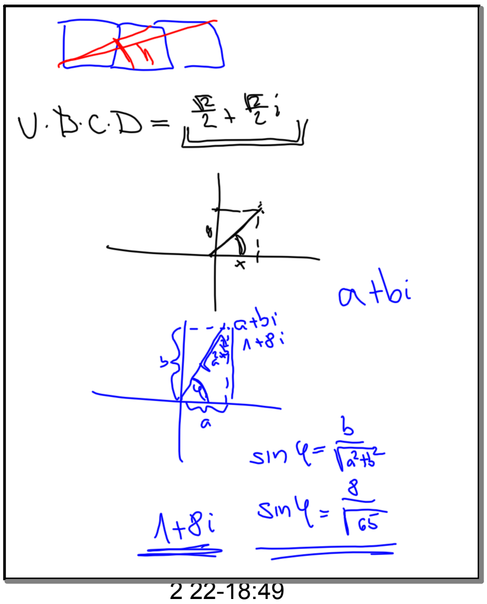


$|z_1z_2| = |z_1|\cdot\overbrace{|z_2|}^{1}$

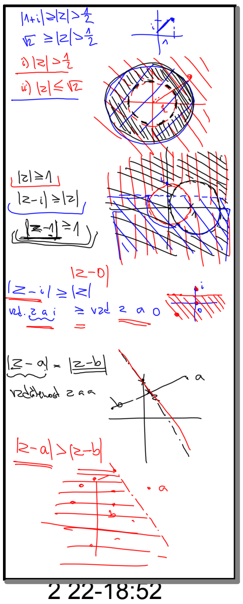
2 22-18:36



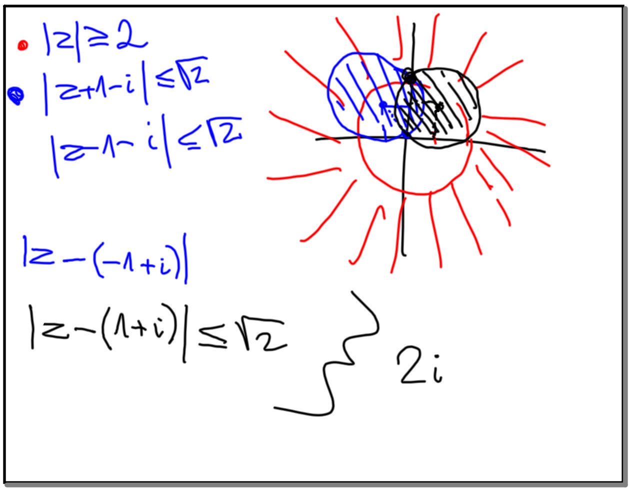
2 22-18:41



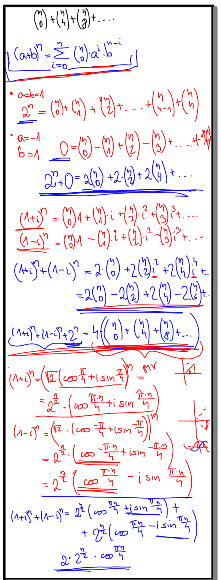
2 22-18:49



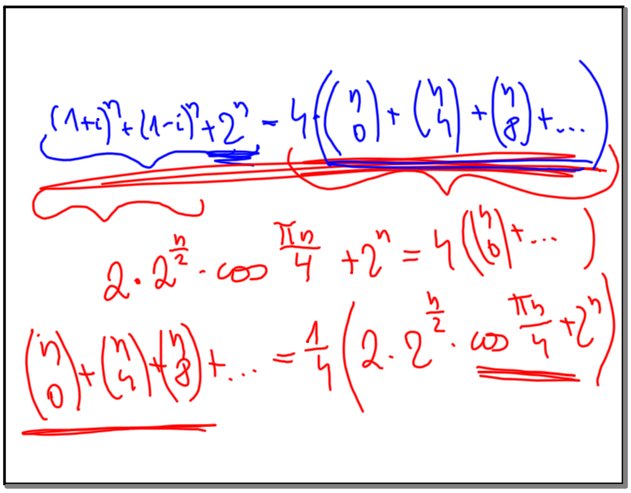
2 22-18:52



2 22-19:04



2 22-19:08



2 22-19:25

$x = \sqrt[n]{z}$

$x = |x| \cdot (\cos\varphi + i\sin\varphi)$

$z = |z| \cdot (\cos\psi + i\sin\psi)$

$x^n = z$

$|x|^n \cdot (\cos n\varphi + i\sin n\varphi) = |z| \cdot (\cos\psi + i\sin\psi)$

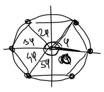
$|x| = \sqrt[n]{|z|}$

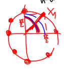
$n \cdot \varphi = \psi + 2k\pi$

$\varphi = \frac{\psi + 2k\pi}{n}$ $\boxed{k = 0, \dots, 5}$

n-té odm. tvoří

vrcholy n-úhelníka





$\underline{\underline{x^n = 1}}$

$x_0, \dots, x_{n-1}$

$x_k = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$

$x_1 = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$

$x_2 = \cos\frac{4\pi}{2} + i\sin\frac{4\pi}{n}$ $\boxed{x_1^n = 1}$

$x_2 = x_1^2$

$1 + x_1^p + (x_1^2)^p + (x_1^3)^p + \dots (x_1^{n-1})^p$

$q = x_1^p$ n jedniček

n

$n | p \Rightarrow p = n \cdot q$

$n|p: x_1^p = 1$

$x_1^{n \cdot q} = (x_1^n)^q = 1$ $\Big|$ $n \nmid p$ $\frac{1 - (x_1^n)^p}{1 - x_1^p} = 0$

2 22-19:27