$\mathbb{R}^n$

$\mathbb{R}[x]$

$\mathbb{R}_n[x]$

$Mat_{22}(\mathbb{R})$

$$\begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \\ x_{n1} & \cdots & x_{nn} \end{pmatrix} \oplus \begin{pmatrix} y_{11} & \cdots & y_{1n} \\ & & \\ y_{n1} & \cdots & y_{nn} \end{pmatrix} = \begin{pmatrix} x_{11}+y_{11} & \cdots & x_{1n}+y_{1n} \\ \vdots & & \\ x_{n1}+y_{n1} & \cdots & x_{nn}+y_{nn} \end{pmatrix}$$

$a \quad -a$

$$\begin{cases} t\odot(u\oplus v) = t\odot u \oplus t\odot v \\ (t+s)\odot u = t\odot u \oplus s\odot u \\ 1\odot u = u \\ t\cdot(s\cdot u) = (t\cdot s)\cdot u \end{cases}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad Mat_{22}(\mathbb{R}) \cong \mathbb{R}^4$

3 28-17:54

$V = \mathbb{R}\times\mathbb{R}$

$(a,b)\oplus(x,y) = (a\cdot x, b+y)$

$t\odot(x,y) = (tx, ty)$

i) uzavřenost

ii) asociativita

$\forall u,v,w \in V: (u\oplus v)\oplus w = u\oplus(v\oplus w)$

$L = (a,b)\oplus((c,d)\oplus(e,f)) = (a,b)\oplus(c\cdot e, d+f)$
$= (a\cdot c\cdot e, b+d+f)$

$P = ((a,b)\oplus(c,d))\oplus(e,f) = (ac, b+d)\oplus(e,f)$
$= (a\cdot c\cdot e, b+d+f)$ $\quad L=P$ ✓

iii) $(a,b)\oplus(c,d) = (c,d)\oplus(a,b)$

$L = (a\cdot c, b+d)$

$P = (c\cdot a, d+b) = (a\cdot c, b+d)$ $\quad$ komyt. ✓

iv) $\underline{o} = (e,f)$

$(a,b)\oplus(e,f) = (a,b)$

$(a\cdot e, b+f) = (a,b)$

$b+f = b$

$f = 0$

$a\cdot e = a$

$a(e-1) = 0$

$e = 1$

$\underline{o} = (1,0)$ ✓

v) $(a,b)\oplus(\bar a, \bar b) = (1,0)$

$(a\cdot\bar a, b+\bar b) = (1,0)$

$b+\bar b = 0$

$\bar b = -b$

$0 \quad a\,\bar a = 1$

$\bar a = \frac{1}{a}$ $\quad (\frac{1}{a}, -b)$

$\nexists\, (0,1)\oplus(e,f) = (0,1)$

3 28-18:16

vi) $(t+s)\odot u = t\odot u \oplus s\odot u$

$(t+s)\odot(a,b) = ((t+s)\cdot a, (t+s)b)$

$t\odot(a,b)\oplus s\odot(a,b) = (t\cdot a, tb)\oplus(s\cdot a, sb)$
$= (t\cdot s\cdot a^2, t\cdot b + s\cdot b)$

v) $t\odot(u\oplus v) = t\odot u \oplus t\odot v$

$L = t\odot((a,b)\oplus(c,d)) = t\odot(ac, b+d) =$
$= (tac, tb+td)$

$P = t\odot(a,b)\oplus t\odot(c,d) = (ta, tb)\oplus(tc, td)$
$= (t^2 ac, tb+td)$

viii) $1\odot(a,b) = (a,b)$

$L = (1\cdot a, 1\cdot b) = (a,b)$

ix) $(t\cdot s)\odot(a,b) = t\odot(s\odot(a,b))$

$L = ((t\cdot s)\cdot a, (t\cdot s)b)$

$P = t\odot(sa, sb) = (tsa, tsb)$

3 28-18:27

a) $\{(x,0,1) \mid x\in\mathbb{R}\}$

$(x,0,1)\oplus(y,0,1) = (x+y, 0, 1)$

$\oplus$

i) $(t+s)\odot u = t\odot u \oplus s\odot u$

$L = (t+s)\odot(x,0,1) = ((t+s)^2\cdot x, 0, 1)$

$P = t\odot(x,0,1)\oplus s\odot(x,0,1)$
$= (t^2 x, 0, 1)\oplus(s^2\cdot x, 0, 1) =$
$= (t^2 x + s^2 x, 0, 1) = ((t^2+s^2)x, 0, 1)$ $\quad L \neq P$

b) $t\odot(x,0,1) = (tx, 0, 1)$

$(t+s)\odot u = t\odot u \oplus s\odot u$

$L = (t+s)\odot(x,0,1) = ((t+s)\cdot x, 0, 1)$

$P = t\odot u\oplus s\odot u = t\odot(x,0,1)\oplus s\odot(x,0,1) =$
$= (tx, 0, 1)\oplus(sx, 0, 1) =$
$= (tx+sx, 0, 1) = ((t+s)x, 0, 1)$

$t\odot(u\oplus v) = t\odot u\oplus t\odot v$

$L = t\odot((x,0,1)\oplus(y,0,1)) = t\odot((x+y, 0, 1)) =$
$= (t\cdot(x+y), 0, 1)$

$P = t\odot(x,0,1)\oplus t\odot(y,0,1) =$
$(t\cdot x, 0, 1)\oplus(t\cdot y, 0, 1) =$
$= (tx+ty, 0, 1) = (t(x+y), 0, 1)$

$t\odot(s\odot u) = (t\cdot s)\odot u$

$L = t\odot(s\odot(x,0,1)) = t\odot(sx, 0, 1) =$
$= (t\cdot s\cdot x, 0, 1)$

$P = (t\cdot s)\odot(x,0,1) = (t\cdot s\cdot x, 0, 1)$

$1\odot u = u$

$L = 1\odot(x,0,1) = (1\cdot x, 0, 1) = (x,0,1)$

$P = (x,0,1)$

3 28-18:33

c) $\{(x,0,1) \mid x\in\mathbb{R}\}$

$2\cdot(x,0,1) = (2\cdot x, 0, 2)$

d) $t\odot(x,0,1) = (x,0,1)$

$(t+s)\odot u = t\odot u \oplus s\odot u$

$L = (t+s)\odot(x,0,1) = (x,0,1)$

$P = t\odot(x,0,1)\oplus s\odot(x,0,1)$
$= (x,0,1)\oplus(x,0,1)$
$= (2x, 0, 1)$

3 28-18:45

$\mathbb{R}^3 \quad t\odot(a,b,c) = (ta, tb, tc)$

αi) $(a,b,c)\oplus(x,y,z) = (a-y, x-b, c+z)$

$(u\oplus v)\oplus w = u\oplus(v+w)$

$L = ((a,b,c)\oplus(p,q,r))\oplus(x,y,z) =$
$= (a-q, p-b, c+r)\oplus(x,y,z) =$
$= (a-q-y, x-p+b, c+r+z)$

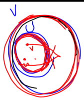
$P = (a,b,c)\oplus((p,q,r)\oplus(x,y,z)) =$
$= (a,b,c)\oplus(p-y, x-q, r+z) =$
$= (a-x+q, p-y-b, c+r+z)$

$(a,b,c)\oplus(x,y,z) = (|a-x|, b+y, 0)$

$L = ((a,b,c)\oplus(p,q,r))\oplus(x,y,z) =$
$= (|a-p|, b+q, 0)\oplus(x,y,z) =$
$= (||a-p|-x|, b+q+y, 0)$

$P = (a,b,c)\oplus((p,q,r)\oplus(x,y,z)) =$
$= (a,b,c)\oplus(|p-x|, q+y, 0)$
$= (|a-|p-x||, b+q+y, 0)$

3 28-18:49



$U \subseteq V$

- $\forall u,v \in U: \ u \oplus v \in U$
- ~~$0 \in U$~~
- ~~$u \Rightarrow -u \in U$~~
- $t, u \Rightarrow \underline{t \odot u \in U}$

$t=0: \ 0 \odot u = 0 \in U$

$t=-1 \quad -u \in U$

$\boxed{\begin{array}{l} 1)\ u,v \in U \Rightarrow u \oplus v \in U \\ 2)\ t \in T, u \in V \Rightarrow t \odot u \in U \end{array}}$

$\boxed{t,s \in T: \ u,v \in V \quad t \odot u \oplus s \odot v \in U}$

$Mat_{22}(\mathbb{R})$

$U = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a+d=0 \right\}$

i) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \begin{pmatrix} e & f \\ g & h \end{pmatrix} \in U \Rightarrow \begin{array}{l} a+d=0 \\ e+h=0 \end{array}$

$\Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \underline{\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}}$

$\Rightarrow (a+e)+(d+h) = (a+d)+(e+h) = 0 \qquad \in U$

$\Downarrow \quad \Downarrow$

$0 \quad 0$

ii) $t, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in U \Rightarrow a+d=0$

$t \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ta & tb \\ tc & td \end{pmatrix} \in U$

$ta+td = t\underbrace{(a+d)}_{=} = 0$

3 28-18:58

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \begin{pmatrix} e & f \\ g & h \end{pmatrix} \in U: \ \underline{a+d=0} \ \wedge \ e+h=0$

$t \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} + s\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ta & tb \\ tc & td \end{pmatrix} + \begin{pmatrix} se & sf \\ sg & sh \end{pmatrix} =$

$= \begin{pmatrix} ta+se & tb+sf \\ tc+sg & td+sh \end{pmatrix}$

$ta+se+td+sh = t\underbrace{(a+d)}_{0} + s\underbrace{(e+h)}_{0} = 0$

3 28-19:08

$U = \left\{ \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \middle| \ a,d \in \mathbb{R} \right\}$

$t \cdot \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} + s \cdot \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} =$

$\begin{pmatrix} ta & 0 \\ 0 & td \end{pmatrix} + \begin{pmatrix} sx & 0 \\ 0 & sy \end{pmatrix} = \begin{pmatrix} ta+sx & 0 \\ 0 & td+sy \end{pmatrix} \in U$

$U = \left\{ \underline{\begin{pmatrix} a & 1 \\ c & d \end{pmatrix}} \middle| \ a,c,d \in \mathbb{R} \right\}$

i) $\begin{pmatrix} a & 1 \\ c & d \end{pmatrix} + \begin{pmatrix} x & 1 \\ y & 2 \end{pmatrix} = \begin{pmatrix} a+x & 2 \\ c+y & d+2 \end{pmatrix}$

$U = \left\{ \begin{pmatrix} a & -a \\ -a & a \end{pmatrix} \middle| \ a \in \mathbb{R} \right\}$

i) $\begin{pmatrix} a & -a \\ -a & a \end{pmatrix} + \begin{pmatrix} x & -x \\ -x & x \end{pmatrix} =$

$= \begin{pmatrix} a+x & -a-x \\ -a-x & a+x \end{pmatrix} =$

$= \begin{pmatrix} a+x & -(a+x) \\ -(a+x) & a+x \end{pmatrix} \in U$

ii) $t\underline{\begin{pmatrix} a & -a \\ -a & a \end{pmatrix}} = \begin{pmatrix} ta & -ta \\ -ta & ta \end{pmatrix} \in U$

3 28-19:09

$\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a+b+c+d=0 \right\}$

$t\begin{pmatrix} a & b \\ c & d \end{pmatrix} + s\begin{pmatrix} x & y \\ z & w \end{pmatrix} =$

$\underline{a+b+c+d=0}; \ \underline{x+y+z+w=0}$

$= \begin{pmatrix} ta & tb \\ tc & td \end{pmatrix} + \begin{pmatrix} sx & sy \\ sz & sw \end{pmatrix} =$

$= \begin{pmatrix} ta+sx & tb+sy \\ tc+sz & td+sw \end{pmatrix} \in U$

$\Rightarrow (ta+sx)+(tb+sy)+(tc+sz)+(td+sw)$

$= t\underbrace{(a+b+c+d)}_{0} + s\underbrace{(x+y+z+w)}_{0}$

$= 0$

3 28-19:16

$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \in U \ \begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix} \in U$

$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \notin U$

$\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$

8) $U = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a \cdot b - c \cdot d = 0 \right\}$

$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in U$

$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \notin U$

3 28-19:20

$\mathbb{R}_5[x]$

$M = \left\{ f \in \mathbb{R}_5[x] \ \middle| \ f(0) \cdot f(1) = 0 \right\}$

$f \in M \Rightarrow \exists g: \ \boxed{f = x \cdot g} \ \boxed{f = (x-1) \cdot g}$

$x \in M$

$x-1 \in M$ $\quad f \ \underline{2x-1} \notin M$

$f(0) \cdot f(1) = (-1) \cdot 1 = -1 \neq 0$

3 28-19:23

$\{f \in \mathbb{R}_5[x] \mid x^2+1 \mid f\}$

$f \in M \Leftrightarrow \exists g : f = (x^2+1) \cdot g$

$x^3 + x \in M$

$\cancel{g}: f = (x^2+1)x$

$f_1, f_2 \in M$

$f_1 = (x^2+1) \cdot g_1$

$f_2 = (x^2+1) \cdot g_2$

$f_1 + f_2 = (x^2+1) \cdot (g_1 + g_2) \in M$

$t \cdot f_1 = t \cdot (x^2+1) g_1 = (x^2+1) \cdot t g_1 \in M$

3) M x^2+1 $\quad -1$ $\quad x^2$

$M = \{f \mid f \text{ nemá } \mathbb{R}\text{-kořen}\}$

1

$(x^2+1)^2 \qquad x^3 - x^2 + x - 1$

$f: x^2+1$

$g: -1 \qquad f + g = x^2$

$M = \{f \mid f(0) = f(1) = f(2) = f(3) = 0\}$

$f = x \cdot (x-1)(x-2)(x-3) \cdot g$

$f_1 = x(x-1)(x-2)(x-3) g_1$

$f_2 = x(x-1)(x-2)(x-3) g_2$

$f_1 + f_2 = x(x-1)(x-2)(x-3)(g_1 + g_2) \in M$

$t \cdot f_1 = t \cdot x(x-1)(x-2)(x-3) \cdot g_1$

$= x(x-1)(x-2)(x-3) \cdot t g_1 \in M$

3 28-19:27

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$-a - b = -e - f$

$a = e + f - b$

$a + b + c = 0 \Rightarrow c = -a - b$

$b + d + e = 0 \Rightarrow e = -b - d$

$c + e + f = 0 \Rightarrow c = -e - f$

$$\begin{pmatrix} a & b & -a-b \\ b & d & -b-d \\ \cancel{e+f} & e & \cancel{f} \end{pmatrix}$$

$$\boxed{\begin{pmatrix} e+f-b & b & -e-f \\ b & d & -b-d \\ -e-f & e & f \end{pmatrix}}$$

$e + f - b + d + f = 0$

$e + 2f - b + d = 0$

$b = e + 2f + d$

3 28-19:37

3 28-18:55