Funkce typu $\sin^m x \cdot \cos^n x$, $m, n \in \mathbb{Z}$:

1. n liché $\longrightarrow$ substituce $\sin x = t$
2. m liché $\longrightarrow$ substituce $\cos x = t$
3. m, n sudá $\longrightarrow$ substituce $\operatorname{tg} x = t$
4. m, n sudá nezáporná $\longrightarrow$ vzorce $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Příklad 1.1.

$$\int \frac{\sin^2 x}{\cos^6 x}\,\mathrm{d}x = \int \operatorname{tg}^2 x \cdot \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\,\mathrm{d}x = \int t^2(t^2+1)\,\mathrm{d}t = \ldots = \frac{1}{5}\operatorname{tg}^5 x + \frac{1}{3}\operatorname{tg}^3 x + C$$

subst. $\operatorname{tg} x = t$, $\frac{1}{\cos^2 x}\,\mathrm{d}x = \mathrm{d}t$, $\cos x = \frac{1}{\sqrt{t^2+1}}$, $\cos^2 x = \frac{1}{t^2+1}$

Funkce typu $R(\sin x, \cos x)$:

1. $R(\sin x, \cos x) = r(\sin x) \cdot \cos x \longrightarrow$ substituce $\sin x = t$
2. $R(\sin x, \cos x) = r(\cos x) \cdot \sin x \longrightarrow$ substituce $\cos x = t$
3. $R(\sin x, \cos x) = r(\operatorname{tg} x) \longrightarrow$ substituce $\operatorname{tg} x = t$
4. kdykoli: substituce $\operatorname{tg} \frac{x}{2} = t$

Příklad 1.2.

$$\int \frac{(\sin x + 2)\cos x}{\sin^2 x - 2\sin x + 5}\,\mathrm{d}x = \int \frac{t+2}{t^2 - 2t + 5}\,\mathrm{d}t = \ldots$$

subst. $\sin x = t$, $\cos x\,\mathrm{d}x = \mathrm{d}t$

Příklad 1.3.

$$\int \frac{\sin x - \cos x}{\sin x + 2\cos x}\,\mathrm{d}x = \int \frac{t-1}{(t+2)(t^2+1)}\,\mathrm{d}t = \ldots$$

subst. $\operatorname{tg} x = t$, $x = \operatorname{arctg} t$, $\mathrm{d}x = \frac{1}{1+t^2}\,\mathrm{d}t$

Příklad 1.4.

$$\int \frac{1-\sin x}{1+\cos x}\,\mathrm{d}x = \int \frac{1-\frac{2t}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} \frac{2}{t^2+1}\,\mathrm{d}t = \int \frac{(t-1)^2}{2}\frac{2}{t^2+1}\,\mathrm{d}t = \int \frac{t^2+1-2t}{t^2+1}\,\mathrm{d}t \dots$$

subst. $\operatorname{tg}\frac{x}{2} = t$, $x = 2\operatorname{arctg} t$, $\mathrm{d}x = \frac{2}{1+t^2}\,\mathrm{d}t$, $\sin\frac{x}{2} = \frac{t}{\sqrt{t^2+1}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{t^2+1}}$, $\sin x = \frac{2t}{t^2+1}$, $\cos x = \frac{1-t^2}{t^2+1}$

Funkce typu $R(x, \sqrt{a^2-x^2}) \to$ substituce $x = a\sin t$
Funkce typu $R(x, \sqrt{x^2-a^2}) \to$ substituce $x = \frac{a}{\sin t}$
Funkce typu $R(x, \sqrt{x^2+a^2}) \to$ substituce $x = a\operatorname{tg} t$

Příklad 1.5.

$$\int \frac{1}{x^2\sqrt{x^2+a^2}}\,\mathrm{d}x = \int \frac{\cos t}{a^2\operatorname{tg}^2 t\cdot a}\cdot\frac{a}{\cos^2 t}\,\mathrm{d}t = \int \frac{1}{a^2}\frac{\cos t}{\sin^2 t}\,\mathrm{d}t = \dots = -\frac{1}{a^2}\frac{1}{\sin(\operatorname{arctg}\frac{x}{a})}+C$$

subst. $x = a\operatorname{tg} t$, $\mathrm{d}x = \frac{a}{\cos^2 t}\,\mathrm{d}t$

Binomický integrál: $\int x^m(a+bx^n)^p\,\mathrm{d}x$, $m, n, p \in \mathbb{Q}$
$p \in \mathbb{Z} \to$ substituce $x = t^s$, s je společný jmenovatel m a n
$p \notin \mathbb{Z}$, $\frac{m+1}{n} \in \mathbb{Z} \to$ substituce $a + bx^n = t^s$, s je jmenovatel p
$p \notin \mathbb{Z}$, $\frac{m+1}{n} \notin \mathbb{Z}$, $\frac{m+1}{n} + p \in \mathbb{Z}$: substituce $ax^{-n} + b = t^s$, s je jmenovatel p

Příklad 1.6.

$$\int \frac{1}{x(1+\sqrt[3]{x})^2}\,\mathrm{d}x = \int \frac{3t^2}{t^3(1+t)^2}\,\mathrm{d}t = 3\int \frac{1}{t(t+1)^2}\,\mathrm{d}t = \dots$$

$m = -1$, $n = \frac{1}{3}$, $p = -2 \to$ *subst.* $x = t^3$, $\mathrm{d}x = 3t^2\,\mathrm{d}t$

Příklad 1.7.

$$\int \frac{\sqrt{1+\sqrt[3]{x}}}{x}\,\mathrm{d}x = \int \frac{t}{(t^2-1)^3}\cdot 3\cdot 2\cdot t\cdot (t^2-1)^2\,\mathrm{d}t = 6\int \frac{t^2}{t^2-1}\,\mathrm{d}t = \dots$$

$m = -1$, $n = \frac{1}{3}$, $p = \frac{1}{2}$, $\frac{m+1}{n} = 0 \in \mathbb{Z} \to$ *subst.* $1 + x^{\frac{1}{3}=t^2}$, $x = (t^2-1)^3$, $\mathrm{d}x = 3(t^2-1)^2\cdot 2t\,\mathrm{d}t$

Příklad 1.8.

$$\int \frac{1}{\sqrt[3]{1+x^3}}\,\mathrm{d}x = \int \frac{(t^3-1)^{\frac{1}{3}}}{(t}\cdot(-1)\cdot(t^3-1)^{-\frac{4}{3}}\cdot t^2\,\mathrm{d}t = -\int \frac{t}{t^3-1}\,\mathrm{d}t = \dots$$

$m = 0$, $n = 3$, $p = -\frac{1}{3}$, $\frac{m+1}{n} + p = 0 \in \mathbb{Z} \to$ *subst.* $x^{-3} + 1 = t^3$, $x^{-3} = t^3 - 1$, $x = (t^3-1)^{-\frac{1}{3}}$, $\mathrm{d}x = -\frac{1}{3}(t^3-1)^{-\frac{4}{3}}\cdot 3t^2\,\mathrm{d}t$, $x^3 = \frac{1}{t^3-1}$, $x^3 + 1 = \frac{t^3}{t^3-1}$