4. zápočtová MB102

① $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+5)} = ?$

Parciální zlomky:

$1 = 2nA + 5A + 2nB - B$

$A = 1/6 \;,\; B = -1/6$

$= \sum \frac{1/6}{2n-1} - \frac{1/6}{2n+5} =$

$= \frac{1}{6}\left(1 - \cancel{\frac{1}{7}} + \frac{1}{3} - \cancel{\frac{1}{9}} + \frac{1}{5} - \cancel{\frac{1}{11}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{13}} + \cancel{\frac{1}{9}} - \cancel{\frac{1}{15}} + \cancel{\frac{1}{11}} - \cancel{\frac{1}{17}} + \cancel{\frac{1}{13}} - \cancel{\frac{1}{19}} + \cancel{\frac{1}{15}} \dots\right)$

$= \frac{1}{6}\left(1 + \frac{1}{3} + \frac{1}{5}\right) = \frac{1}{6} \cdot \frac{15+5+3}{15} = \underline{\underline{\frac{23}{90}}}$

② $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverguje protože $\underbrace{\frac{1}{n}}_{\text{diverguje}} \leq \frac{1}{\sqrt{n}}$ (srovnávací)

$\sum_{n=1}^{\infty} \frac{2^n}{n}$ diverguje, protože $\frac{a_{n+1}}{a_n} = \frac{2^{n+1} \cdot n}{(n+1) \cdot 2^n} = \frac{2n}{n+1} > 1$ (podílové)

$\sum_{n=1}^{\infty} \frac{n^2}{\left(2+\frac{1}{n}\right)^n}$ konverguje, ptže $\lim_{n\to\infty} \sqrt[n]{\frac{n^2}{\left(2+\frac{1}{n}\right)^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^2}}{2+\frac{1}{n}} = \frac{1}{2} < 1$

$\sqrt[n]{n^2} = n^{\frac{2}{n}} \to 1$, $\frac{2}{n} \to 0$; $2 \to 2$, $\frac{1}{n} \to 0$

(odmocninové)

③ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$, $(-1)^{n+1} \frac{1}{n} = a_n$

$\frac{a_{n+1}}{a_n} = \frac{(-1)^{n+1} n}{(n+1)(-1)^n} = \frac{(-1)n}{n+1} \Rightarrow \lambda = \lim_{n\to\infty} \frac{-n}{n+1} \overset{LP}{=} -1$

poloměr $= \frac{1}{\lambda}$

$\Rightarrow$ obor konv.: $\underline{\underline{(-1,1)}}$, $\underline{\underline{\text{poloměr} = 1}}$

(4) $y' = \frac{-xy}{x+1}$

$$\frac{dy}{dx} = -\frac{xy}{x+1}$$

vydělit $= 1 - \frac{1}{x+1}$

$$\int \frac{1}{y}\,dy = -\int \frac{x}{x+1}\,dx$$

$$\ln y = -x + \ln|x+1| + c \quad / e$$

$$y = e^{-x}(x+1) \cdot C$$

(5) $e^{-y}(1+y') = 1$

$$e^{-y}\frac{dy}{dx} = 1 - e^{-y}$$

$$\frac{f'}{f} \to \int \frac{e^{-y}}{1-e^{-y}}\,dy = \int dx$$

$$\ln|1-e^{-y}| = x + c \quad / e$$

$$1 - e^{-y} = e^{x+c} \qquad e^{x+c} = e^{x} \cdot e^{c} = e^{x} K$$

$= \text{konstanta } K$

$$e^{-y} = 1 - e^{x} K$$