$4x^4 + x^2 - 11x - 6$ nad $\mathbb{R}$

$= 4(x+\frac{1}{2})(x-\frac{3}{2})(x^2+x+2)$

$= 4(x-\frac{1}{2})(x-\frac{3}{2})(x-(-\frac{1}{2}+\frac{\sqrt{7}}{2}i))(x-(-\frac{1}{2}-\frac{\sqrt{7}}{2}i))$

(nad $\mathbb{C}$)

$D = -7$

	4	0	1	-11	-6
$\frac{1}{2}$	4	2	2	-10	-11
$-\frac{1}{2}$	4	-2	2	-12	0
$\frac{3}{2}$	4	4	8	0	

$4x^5 + x^2 - 11x - 6 = x^5 + x^2 + x =$ nad $\mathbb{Z}_3$

$= x(x^3 + x + 1) =$

$= x(x-1)(x^2+x+2)$

	4	0	1	-11	-6

	1	0	1	1
1	1	1	2	0

$x^5 + 3x^3 + 3 = (x-1)(x^3+4x^2+5x+4)$

Podle Eisensteinova kritéria je polynom ireduc. nad $\mathbb{Q}$ (samozřejmě i nad $\mathbb{Z}$)

	1	3	0	0	3
1	1	4	4	4	0
1	1	5	2	6	
2	1	-1	2	1	
3	1	0	4	2	
4				1	
5				2	
6				4	

3 13-8:22

Rozložte polynom

$$x^4+2x^3+2x^2+x+1$$

nad $\mathbb{Z}_3$:

$$x^4+2x^3+2x^2+x+1 = (x^2+ax+b)(x^2+cx+d)$$

x^0: $bd = 1$

x^1: $bc+ad = 1$

x^2: $d+b+ac = 2$

x^3: $a+c = 2$

i) $b = d = 1$

$c+a = 1$

$a+c = 2$

↳

ii) $b = d = 2$

$2c+2a = 1$

$1+ac+c = 2 \leftrightarrow ac = 1$

$a+c = 2$

$\Rightarrow a = c = 1$

$$x^4+2x^3+2x^2+x+1 = (x^2+x+2)^2$$

$x^5 - 3x^4 + 7x^3 - 9x^2 + 8x - 5 = (x-1)(x^4 - 2x^3 + 5x^2 - 4x + 5)$

	1	-3	7	-9	8	-5
1	1	-2	5	-4	4	0
1						>0
-2						>0
-5						>0
2	1	0	5	6	16	>0
4	1	2	13			>0

$P(x)$

Polynom $x^4 - 2x^3 + 5x^2 - 4x + 5$ nemá racionální kořeny.

Vyšetříme, jestli nemá násobné kořeny:

derivace $P'(x) = 4x^3 - 6x^2 + 10x - 4$

$2x^4 - 4x^3 + 10x^2 - 8x + 8 : 2x^3 - 3x^2 + 5x - 2 = x - 1$

$-(2x^4 - 3x^3 + 5x^2 - 2x)$

$-x^3 + 5x^2 - 6x + 8 \qquad 7x^2 - 7x + 14$

$-(-2x^3 + 3x^2 - 5x + 2)$

$2x^3 - 3x^2 + 5x - 2 : x^2 - x + 2 = 2x - 1$

$2x^3 - 2x^2 + 5x$

$-x^2 + x - 2$

Kořeny $x^2 - x + 2$ jsou $\frac{1 \pm \sqrt{7}i}{2}$. Oba jsou kořeny dvojnásobné kořeny původního polynomu, tedy

$(x^4 - 2x^3 + 5x^2 - 5x + 5) = \left(x - \left(\frac{1+\sqrt{7}i}{2}\right)\right)^2 \left(x - \left(\frac{1-\sqrt{7}i}{2}\right)\right)^2 =$

$= (x^2 - x + 2)^2$

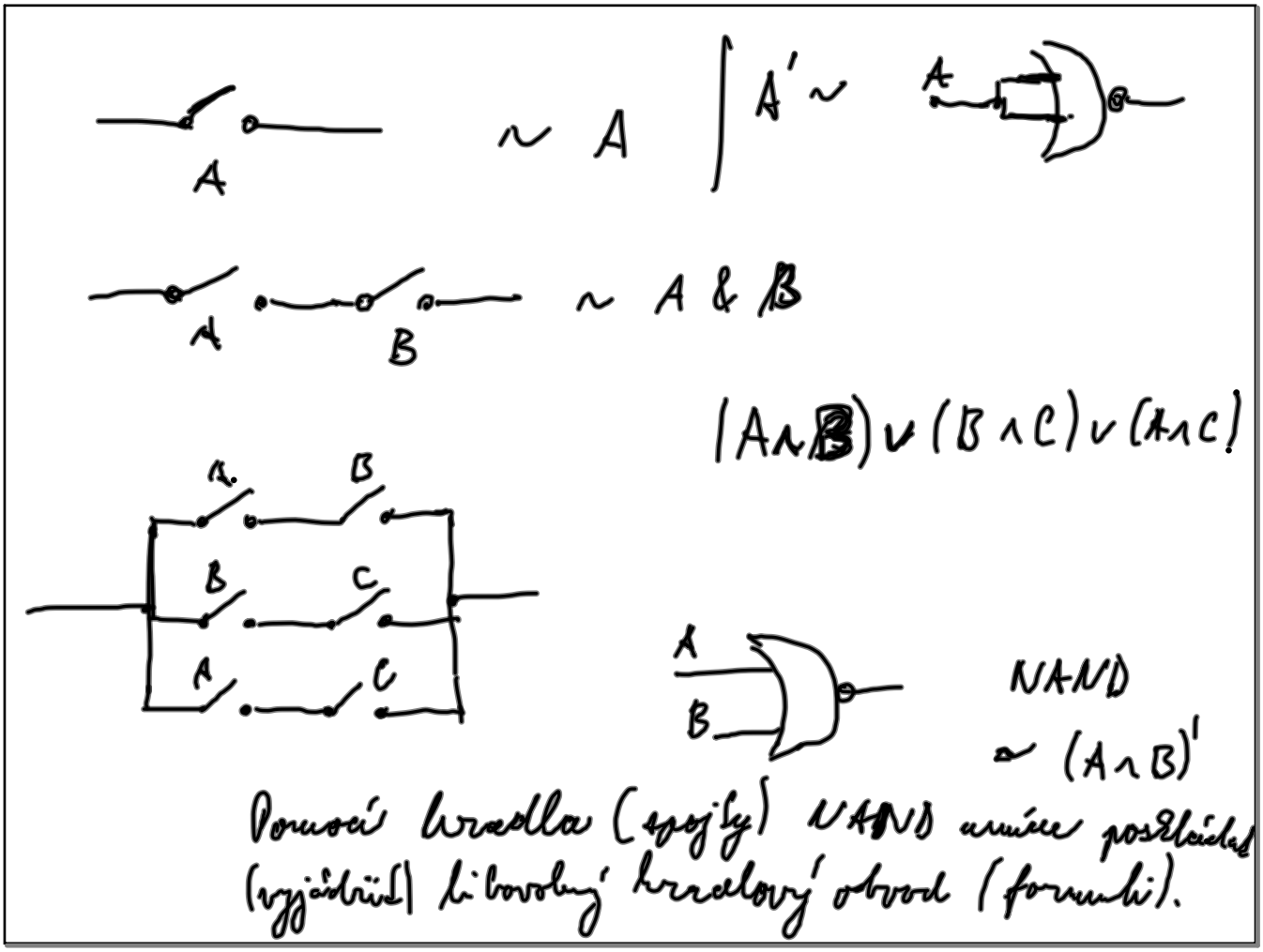
---

nad $\mathbb{R}$

$P(x) = (x-1)(x^2 - x + 2)^2$

$= (x-1)\left(x - \left(\frac{1}{2} + \frac{\sqrt{7}}{2}i\right)\right)^2 \left(x - \left(\frac{1}{2} - \frac{\sqrt{7}}{2}i\right)\right)^2$

nad $\mathbb{C}$


A
~ A
A' ~
A
A
B
~ A & B
(A ∧ B) ∨ (B ∧ C) ∨ (A ∧ C)
A
B
B
C
A
C
A
B
NAND
~ (A ∧ B)'
Pomocí hradla (spojky) NAND umíme postavit
(vyjádřit) libovolný hradlový obvod (formuli).

$(A \wedge B \wedge \bar{C}) \vee (\bar{A} \wedge B \wedge C)$

$(A \wedge B) \vee C = (A \wedge B \wedge C) \vee (A \wedge B \wedge \bar{C}) \vee$
$\vee (A \wedge B \wedge C) \vee (\bar{A} \wedge B \wedge C) \vee$
$(A \wedge \bar{B} \wedge C) \vee (\bar{A} \wedge \bar{B} \wedge C)$
$= [(A \wedge B \wedge C) \vee (A \wedge B \wedge \bar{C}) \vee (\bar{A} \wedge B \wedge C)$
$\vee (A \wedge \bar{B} \wedge C) \vee (\bar{A} \wedge \bar{B} \wedge C)$

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$F = (B' \Rightarrow C) \wedge \overline{(A \vee C) \wedge B}'$
$= (B' \Rightarrow C) \wedge ((A \vee C)' \vee B') =$
$= (B' \Rightarrow C) \wedge ((A' \wedge C') \vee B')$
$= (A' \wedge B' \wedge C) \vee (A' \wedge B \wedge C')$
$\vee (A \wedge B' \wedge C)$

$(K, \leq) \sim (K, \wedge, \vee)$

$a \leq b \Leftrightarrow a \wedge b = a$

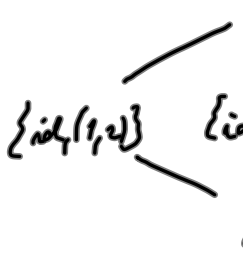
$a \wedge b := \inf(a, b)$

$a(b+c) = ab + bc$

$a + bc \neq (a+b)(a+c)$

Nějaký distributivní svaz $(K, \wedge, \vee)$ je Booleova algebra:

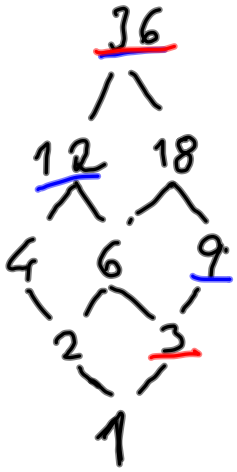
$(K, \wedge, \vee, 0, 1, ')$



S_3 $\quad$ $(a \vee b) \wedge c = (a \wedge b) \vee (a \wedge c)$

- je svaz
- není Booleova algebra

(musí splňovat $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$)



$(a \wedge b) := NSD(a, b)$

$(a \vee b) := NSN(a, b)$

$a \leq b :\Leftrightarrow a / b$

je svaz
není Boolova algebra