$G = GL(n, \mathbb{R})$ (reálné invertibilní matice $n\times n$)

$\det : G \to \mathbb{R}\setminus\{0\}$

$|A\cdot B| = |A|\cdot|B|$

$\exp: \mathbb{R} \to \mathbb{R}_+ \qquad x \mapsto e^x$

$x+y \mapsto e^x\cdot e^y$

2 27-17:55

$f: G \to H$ hom

1) $f(e\cdot a) = f(a) = f(e)\cdot f(a)$

2) $f(e_G) = e_H \qquad \underbrace{f(a)}_{\in K}^{\in f(K)} \cdot \underbrace{f(b)}_{\in K}^{\in f(K)} = f(\underbrace{a\cdot b}_{\in K})^{\in f(K)}$

$f(a\cdot a^{-1}) = f(e_G) = e_H = f(a)\cdot \underbrace{f(a^{-1})}_{=(f(a))^{-1}}$

3) $e \in f^{-1}(K) \quad a \in f^{-1}(K),\ b \in f^{-1}(K)$

$f(a^{-1}) = (f(a))^{-1} \in K, \quad f(a\cdot b) = f(a)\cdot f(b)$

6) $f(a) = f(b) \Rightarrow e_H = f(a^{-1}\cdot b)$

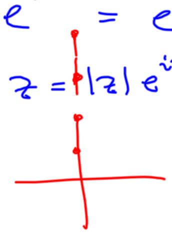
2 27-18:11

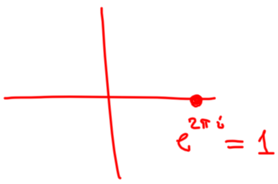
$z \mapsto e^z$ pro $z \in \mathbb{C}$

$e^z = \sum_{i=0}^{\infty} \frac{z^i}{i!}$

$e^{z+w} = e^z\cdot e^w$

$z = |z|\, e^{it}$

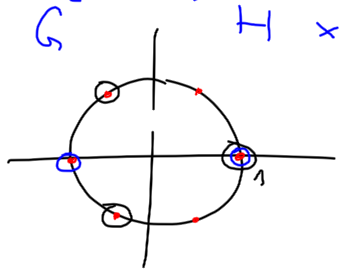




2 27-18:18

$(a,x) \qquad G\times H$

$a \quad G \qquad H \quad x$



$\mathbb{Z}_2$

$\mathbb{Z}_3$

2 27-18:30

$a^{k+1} = a \qquad k$ nejmenší

$\mathbb{Z}_4 = \{0,1,2,3\}$

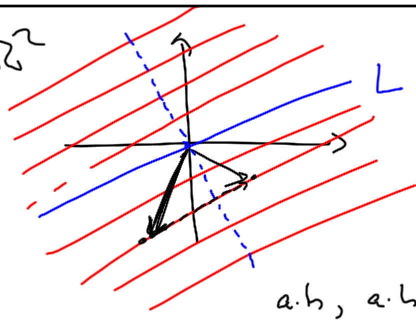
$\{0\} \quad \{1, 1+1, 1+1+1, 0\}$

$\{2,0\}$

$\{3,2,1,0\}$

2 27-18:38

$\mathbb{R}^2$



$a\cdot h, \quad a\cdot h'$

$a\cdot H \qquad (a\cdot h')^{-1}\cdot a\cdot h = (h')^{-1}\cdot a^{-1}\cdot a\cdot h$

$= (h')^{-1}\cdot h \in H$

2 27-18:45

$f: V \to V$

Schurova věta o triangularizaci

$f(v_1) = \lambda_1 \cdot v_1$

$V = \langle v_1 \rangle \oplus \langle v_1 \rangle^{\perp}$

$V/\langle v_1 \rangle \cong \langle v_1 \rangle^{\perp}$

f má matici $\left(\begin{array}{c|c} \lambda_1 & * \\ 0 & \\ \vdots & * \\ 0 & \end{array}\right)$

$f_1: V/\langle v_1 \rangle \to V/\langle v_1 \rangle$

f_1 má matici $\left(\begin{array}{cc|c} \lambda_1 & * & * \\ 0 & \lambda_2 & \\ \vdots & \vdots & * \\ 0 & 0 & \end{array}\right)$

$\dots \to \begin{pmatrix} \lambda_1 & * & \dots & * \\ 0 & \lambda_2 & \dots & * \\ \vdots & & \ddots & \\ 0 & 0 & \dots & \lambda_n \end{pmatrix}$

2 27-18:52

1) $a \in G$, $h \in H$ libovolné, ex. h'

$h \cdot a = a \cdot h' \iff a^{-1} \cdot h \cdot a = h' \in H$

$a \cdot h = a \cdot h' \Rightarrow \underbrace{a^{-1} \cdot a}_{e} \cdot h = \underbrace{a^{-1} \cdot a}_{e} \cdot h'$

$f: G \to H$ $\quad a \in \ker f$

$b \in G$ lib. $\quad \underbrace{f(b \cdot a \cdot b^{-1})}_{\in \ker f} = \underbrace{f(b) \cdot \cancel{f(a)} \cdot f(b^{-1})}_{e_H}$, $\quad f(a) = e_H$

$= e_H$

$\tilde{f}: G/\ker f \to H \quad \tilde{f}(a \cdot \ker f) = f(a)$

2 27-18:59

$(z \cdot w)^k = z^k \cdot w^k$

$f(z) = z^k$

$\tilde{f}: \mathbb{C}^*/\mathbb{Z}_k \to \mathbb{C}^*$

2 27-19:19

Akce grup

G grupa, S množina

akce (levá) $\varphi: G \times S \to S$

$\varphi(a \cdot b, s) = \varphi(a, \varphi(b, s))$

$(a \cdot b) \cdot s = a \cdot (b \cdot s)$

$x \in S$, $\quad G \cdot x \subset S$ orbita prvku x

$G_x = \{a \in G;\ a \cdot x = x\}$ stabilizátor podgrupa

tranzitivní akce $= S = G \cdot x$.

2 27-19:26

pro tranzitivní akce je $S = G/G_x$

Věta $\varphi: G \times S \to S$ levá akce, $|G| = n$.

(1) $\forall x \in S$ je $|G| = |G_x| \cdot |S_x|$

(2) Je-li N počet orbit akce G na S pak

$$|G| = \frac{1}{N} \sum_{a \in G} S_a$$

$S_a = \{x \in S;\ a \cdot x = x\}$

2 27-19:34