TOPOLOGY

- a pair (X, Ω), $\Omega \subseteq 2^X$ s.t.
  $A, B \in \Omega \Rightarrow A \cap B \in \Omega$,
  $(A_i : i \in I) \subseteq \Omega \Rightarrow \bigcup_{i \in I} A_i \in \Omega$
- $A \in \Omega$ is called <u>open</u>
  B s.t. $\overline{B} \in \Omega$, then B is <u>closed</u>
- Ω_0 is a generator of topology (X, Ω)
  if each $A \in \Omega$ is obtained as $A = \bigcup_{i \in I} B_i$
  where $B_i \in \Omega_0$.

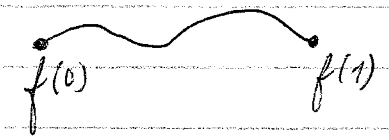
DEF: <u>Euclidean plane topology</u> is generated
by $\{ B_\varepsilon(x) ; x \in \mathbb{R}^2, \varepsilon > 0 \}$
where $B_\varepsilon(x) = \{ y \in \mathbb{R}^2 ; \| y - x \| < \varepsilon \}$

For <u>real line</u>, the same topology with $\mathbb{R}$ instead of $\mathbb{R}^2$.

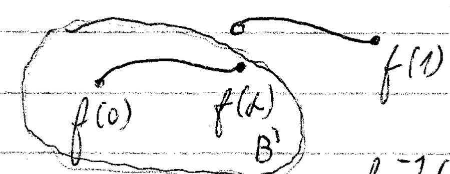
DEF: a mapping $f : X \to Y$ is <u>continuous</u> in topologies
$(X, \Omega), (Y, \Omega')$ if $\forall B \in \Omega' ; f^{-1}(B) \in \Omega$.

EXAMPLE : $f : [0,1] \to \mathbb{R}^2$ simple

a)



b)



$f^{-1}(B) = [0, \alpha]$ - not open

DEF: an *arc* is $f([0,1])$ where f is a continuous simple map ($[0,1] \to \mathbb{R}^2$).
The ends of f are $f(0)$, $f(1)$ (points in $\mathbb{R}^2$)

a closed arc (*loop*) is $f([0,1])$ again, but $f(0) = f(1)$ and f is simple on $[0,1)$.

GRAPHS

- we consider *multigraphs*!

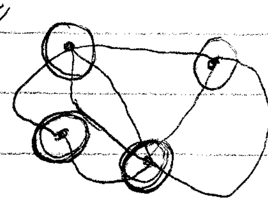
- $G = (V, E, \varepsilon)$ where $V \cap E = \emptyset$ and $\varepsilon : E \to \binom{V}{2} \cup V$, $\varepsilon^{-1}(V)$ are the *loops* of G, $\varepsilon^{-1}(\binom{V}{2})$ are the *edges*.

DEF: Graph G is *embedded* in $\mathbb{R}^2$ if
- $V \subseteq \mathbb{R}^2$, $E = \{\beta_i \,;\, i \in I, \beta_i \text{ is an arc/loop in } \mathbb{R}^2\}$, $\varepsilon(\beta_i) = \{\text{the ends } \beta_i(0), \beta_i(1)\}$, and
- every open arc ($\beta_i((0,1))$) is disjoint from all β_j, $j \neq i$ and from V

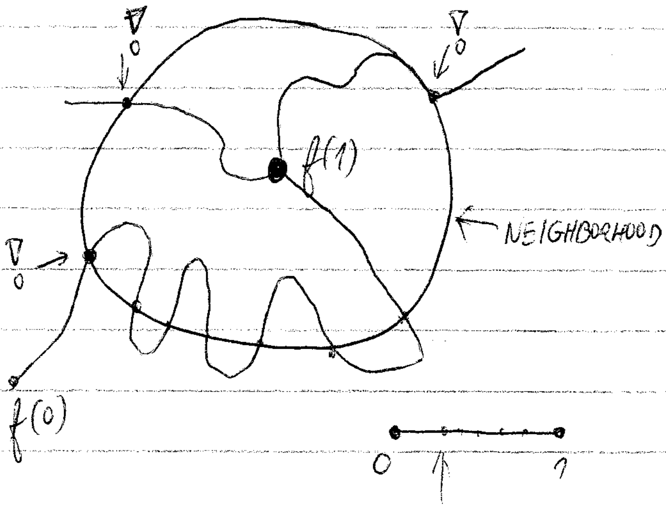
DEF: An arc (loop) is *piecewise-linear* (*polygonal*) if it is the union of finitely many line segments.

THEOREM: Every graph embedded in $\mathbb{R}^2$ has an embedding (in $\mathbb{R}^2$) with all the edges as polygonal arcs.

Proof: (1) find "small" neighbourhoods of the vertex points which are disjoint from all other edges (than of this vertex)

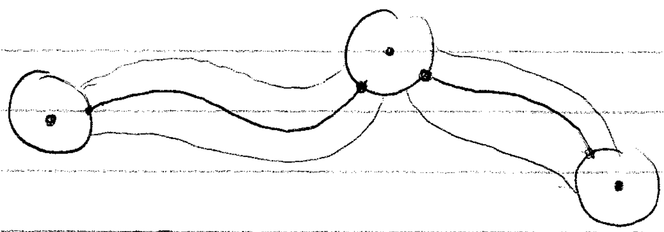


(2) for every vertex v and incident edge e, find the *last* point e leaves the neighb. of v

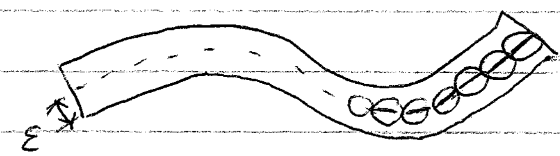


TAKE MINIMUM (BY COMPACTNESS)
(and connect the three points in picture with f(1) by straight lines)

(3) consider only the sections of ~~t~~ edges <u>between</u> the neighborhood.



(4) redraw the edges as polygonal!



balls with
ε/2 diameter
(finitely many of them)

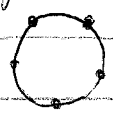
<u>SPQR trees</u>: for graph G:

- a tree with S-, P-, R-, Q-nodes

the edges of G

<u>rigid</u> (3-connected) structures

cycles



parallel