Digital Signal Processing

Decibels (dB and dBm)

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Using Logarithms to Determine Relative Signal Power

- Decibels evolution
  - When comparing analog signal levels, early specialists found it useful to define a measure of difference in powers of two signals
  - If that difference was treated as logarithm of a ratio of powers, it could be used as a simple additive measure to determine overall gain or loss of cascaded electronic circuits
  - Positive logarithms associated with system components having gain could be added to negative logarithms of those components having loss quickly to determine overall gain or loss of system

Using Logarithms to Determine Relative Signal Power

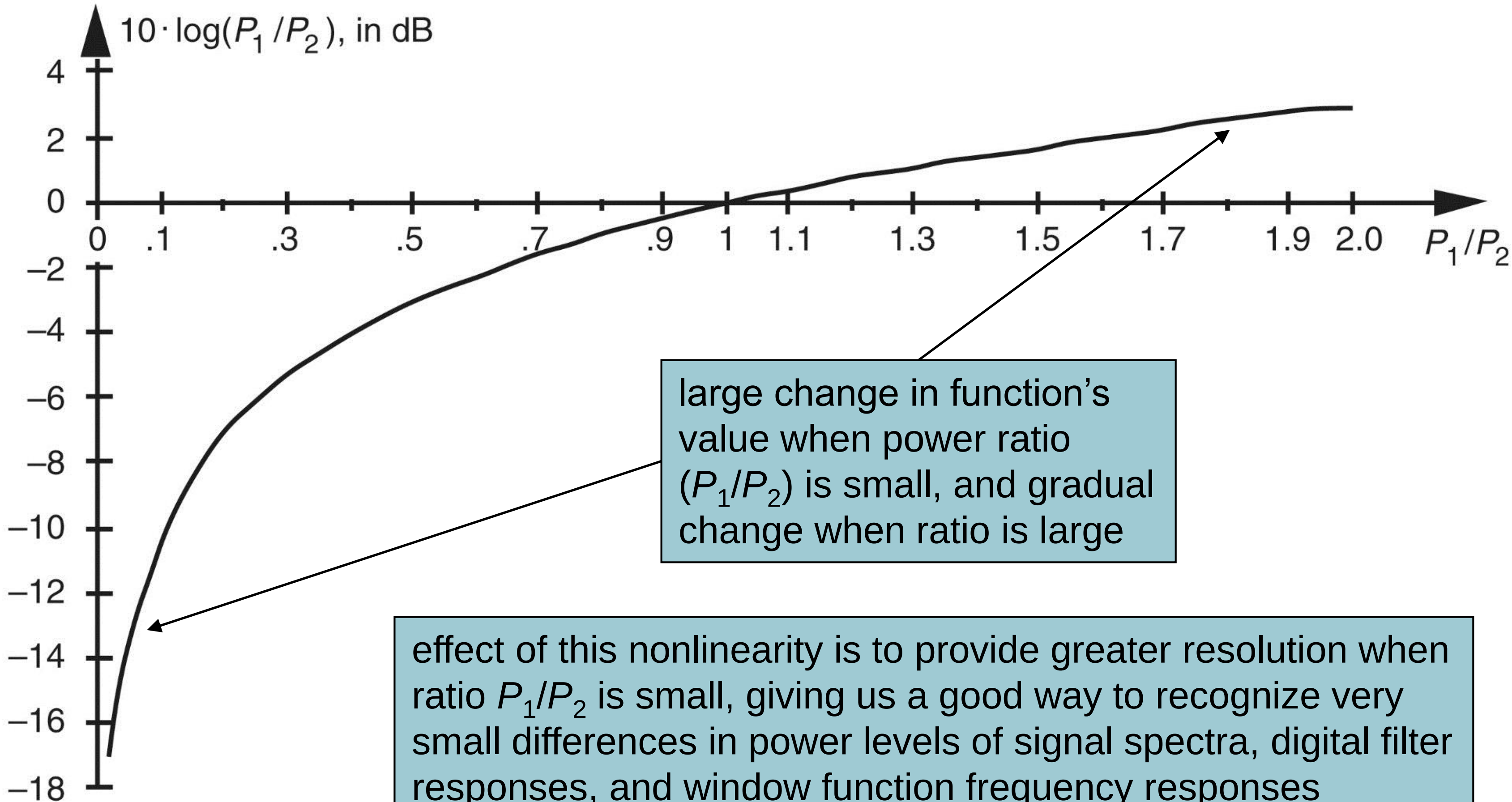
- Difference between two signal power levels

$$\text{Power difference } = \log_{10}\left(\frac{P_1}{P_2}\right) \text{bels}$$

  - Measured power differences smaller than one bel were so common that it led to the use of decibel (bel/10)

$$\text{Power difference } = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) \text{dB}$$

Using Logarithms to Determine Relative Signal Power



large change in function's value when power ratio (P_1/P_2) is small, and gradual change when ratio is large

effect of this nonlinearity is to provide greater resolution when ratio P_1/P_2 is small, giving us a good way to recognize very small differences in power levels of signal spectra, digital filter responses, and window function frequency responses

Figure E–1 Logarithmic decibel function of Eq. (E–2).

Using Logarithms to Determine Relative Signal Power

- For any frequency-domain sequence *X(m)*

$$\text{discrete power spectrum of } X(m) = |X(m)|^2$$

$$X_{dB}(m) = 10 \cdot \log_{10}(|X(m)|^2) = 20 \cdot \log_{10}(|X(m)|)\, dB$$

  - These expressions are used to convert a linear magnitude axis to a logarithmic magnitude-squared, or power, axis measured in dB
  - Without the need for squaring operation, we calculate $X_{\mathrm{dB}}(m)$ power spectrum sequence from $X(m)$ sequence

Using Logarithms to Determine Relative Signal Power

- Normalized log magnitude spectral plots

$$\text{normalized } X_{dB}(m) = 10 \cdot \log_{10}\left(\frac{|X(m)|^2}{|X(0)|^2}\right) = 20 \cdot \log_{10}\left(\frac{|X(m)|}{|X(0)|}\right)\ dB$$

  - Division by $|X(0)|^2$ or $|X(0)|$ value forces the first value in normalized log magnitude sequence $X_{\mathrm{dB}}(m)$ equal to 0 dB
    - This makes it easy to compare multiple log magnitude spectral plots

Using Logarithms to Determine Relative Signal Power

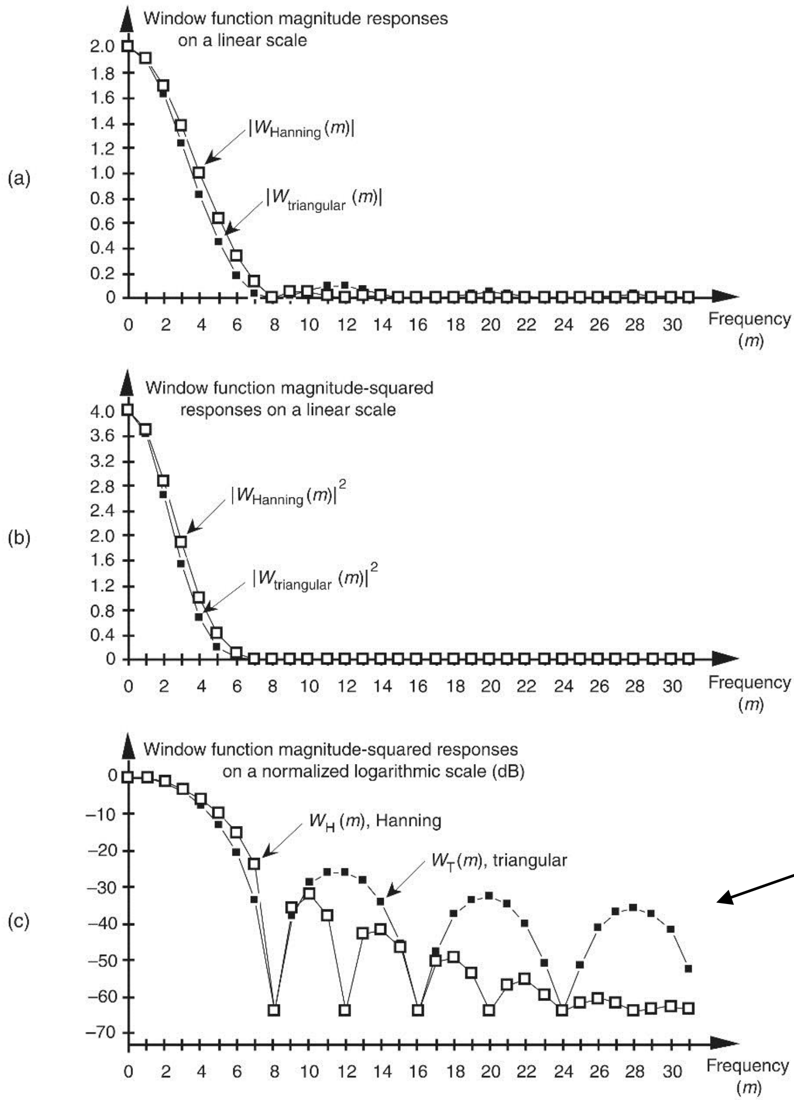


Figure E-2 Hanning (white squares) and triangular (black squares) window functions in the frequency domain: (a) magnitude responses using a linear scale; (b) magnitude-squared responses using a linear scale; (c) log magnitude responses using a normalized dB scale.

normalization:

$$W_H(m) = 10 \cdot \log_{10}\left(\frac{|W_{Hanning}(m)|^2}{|W_{Hanning}(0)|^2}\right)$$

$$= 20 \cdot \log_{10}\left(\frac{|X_{Hanning}(m)|}{|X_{Hanning}(0)|}\right) dB$$

we can clearly see the difference in magnitude-squared window functions in (c) as compared to linear plots in (b)

Using Logarithms to Determine Relative Signal Power

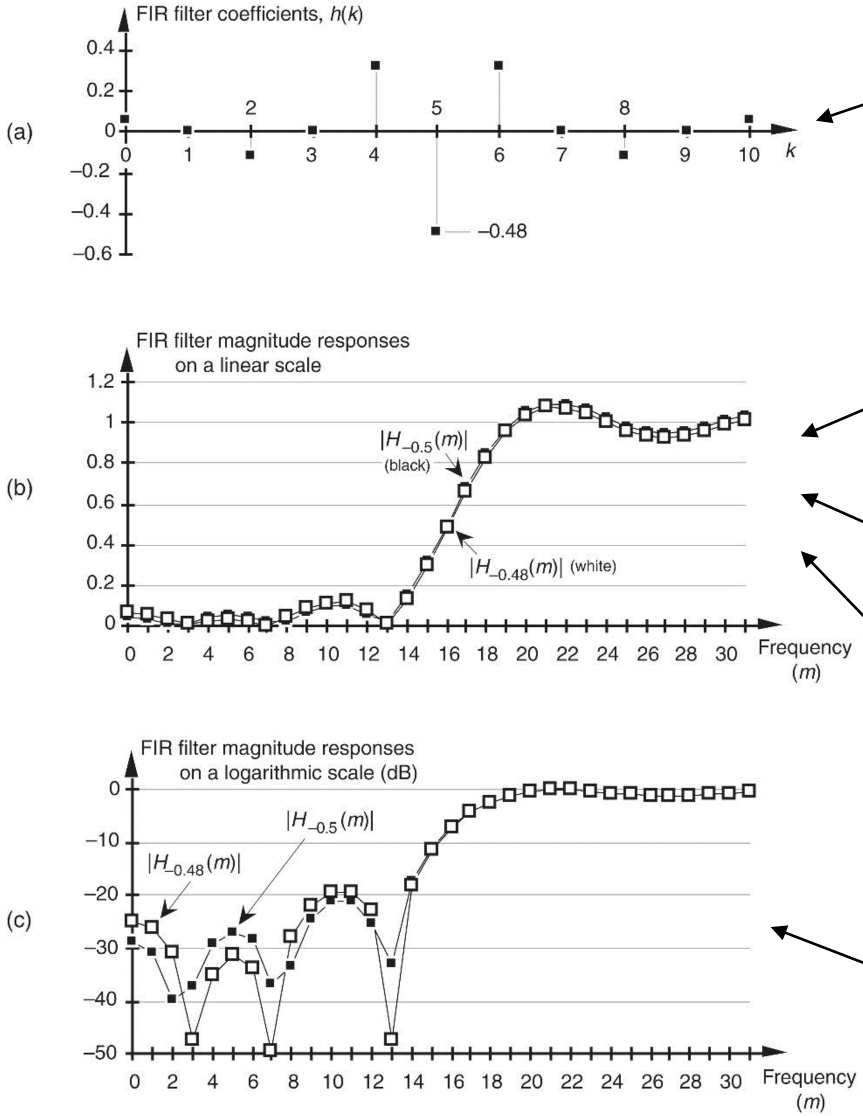


Figure E–3 FIR filter magnitude responses: (a) FIR filter time-domain coefficients; (b) magnitude responses using a linear scale; (c) log magnitude responses using the dB scale.

we're designing an 11-tap highpass FIR filter whose coefficients are shown in (a)

if center coefficient $h(5)$ is –0.48, filter's frequency magnitude response $|H_{-0.48}(m)|$ can be plotted as white dots on linear scale

$h(5)$: –0.48 → –0.5, new frequency magnitude response $|H_{-0.5}(m)|$ are black dots

difficult to see much difference on a linear scale

calculating two normalized log magnitude sequences, filter sidelobe effects of changing $h(5)$ are now easy to see

Some Useful Decibel Numbers

- A few constants to memorize
  - A power difference of 3 dB corresponds to a power factor of two
    - That is, if magnitude-squared ratio of two different frequency components is 2, then

$$\text{power difference } = 10 \cdot \log_{10}\left(\frac{2}{1}\right) = 10 \cdot \log_{10}(2) = 3.01 \approx 3\,\text{dB}$$

  - If magnitude-squared ratio of two different frequency components is 1/2

$$\text{power difference } = 10 \cdot \log_{10}\left(\frac{1}{2}\right) = 10 \cdot \log_{10}(0.5) = -3.01 \approx -3\,\text{dB}$$

Some Useful Decibel Numbers

Magnitude ratio	**Magnitude-squared power (P_1/P_2) ratio**	**Relative dB (approximate)**	
$10^{-1/2}$	10^{-1}	-10	P_1 is one-tenth P_2
2^{-1}	$2^{-2} = 1/4$	-6	P_1 is one-fourth P_2
$2^{-1/2}$	$2^{-1} = 1/2$	-3	P_1 is one-half P_2
2^{0}	$2^{0} = 1$	0	P_1 is equal to P_2
$2^{1/2}$	$2^{1} = 2$	3	P_1 is twice P_2
2^{1}	$2^{2} = 4$	6	P_1 is four times P_2
$10^{1/2}$	$10^{1} = 10$	10	P_1 is ten times P_2
10^{1}	$10^{2} = 100$	20	P_1 is one hundred times P_2
$10^{3/2}$	$10^{3} = 1000$	30	P_1 is one thousands times P_2

Absolute Power Using Decibels

- Another use of decibels
  - To measure signal-power levels referenced to a specific absolute power level
    - In this way, we can speak of absolute power levels in watts while taking advantage of convenience of decibels
  - The most common absolute power reference level used is milliwatt

$$\text{absolute power of } P_1 = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) = 10 \cdot \log_{10}\left(\frac{P_1 \text{ in watts}}{1\,\text{milliwatt}}\right) \text{dBm}$$

  - dBm = dB relative to a milliwatt