J003 - Fundamental Concepts of Computer Science

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Exercise sheet 1

deadline: **29.5.2015**

Common definitions

$$\begin{aligned} \Sigma_{bool} &= \{0,1\} \\ L_U &= \{\text{Kod}(M)\#w \mid w \in \Sigma_{bool}^* \text{ and } M \text{ accepts } w\} \\ L_H &= \{\text{Kod}(M)\#w \mid w \in \Sigma_{bool}^* \text{ and } M \text{ halts on } w\} \end{aligned}$$

A function $t : \mathbb{N} \to \mathbb{N}$ is called *time constructible*, if there exists an MTM (multi-tape Turing machine) A, such that

(i) $Time_A \in \mathcal{O}(t(n))$

(ii) for any input 0^n, $n \in \mathbb{N}$, A generates the word $0^{t(n)}$ on the first working tape and halts in q_{accept}.

Exercise 1.
Prove that $L_H \leq_R L_U$.

Exercise 2.
Describe how to find, for any infinite language $L \subseteq \{0,1\}^*$, a subset of L that is not recursively enumerable. Justify your claim.

Exercise 3.
Consider the languages

$$\begin{aligned} L_{H,001} &= \{\text{Kod}(M) \mid M \text{ halts on } 001\} \text{ and} \\ L_{EQ} &= \{\text{Kod}(M)\#\text{Kod}(M') \mid L(M) = L(M')\} \end{aligned}$$

Prove the following claims:

(a) $L_{H,001} \leq_R L_U$

(b) $L_U \leq_R L_{EQ}$

Exercise 4.
Let w_i be the i-th word over Σ_{bool} in canonical order and let M_i be the i-th Turing machine in canonical order. We consider two languages

(a) $L_1 = \{w \in \Sigma_{bool}^* \mid w = w_{5i+3}$ for some $i \in \mathbb{N}$ and M_i does not accept $w_{5i+3}\}$ and

(b) $L_2 = \{w \in \Sigma_{bool}^* \mid w = w_i$ for some $i \in \mathbb{N}$ and M_{5i+3} does not accept $w_i\}$

Prove, analogously to the proof for the diagonal language, that one of the two languages is not recursive and argue why such a proof is not possible for the other language.

Exercise 5.
We consider the language

$$L_{\text{all}} = \{\text{Kod}(M) \mid L(M) = \Sigma_{bool}^*\}$$

Prove that $L_{\text{all}} \notin \mathcal{L}_R$.

Exercise 6.
Consider the language

$$L_{\text{disjoint}} = \{\text{Kod}(M)\#\text{Kod}(M') \mid L(M) \cap L(M') = \emptyset\}$$

Prove that $(L_{\text{disjoint}})^{\complement} \in \mathcal{L}_{RE}$.

Exercise 7.
Prove that the following two functions are time-constructible:

(a) $e(n) = 2^n$,

(b) $f(n) = \mathit{fib}_n$.

Here fib_n denotes the n-th Fibonacci number, defined by $\mathit{fib}_0 = 0, \mathit{fib}_1 = 1$, and $\mathit{fib}_i = \mathit{fib}_{i-2} + \mathit{fib}_{i-1}$ for $i \in \mathbb{N}_{\geq 2}$.